Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May	2019	MATH1541-WE01

Title:
Statistics

Time Allowed:	3 hours	
Additional Material provided:	Graph paper Tables: Normal distribution, t-distribution, Chi-squared distribution, F-distribution, Wilcoxon test, Mann-Whitney test.	
Materials Permitted:	You may keep one folder of notes at your desk.	
Calculators Permitted:	Yes	Models Permitted: Casio fx-83 GTPLUS or Casio fx-85 GTPLUS.
Visiting Students may use dictionaries: No		

Instructions to Candidates:	Credit will be given for the best **SIX** answers. All questions carry the same marks. This is an open-book examination: you may keep one folder of notes at your desk.

Revision:	

ED01/2019

1. The uptake of a specific fertiliser by the roots of a wheat plant was measured throughout a 10 hour period and the results shown below. Theoretical arguments suggest that the law governing uptake should satisfy a hyperbolic relation of the form $w = ct/(d + t)$, where t is time and w represents uptake.

Time t	1.00	1.11	1.25	1.43	1.67	2.00	2.50	3.33	5.00	10.00
Uptake w	1.44	1.52	1.53	1.61	1.60	1.66	1.75	1.85	1.86	1.92

   (a) Find a suitable transformation from w and t to y and x to convert the hyperbolic law into the standard regression equation.

   (b) Apply the transformation from part (a) to the data and assess whether this transformation is appropriate for use in this context.

   (c) Estimate c and d.

   (d) Predict fertiliser uptake at $t = 4$ and $t = 40$ hours and comment on these predictions, including the assumptions used.

   (e) The biologists plan to repeat this experiment a large number of times. Find an interval on w that should contain approximately 90% of the uptake values of the proposed experiments at $t = 4$.

   (f) State the additional assumptions used for part (e) and describe how you would check these (you do not need to perform these checks).

   (g) Comment on whether the times t, at which the uptake was measured, were well chosen by the biologists.

2. A marine biologist is studying the number of cod in the north sea. She measures the number of cod caught in 4 regions of the north sea labelled A, B, C and D, in six separate catches for each region. The table below gives the results of these experiments.

Region	Number of cod caught						Sum	Sum of squares
A	20	32	37	29	32	37	187	6027
B	38	47	48	46	45	59	283	13579
C	7	11	20	21	14	16	89	1463
D	13	12	20	17	21	14	97	1639

   (a) Decompose the data into "overall mean", "effects" and "residuals" and produce an "effects and residuals" plot.

   (b) Construct the sum of squares table. Interpret the table in relation to your plot.

   (c) Explain briefly how you test if the underlying populations are homogeneous.

3. The following data give the length L (the longest diameter), the width W (the diameter of the largest circular cross-section), and the volume V of 12 hens' eggs.

L	2.112	2.151	2.086	2.099	2.138	2.195	2.125	2.170	2.164	2.201	2.151	2.157
W	1.866	1.889	1.859	1.874	1.874	1.866	1.851	1.851	1.866	1.843	1.835	1.851
V	5.618	5.755	5.479	5.551	5.686	5.755	5.339	5.618	5.551	5.551	5.551	5.551

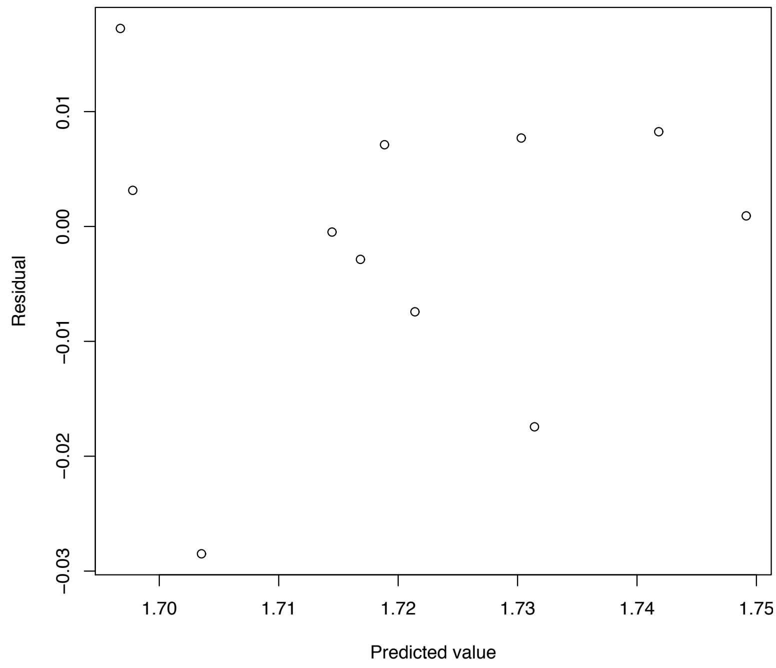
Multiple regression was used to construct an equation for predicting the natural logarithm of V from the natural logarithms of L and W. **Some output from R is shown on the next page: please look at it before attempting the questions below**.

(a) Why does it make sense to do the regression using logarithms? Are the coefficients in the regression equation compatible with your explanation?

(b) Using the output from R calculate an estimate of the volume of another egg for which $L = 2.2$ and $W = 1.89$.

(c) Give a formula for computing a measure of overall quality for a multiple linear regression model. Describe the characteristics and properties of such a measure.

(d) A plot of residuals versus predicted values for the regression is shown below. The point corresponding to the first egg has been omitted. Calculate the coordinates of the missing point and, having seen where it lies in the plot, interpret the residual plot. If there are any apparent outliers, identify to which eggs they correspond, and assess the extent to which those eggs are outliers in the original data.



```
> fit<-lm(log(V)~log(L)+log(W))
> summary(fit)

Call:
lm(formula = log(V) ~ log(L) + log(W))

Residuals:
      Min        1Q    Median        3Q       Max
-0.028495 -0.004006  0.002033  0.007837  0.017238

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.03967    0.44059   0.090  0.93022
log(L)       0.73138    0.26716   2.738  0.02294 *
log(W)       1.80692    0.54867   3.293  0.00933 **
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Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1  1

Residual standard error: 0.01421 on 9 degrees of freedom
Multiple R-squared: 0.6188, Adjusted R-squared: 0.5341
F-statistic: 7.305 on 2 and 9 DF,  p-value: 0.01303
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4. (a) A new and cheaper insulating material was tested for electrical resistance. Forty resistance measurements on the material were made, and the results shown below:

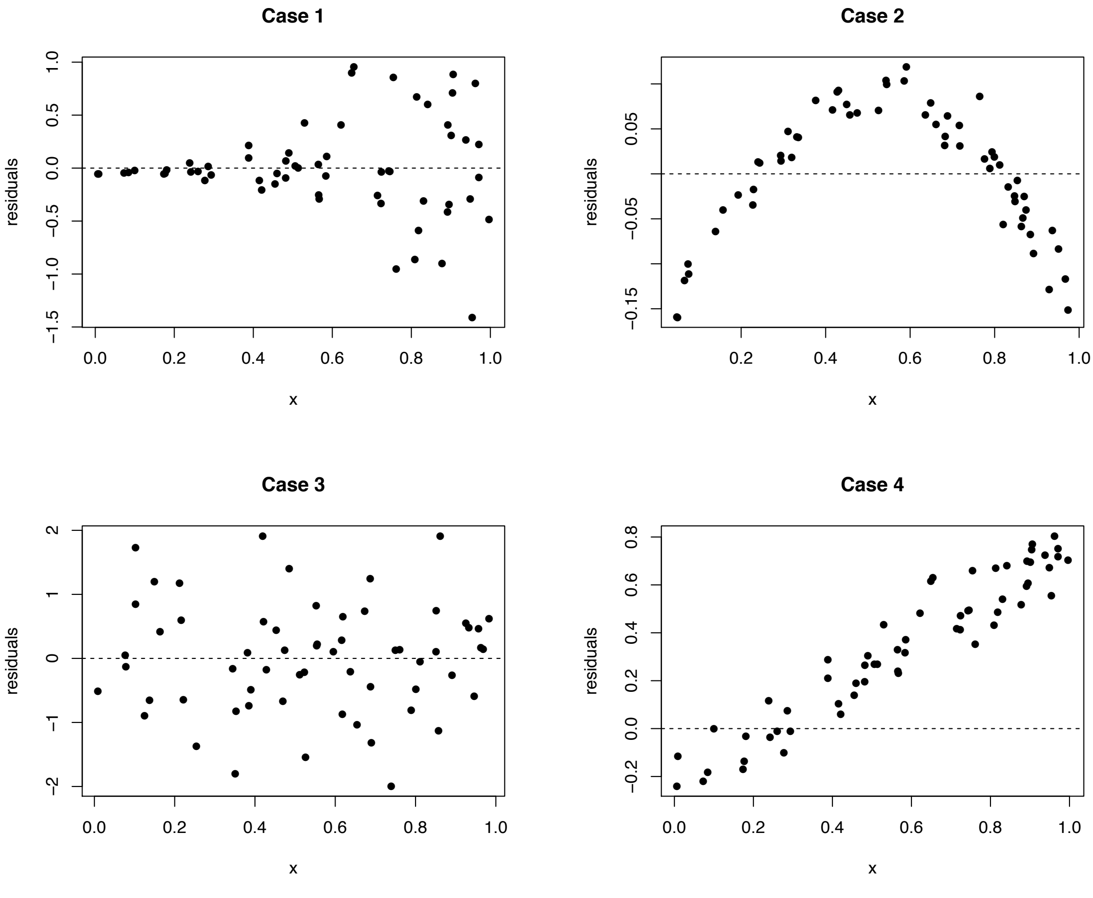
14.0	15.6	16.1	16.8	16.9	17.0	17.1	17.2	17.7	17.7
17.9	18.1	18.4	18.5	18.6	18.9	19.1	19.1	19.1	19.2
19.2	19.4	19.5	19.7	20.0	20.4	20.4	20.7	20.8	21.0
21.1	21.3	21.5	22.0	22.4	23.0	23.6	24.7	26.0	30.3

(i) Construct a stem and leaf plot of the 40 measurements, choosing an appropriate class width and clearly stating the endpoint convention you have used.

(ii) Construct a box-plot of the data, making the usual modification to show potential outliers. Show your working.

(iii) What effect would you expect on each of the following if any potential outliers were moved to just fall within the range of non-outlying observations: median, mean, inter-quartile range, and standard deviation? Explain.

(b) The residual plots generated by students for four separate cases of linear regression are shown below. In each case, comment on the plots, identifying any problems if present, and suggest possible remedies if necessary.



5. Umbrella Pharmaceuticals reported a new test detecting if an individual is infected by the lethal Tyrant Virus (T-virus). Individuals infected by T-virus present murderous aggression and an obsessive hunger to the state of cannibalism. Umbrella performed a diagnostic test evaluation: 200 individuals were classified according to the presence or absence of the disease (as diagnosed by a gold standard test) and according to the results of the new blood test (see Table 1). Previous reports suggest that the odds in favor of a person having been infected by T-Virus are 40/100 for a North American resident, and 2/100 for a South American resident.

		T-virus	
		Infected	Not infected
Result of new test	Positive	60	40
	Negative	20	80

Table 1: Dataset

(a) Compute the sensitivity, and specificity of the new test.

(b) Compute the probabilities for a false positive, and a false negative. Which of the two probabilities refers to a false diagnostic test outcome which may lead one to ignore the disease?

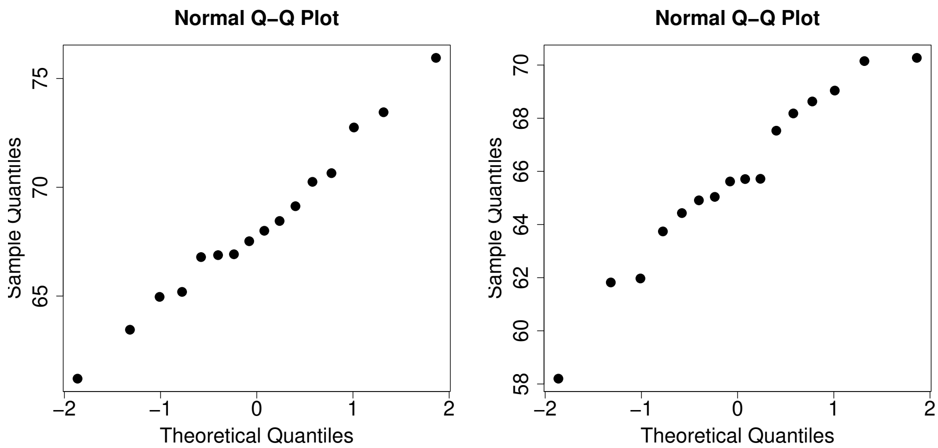
(c) A person from North America is randomly chosen and undergoes the new test which gives a positive result. What is the probability that this person is infected by the T-virus?

(d) For a positive test result, calculate and interpret the likelihood ratio. Explain why it is the same for each patient.

6. The concentration of active ingredient in a liquid laundry detergent is thought to be affected by the type of catalyst used in the process. A company randomly collected 16 observations on concentration from each catalyst, the data are presented in Table 2. In Figure 1, we present the qq-plots of the concentration for each catalyst.

Catalyst 1		Catalyst 2	
64.96	66.79	61.82	64.91
68.45	73.45	65.72	68.63
75.95	69.13	68.18	70.15
70.65	66.88	69.04	65.62
70.25	63.45	63.74	58.20
67.52	61.20	65.71	70.27
68.00	65.19	64.43	61.97
72.75	66.92	65.04	67.53

Table 2: Dataset



(a) Catalyst 1 (b) Catalyst 2

Figure 1: qq-plots for each catalyst

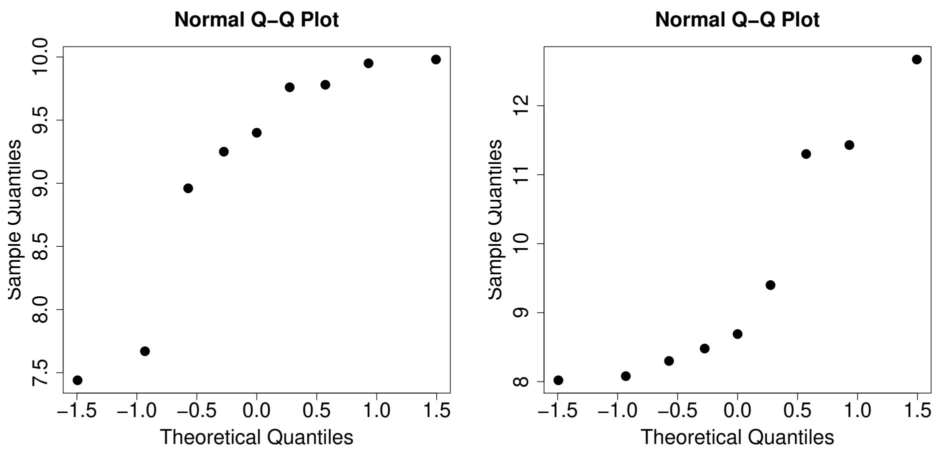
(a) Are the two samples drawn from populations with equal variances? Use significance level 0.05. Justify any assumptions you make.

(b) By constructing a confidence interval, test the hypothesis that the mean active concentrations are the same for either catalyst? Use significance level 0.05. Justify any assumptions you make.

7. The manufacturer of a hot tub is interested in testing two different heating elements for his product. The element that produces the maximum heat gain after 15 minutes would be preferable. He obtains 9 samples of each heating element unit and tests each one. The heat gain after 15 minutes (in F) are presented in Table 3. Interest lies in finding out whether the two units present similar performance or not. Address the scientific question by analyzing the data, and report your results in an appropriate statistical manner. Use significance level 0.05. Justify your choice of test. In Figure 2 we present the qq-plots of the two samples against the Normal quantiles.

	Heating gain 15 minutes								
Heating element unit 1	9.78	9.25	9.98	9.76	9.95	8.96	7.67	9.40	7.44
Heating element unit 2	11.30	8.69	8.48	8.08	9.18	8.30	12.67	11.43	8.02

Table 3: Dataset



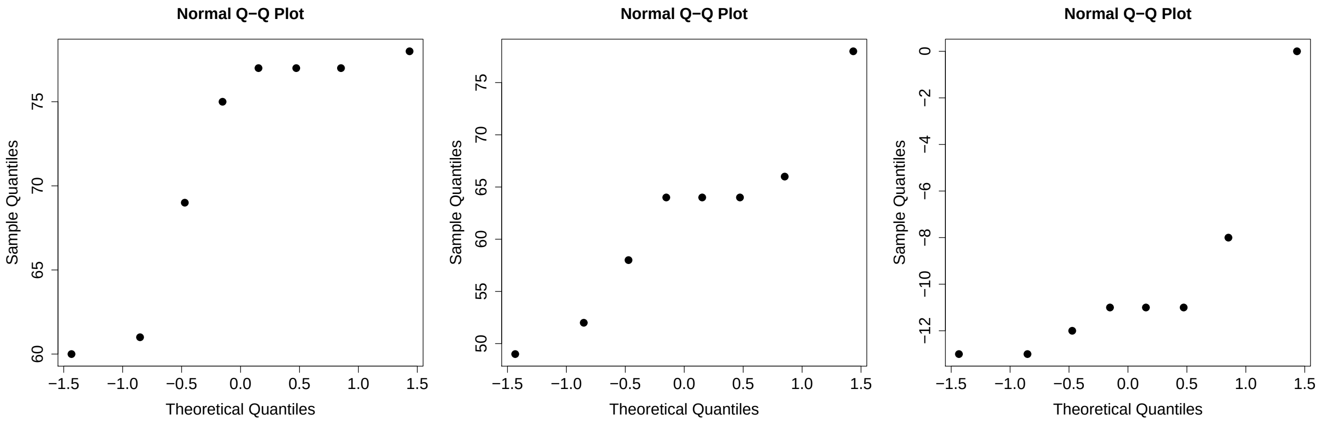
(a) Heating element unit 1 (b) Heating element unit 2

Figure 2: qq-plots

8. Two different types of tips can be used in a Rockwell hardness tester. Eight coupons from test ingots of a nickel-based alloy are selected, and each coupon is tested twice, once with each tip. The Rockwell C-scale hardness readings are shown in Table 4. qq-plots of the two samples and their differences against the Normal theoretical quantiles are displayed in Figure 3. We are interested in finding whether or not the two tips produce equivalent hardness readings. Address the scientific question by analyzing the data, and report your results in an appropriate statistical manner. Use significance level 0.05. Justify any assumptions you make.

Coupon (i)	Type 1 (x_i)	Type 2 (y_i)
1	77	64
2	75	64
3	61	49
4	77	66
5	78	78
6	77	64
7	69	58
8	60	52

Table 4: Dataset

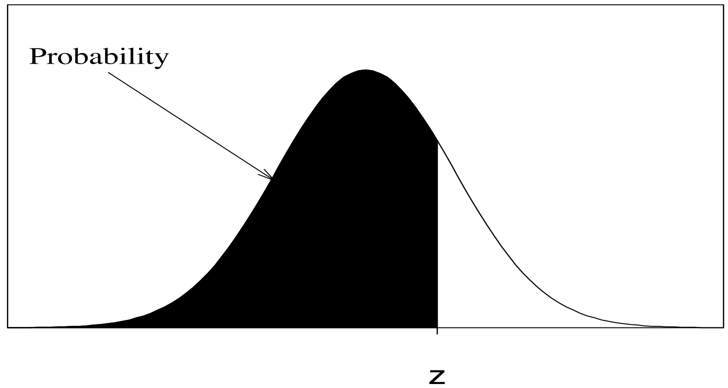


(a) Type 1 (x_i) (b) Type 2 (y_i) (c) Difference ($d_i = y_i - x_i$)

Figure 3: qq-plots

Probabilities for the standard normal distribution

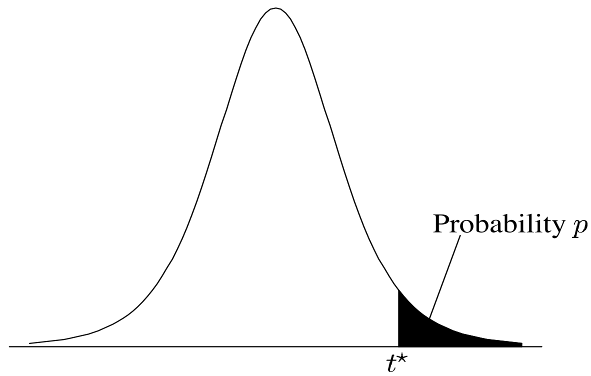
Table entry for z is the probability lying to the left of z



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Probabilities for the t-distribution

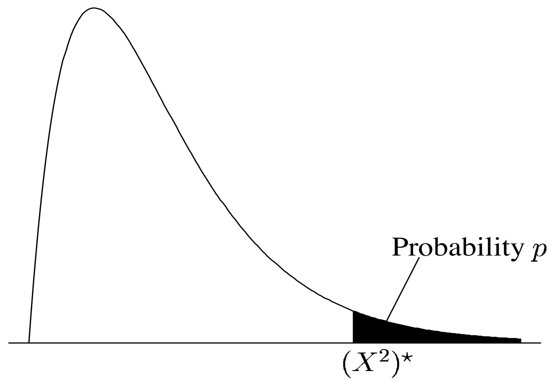
Table entry for p and C is the point $t^\star$ with probability p lying above it and probability C lying between $-t^\star$ and $t^\star$



	Tail probability p											
df	.25	.2	.15	.1	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.706	15.895	31.821	63.657	127.321	318.309	636.619
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.089	22.327	31.599
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.215	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.610	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	0.674	0.842	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.090	3.291
	50%	60%	70 %	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

Probabilities for the χ^2-distribution

Table entry for p is the point $(X^2)^\star$ with probability p lying above it



	Tail probability p											
df	.25	.2	.15	.1	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.52	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.69
80	88.13	90.41	93.11	96.58	101.88	106.63	108.07	112.33	116.32	120.10	124.84	128.26
100	109.14	111.67	114.66	118.50	124.34	129.56	131.14	135.81	140.17	144.29	149.45	153.17

Values for the Wilcoxon signed-rank Test

Reject the hypothesis of identical populations if the test statistic is *less than* the value T shown in the following table.

Sample size	Level of significance for a two-tailed test			
n	10%	5%	2%	1%
5	1	–	–	–
6	3	1	–	–
7	4	3	1	–
8	6	4	2	1
9	9	6	4	2
10	11	9	6	4
11	14	11	8	6
12	18	14	10	8
13	22	18	13	10
14	26	22	16	13
15	31	26	20	16
16	36	30	24	20
17	42	35	28	24
18	48	41	33	28
19	54	47	38	33
20	61	53	44	38
21	68	59	50	43
22	76	66	56	49
23	84	74	63	55
24	92	82	70	62
25	101	90	77	69

Values for the Mann-Whitney-Wilcoxon Test

Reject the hypothesis of identical populations if the test statistic is *less than* the value T_L shown in the following table or *greater than* the value T_U where

$$T_U = n_1(n_1 + n_2 + 1) - T_L$$

$\alpha = .01$		$\mathbf{n_2}$: 2	3	4	5	6	7	8	9	10
	2	–	–	–	–	–	–	–	–	–
	3	–	–	–	–	–	–	–	7	7
	4	–	–	–	–	11	11	12	12	13
	5	–	–	–	16	17	17	18	19	20
$\mathbf{n_1}$	6	–	–	22	23	24	25	26	27	28
	7	–	–	29	30	32	33	35	36	38
	8	–	–	38	39	41	43	44	46	48
	9	–	46	47	49	51	53	55	57	59
	10	–	56	58	60	62	65	67	69	72

$\alpha = .05$		$\mathbf{n_2}$: 2	3	4	5	6	7	8	9	10
	2	–	–	–	–	–	–	4	4	4
	3	–	–	–	7	8	8	9	9	10
	4	–	–	11	12	13	14	15	15	16
	5	–	16	17	18	19	21	22	23	24
$\mathbf{n_1}$	6	–	23	24	25	27	28	30	32	33
	7	–	30	32	34	35	37	39	41	43
	8	37	39	41	43	45	47	50	52	54
	9	46	48	50	53	56	58	61	63	66
	10	56	59	61	64	67	70	73	76	79

$\alpha = .10$		$\mathbf{n_2}$: 2	3	4	5	6	7	8	9	10
	2	–	–	–	4	4	4	5	5	5
	3	–	7	7	8	9	9	10	11	11
	4	–	11	12	13	14	15	16	17	18
	5	16	17	18	20	21	22	24	25	27
$\mathbf{n_1}$	6	22	24	25	27	29	30	32	34	36
	7	29	31	33	35	37	40	42	44	46
	8	38	40	42	45	47	50	52	55	57
	9	47	50	52	55	58	61	64	67	70
	10	57	60	63	67	70	73	76	80	83

F distribution critical values

Denominator degrees of freedom	p	Degrees of freedom in the numerator 1	2	3	4	5	6	7	8	9
	0.1	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
	0.05	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
2	0.025	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39
	0.01	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
	0.001	998.50	999.00	999.17	999.25	999.30	999.33	999.36	999.37	999.39
	0.1	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
	0.05	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
3	0.025	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47
	0.01	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
	0.001	167.03	148.50	141.11	137.10	134.58	132.85	131.58	130.62	129.86
	0.1	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
	0.05	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
4	0.025	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90
	0.01	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
	0.001	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.47
	0.1	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
	0.05	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
5	0.025	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68
	0.01	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
	0.001	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	27.24
	0.1	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
	0.05	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
6	0.025	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52
	0.01	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98
	0.001	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.69
	0.1	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
	0.05	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
7	0.025	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82
	0.01	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
	0.001	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.33
	0.1	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
	0.05	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
8	0.025	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36
	0.01	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
	0.001	25.41	18.49	15.83	14.39	13.48	12.86	12.40	12.05	11.77
	0.1	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
	0.05	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
9	0.025	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
	0.01	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
	0.001	22.86	16.39	13.90	12.56	11.71	11.13	10.70	10.37	10.11
	0.1	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
	0.05	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
10	0.025	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78
	0.01	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
	0.001	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.96
	0.1	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27
	0.05	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
11	0.025	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59
	0.01	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63
	0.001	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35	8.12
	0.1	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
	0.05	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
12	0.025	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44
	0.01	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
	0.001	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.48
	0.1	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16
	0.05	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
13	0.025	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31
	0.01	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19
	0.001	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.98

F distribution critical values

Denominator degrees of freedom	p	Degrees of freedom in the numerator: 1	2	3	4	5	6	7	8	9
14	0.1	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12
	0.05	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
	0.025	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21
	0.01	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03
	0.001	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.58
15	0.1	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
	0.05	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
	0.025	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12
	0.01	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
	0.001	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.26
16	0.1	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06
	0.05	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
	0.025	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05
	0.01	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78
	0.001	16.12	10.97	9.01	7.94	7.27	6.80	6.46	6.19	5.98
17	0.1	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03
	0.05	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
	0.025	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98
	0.01	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68
	0.001	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96	5.75
18	0.1	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00
	0.05	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
	0.025	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93
	0.01	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60
	0.001	15.38	10.39	8.49	7.46	6.81	6.35	6.02	5.76	5.56
19	0.1	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98
	0.05	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
	0.025	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88
	0.01	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52
	0.001	15.08	10.16	8.28	7.27	6.62	6.18	5.85	5.59	5.39
20	0.1	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
	0.05	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
	0.025	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84
	0.01	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
	0.001	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24
21	0.1	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95
	0.05	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
	0.025	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80
	0.01	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40
	0.001	14.59	9.77	7.94	6.95	6.32	5.88	5.56	5.31	5.11
22	0.1	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93
	0.05	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
	0.025	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76
	0.01	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35
	0.001	14.38	9.61	7.80	6.81	6.19	5.76	5.44	5.19	4.99
23	0.1	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92
	0.05	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
	0.025	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73
	0.01	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30
	0.001	14.20	9.47	7.67	6.70	6.08	5.65	5.33	5.09	4.89
24	0.1	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91
	0.05	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
	0.025	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70
	0.01	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26
	0.001	14.03	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.80
25	0.1	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89
	0.05	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
	0.025	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68
	0.01	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22
	0.001	13.88	9.22	7.45	6.49	5.89	5.46	5.15	4.91	4.71

F distribution critical values

Denominator degrees of freedom	p	1	2	3	4	5	6	7	8	9
		Degrees of freedom in the numerator								
26	0.1	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88
	0.05	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
	0.025	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65
	0.01	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18
	0.001	13.74	9.12	7.36	6.41	5.80	5.38	5.07	4.83	4.64
27	0.1	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87
	0.05	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
	0.025	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63
	0.01	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15
	0.001	13.61	9.02	7.27	6.33	5.73	5.31	5.00	4.76	4.57
28	0.1	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87
	0.05	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
	0.025	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61
	0.01	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12
	0.001	13.50	8.93	7.19	6.25	5.66	5.24	4.93	4.69	4.50
29	0.1	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86
	0.05	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
	0.025	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59
	0.01	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09
	0.001	13.39	8.85	7.12	6.19	5.59	5.18	4.87	4.64	4.45
30	0.1	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85
	0.05	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
	0.025	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57
	0.01	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
	0.001	13.29	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39
40	0.1	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79
	0.05	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
	0.025	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45
	0.01	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89
	0.001	12.61	8.25	6.59	5.70	5.13	4.73	4.44	4.21	4.02
50	0.1	2.81	2.41	2.20	2.06	1.97	1.90	1.84	1.80	1.76
	0.05	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07
	0.025	5.34	3.97	3.39	3.05	2.83	2.67	2.55	2.46	2.38
	0.01	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78
	0.001	12.22	7.96	6.34	5.46	4.90	4.51	4.22	4.00	3.82
60	0.1	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
	0.05	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
	0.025	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33
	0.01	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
	0.001	11.97	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69
100	0.1	2.76	2.36	2.14	2.00	1.91	1.83	1.78	1.73	1.69
	0.05	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97
	0.025	5.18	3.83	3.25	2.92	2.70	2.54	2.42	2.32	2.24
	0.01	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59
	0.001	11.50	7.41	5.86	5.02	4.48	4.11	3.83	3.61	3.44
200	0.1	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66
	0.05	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93
	0.025	5.10	3.76	3.18	2.85	2.63	2.47	2.35	2.26	2.18
	0.01	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50
	0.001	11.15	7.15	5.63	4.81	4.29	3.92	3.65	3.43	3.26
1000	0.1	2.71	2.31	2.09	1.95	1.85	1.78	1.72	1.68	1.64
	0.05	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89
	0.025	5.04	3.70	3.13	2.80	2.58	2.42	2.30	2.20	2.13
	0.01	6.66	4.63	3.80	3.34	3.04	2.82	2.66	2.53	2.43
	0.001	10.89	6.96	5.46	4.65	4.14	3.78	3.51	3.30	3.13

F distribution critical values

Denominator degrees of freedom	p	Degrees of freedom in the numerator									
		10	11	12	13	14	15	16	17	18	19
2	0.1	9.39	9.40	9.41	9.41	9.42	9.42	9.43	9.43	9.44	9.44
	0.05	19.40	19.40	19.41	19.42	19.42	19.43	19.43	19.44	19.44	19.44
	0.025	39.40	39.41	39.41	39.42	39.43	39.43	39.44	39.44	39.44	39.45
	0.01	99.40	99.41	99.42	99.42	99.43	99.43	99.44	99.44	99.44	99.45
	0.001	999.40	999.41	999.42	999.42	999.43	999.43	999.44	999.44	999.44	999.45
3	0.1	5.23	5.22	5.22	5.21	5.20	5.20	5.20	5.19	5.19	5.19
	0.05	8.79	8.76	8.74	8.73	8.71	8.70	8.69	8.68	8.67	8.67
	0.025	14.42	14.37	14.34	14.30	14.28	14.25	14.23	14.21	14.20	14.18
	0.01	27.23	27.13	27.05	26.98	26.92	26.87	26.83	26.79	26.75	26.72
	0.001	129.25	128.74	128.32	127.96	127.64	127.37	127.14	126.93	126.74	126.57
4	0.1	3.92	3.91	3.90	3.89	3.88	3.87	3.86	3.86	3.85	3.85
	0.05	5.96	5.94	5.91	5.89	5.87	5.86	5.84	5.83	5.82	5.81
	0.025	8.84	8.79	8.75	8.71	8.68	8.66	8.63	8.61	8.59	8.58
	0.01	14.55	14.45	14.37	14.31	14.25	14.20	14.15	14.11	14.08	14.05
	0.001	48.05	47.70	47.41	47.16	46.95	46.76	46.60	46.45	46.32	46.21
5	0.1	3.30	3.28	3.27	3.26	3.25	3.24	3.23	3.22	3.22	3.21
	0.05	4.74	4.70	4.68	4.66	4.64	4.62	4.60	4.59	4.58	4.57
	0.025	6.62	6.57	6.52	6.49	6.46	6.43	6.40	6.38	6.36	6.34
	0.01	10.05	9.96	9.89	9.82	9.77	9.72	9.68	9.64	9.61	9.58
	0.001	26.92	26.65	26.42	26.22	26.06	25.91	25.78	25.67	25.57	25.48
6	0.1	2.94	2.92	2.90	2.89	2.88	2.87	2.86	2.85	2.85	2.84
	0.05	4.06	4.03	4.00	3.98	3.96	3.94	3.92	3.91	3.90	3.88
	0.025	5.46	5.41	5.37	5.33	5.30	5.27	5.24	5.22	5.20	5.18
	0.01	7.87	7.79	7.72	7.66	7.60	7.56	7.52	7.48	7.45	7.42
	0.001	18.41	18.18	17.99	17.82	17.68	17.56	17.45	17.35	17.27	17.19
7	0.1	2.70	2.68	2.67	2.65	2.64	2.63	2.62	2.61	2.61	2.60
	0.05	3.64	3.60	3.57	3.55	3.53	3.51	3.49	3.48	3.47	3.46
	0.025	4.76	4.71	4.67	4.63	4.60	4.57	4.54	4.52	4.50	4.48
	0.01	6.62	6.54	6.47	6.41	6.36	6.31	6.28	6.24	6.21	6.18
	0.001	14.08	13.88	13.71	13.56	13.43	13.32	13.23	13.14	13.06	12.99
8	0.1	2.54	2.52	2.50	2.49	2.48	2.46	2.45	2.45	2.44	2.43
	0.05	3.35	3.31	3.28	3.26	3.24	3.22	3.20	3.19	3.17	3.16
	0.025	4.30	4.24	4.20	4.16	4.13	4.10	4.08	4.05	4.03	4.02
	0.01	5.81	5.73	5.67	5.61	5.56	5.52	5.48	5.44	5.41	5.38
	0.001	11.54	11.35	11.19	11.06	10.94	10.84	10.75	10.67	10.60	10.54
9	0.1	2.42	2.40	2.38	2.36	2.35	2.34	2.33	2.32	2.31	2.30
	0.05	3.14	3.10	3.07	3.05	3.03	3.01	2.99	2.97	2.96	2.95
	0.025	3.96	3.91	3.87	3.83	3.80	3.77	3.74	3.72	3.70	3.68
	0.01	5.26	5.18	5.11	5.05	5.01	4.96	4.92	4.89	4.86	4.83
	0.001	9.89	9.72	9.57	9.44	9.33	9.24	9.15	9.08	9.01	8.95
10	0.1	2.32	2.30	2.28	2.27	2.26	2.24	2.23	2.22	2.22	2.21
	0.05	2.98	2.94	2.91	2.89	2.86	2.85	2.83	2.81	2.80	2.79
	0.025	3.72	3.66	3.62	3.58	3.55	3.52	3.50	3.47	3.45	3.44
	0.01	4.85	4.77	4.71	4.65	4.60	4.56	4.52	4.49	4.46	4.43
	0.001	8.75	8.59	8.45	8.32	8.22	8.13	8.05	7.98	7.91	7.86
11	0.1	2.25	2.23	2.21	2.19	2.18	2.17	2.16	2.15	2.14	2.13
	0.05	2.85	2.82	2.79	2.76	2.74	2.72	2.70	2.69	2.67	2.66
	0.025	3.53	3.47	3.43	3.39	3.36	3.33	3.30	3.28	3.26	3.24
	0.01	4.54	4.46	4.40	4.34	4.29	4.25	4.21	4.18	4.15	4.12
	0.001	7.92	7.76	7.63	7.51	7.41	7.32	7.24	7.17	7.11	7.06
12	0.1	2.19	2.17	2.15	2.13	2.12	2.10	2.09	2.08	2.08	2.07
	0.05	2.75	2.72	2.69	2.66	2.64	2.62	2.60	2.58	2.57	2.56
	0.025	3.37	3.32	3.28	3.24	3.21	3.18	3.15	3.13	3.11	3.09
	0.01	4.30	4.22	4.16	4.10	4.05	4.01	3.97	3.94	3.91	3.88
	0.001	7.29	7.14	7.00	6.89	6.79	6.71	6.63	6.57	6.51	6.45
13	0.1	2.14	2.12	2.10	2.08	2.07	2.05	2.04	2.03	2.02	2.01
	0.05	2.67	2.63	2.60	2.58	2.55	2.53	2.51	2.50	2.48	2.47
	0.025	3.25	3.20	3.15	3.12	3.08	3.05	3.03	3.00	2.98	2.96
	0.01	4.10	4.02	3.96	3.91	3.86	3.82	3.78	3.75	3.72	3.69
	0.001	6.80	6.65	6.52	6.41	6.31	6.23	6.16	6.09	6.03	5.98

F distribution critical values

Denominator degrees of freedom	p	Degrees of freedom in the numerator: 10	11	12	13	14	15	16	17	18	19
	0.1	2.10	2.07	2.05	2.04	2.02	2.01	2.00	1.99	1.98	1.97
	0.05	2.60	2.57	2.53	2.51	2.48	2.46	2.44	2.43	2.41	2.40
14	0.025	3.15	3.09	3.05	3.01	2.98	2.95	2.92	2.90	2.88	2.86
	0.01	3.94	3.86	3.80	3.75	3.70	3.66	3.62	3.59	3.56	3.53
	0.001	6.40	6.26	6.13	6.02	5.93	5.85	5.78	5.71	5.66	5.60
	0.1	2.06	2.04	2.02	2.00	1.99	1.97	1.96	1.95	1.94	1.93
	0.05	2.54	2.51	2.48	2.45	2.42	2.40	2.38	2.37	2.35	2.34
15	0.025	3.06	3.01	2.96	2.92	2.89	2.86	2.84	2.81	2.79	2.77
	0.01	3.80	3.73	3.67	3.61	3.56	3.52	3.49	3.45	3.42	3.40
	0.001	6.08	5.94	5.81	5.71	5.62	5.54	5.46	5.40	5.35	5.29
	0.1	2.03	2.01	1.99	1.97	1.95	1.94	1.93	1.92	1.91	1.90
	0.05	2.49	2.46	2.42	2.40	2.37	2.35	2.33	2.32	2.30	2.29
16	0.025	2.99	2.93	2.89	2.85	2.82	2.79	2.76	2.74	2.72	2.70
	0.01	3.69	3.62	3.55	3.50	3.45	3.41	3.37	3.34	3.31	3.28
	0.001	5.81	5.67	5.55	5.44	5.35	5.27	5.20	5.14	5.09	5.04
	0.1	2.00	1.98	1.96	1.94	1.93	1.91	1.90	1.89	1.88	1.87
	0.05	2.45	2.41	2.38	2.35	2.33	2.31	2.29	2.27	2.26	2.24
17	0.025	2.92	2.87	2.82	2.79	2.75	2.72	2.70	2.67	2.65	2.63
	0.01	3.59	3.52	3.46	3.40	3.35	3.31	3.27	3.24	3.21	3.19
	0.001	5.58	5.44	5.32	5.22	5.13	5.05	4.99	4.92	4.87	4.82
	0.1	1.98	1.95	1.93	1.92	1.90	1.89	1.87	1.86	1.85	1.84
	0.05	2.41	2.37	2.34	2.31	2.29	2.27	2.25	2.23	2.22	2.20
18	0.025	2.87	2.81	2.77	2.73	2.70	2.67	2.64	2.62	2.60	2.58
	0.01	3.51	3.43	3.37	3.32	3.27	3.23	3.19	3.16	3.13	3.10
	0.001	5.39	5.25	5.13	5.03	4.94	4.87	4.80	4.74	4.68	4.63
	0.1	1.96	1.93	1.91	1.89	1.88	1.86	1.85	1.84	1.83	1.82
	0.05	2.38	2.34	2.31	2.28	2.26	2.23	2.21	2.20	2.18	2.17
19	0.025	2.82	2.76	2.72	2.68	2.65	2.62	2.59	2.57	2.55	2.53
	0.01	3.43	3.36	3.30	3.24	3.19	3.15	3.12	3.08	3.05	3.03
	0.001	5.22	5.08	4.97	4.87	4.78	4.70	4.64	4.58	4.52	4.47
	0.1	1.94	1.91	1.89	1.87	1.86	1.84	1.83	1.82	1.81	1.80
	0.05	2.35	2.31	2.28	2.25	2.22	2.20	2.18	2.17	2.15	2.14
20	0.025	2.77	2.72	2.68	2.64	2.60	2.57	2.55	2.52	2.50	2.48
	0.01	3.37	3.29	3.23	3.18	3.13	3.09	3.05	3.02	2.99	2.96
	0.001	5.08	4.94	4.82	4.72	4.64	4.56	4.49	4.44	4.38	4.33
	0.1	1.92	1.90	1.87	1.86	1.84	1.83	1.81	1.80	1.79	1.78
	0.05	2.32	2.28	2.25	2.22	2.20	2.18	2.16	2.14	2.12	2.11
21	0.025	2.73	2.68	2.64	2.60	2.56	2.53	2.51	2.48	2.46	2.44
	0.01	3.31	3.24	3.17	3.12	3.07	3.03	2.99	2.96	2.93	2.90
	0.001	4.95	4.81	4.70	4.60	4.51	4.44	4.37	4.31	4.26	4.21
	0.1	1.90	1.88	1.86	1.84	1.83	1.81	1.80	1.79	1.78	1.77
	0.05	2.30	2.26	2.23	2.20	2.17	2.15	2.13	2.11	2.10	2.08
22	0.025	2.70	2.65	2.60	2.56	2.53	2.50	2.47	2.45	2.43	2.41
	0.01	3.26	3.18	3.12	3.07	3.02	2.98	2.94	2.91	2.88	2.85
	0.001	4.83	4.70	4.58	4.49	4.40	4.33	4.26	4.20	4.15	4.10
	0.1	1.89	1.87	1.84	1.83	1.81	1.80	1.78	1.77	1.76	1.75
	0.05	2.27	2.24	2.20	2.18	2.15	2.13	2.11	2.09	2.08	2.06
23	0.025	2.67	2.62	2.57	2.53	2.50	2.47	2.44	2.42	2.39	2.37
	0.01	3.21	3.14	3.07	3.02	2.97	2.93	2.89	2.86	2.83	2.80
	0.001	4.73	4.60	4.48	4.39	4.30	4.23	4.16	4.10	4.05	4.00
	0.1	1.88	1.85	1.83	1.81	1.80	1.78	1.77	1.76	1.75	1.74
	0.05	2.25	2.22	2.18	2.15	2.13	2.11	2.09	2.07	2.05	2.04
24	0.025	2.64	2.59	2.54	2.50	2.47	2.44	2.41	2.39	2.36	2.35
	0.01	3.17	3.09	3.03	2.98	2.93	2.89	2.85	2.82	2.79	2.76
	0.001	4.64	4.51	4.39	4.30	4.21	4.14	4.07	4.02	3.96	3.92
	0.1	1.87	1.84	1.82	1.80	1.79	1.77	1.76	1.75	1.74	1.73
	0.05	2.24	2.20	2.16	2.14	2.11	2.09	2.07	2.05	2.04	2.02
25	0.025	2.61	2.56	2.51	2.48	2.44	2.41	2.38	2.36	2.34	2.32
	0.01	3.13	3.06	2.99	2.94	2.89	2.85	2.81	2.78	2.75	2.72
	0.001	4.56	4.42	4.31	4.22	4.13	4.06	3.99	3.94	3.88	3.84

F distribution critical values

Denominator degrees of freedom		Degrees of freedom in the numerator									
	p	10	11	12	13	14	15	16	17	18	19
26	0.1	1.86	1.83	1.81	1.79	1.77	1.76	1.75	1.73	1.72	1.71
	0.05	2.22	2.18	2.15	2.12	2.09	2.07	2.05	2.03	2.02	2.00
	0.025	2.59	2.54	2.49	2.45	2.42	2.39	2.36	2.34	2.31	2.29
	0.01	3.09	3.02	2.96	2.90	2.86	2.81	2.78	2.75	2.72	2.69
	0.001	4.48	4.35	4.24	4.14	4.06	3.99	3.92	3.86	3.81	3.77
27	0.1	1.85	1.82	1.80	1.78	1.76	1.75	1.74	1.72	1.71	1.70
	0.05	2.20	2.17	2.13	2.10	2.08	2.06	2.04	2.02	2.00	1.99
	0.025	2.57	2.51	2.47	2.43	2.39	2.36	2.34	2.31	2.29	2.27
	0.01	3.06	2.99	2.93	2.87	2.82	2.78	2.75	2.71	2.68	2.66
	0.001	4.41	4.28	4.17	4.08	3.99	3.92	3.86	3.80	3.75	3.70
28	0.1	1.84	1.81	1.79	1.77	1.75	1.74	1.73	1.71	1.70	1.69
	0.05	2.19	2.15	2.12	2.09	2.06	2.04	2.02	2.00	1.99	1.97
	0.025	2.55	2.49	2.45	2.41	2.37	2.34	2.32	2.29	2.27	2.25
	0.01	3.03	2.96	2.90	2.84	2.79	2.75	2.72	2.68	2.65	2.63
	0.001	4.35	4.22	4.11	4.01	3.93	3.86	3.80	3.74	3.69	3.64
29	0.1	1.83	1.80	1.78	1.76	1.75	1.73	1.72	1.71	1.69	1.68
	0.05	2.18	2.14	2.10	2.08	2.05	2.03	2.01	1.99	1.97	1.96
	0.025	2.53	2.48	2.43	2.39	2.36	2.32	2.30	2.27	2.25	2.23
	0.01	3.00	2.93	2.87	2.81	2.77	2.73	2.69	2.66	2.63	2.60
	0.001	4.29	4.16	4.05	3.96	3.88	3.80	3.74	3.68	3.63	3.59
30	0.1	1.82	1.79	1.77	1.75	1.74	1.72	1.71	1.70	1.69	1.68
	0.05	2.16	2.13	2.09	2.06	2.04	2.01	1.99	1.98	1.96	1.95
	0.025	2.51	2.46	2.41	2.37	2.34	2.31	2.28	2.26	2.23	2.21
	0.01	2.98	2.91	2.84	2.79	2.74	2.70	2.66	2.63	2.60	2.57
	0.001	4.24	4.11	4.00	3.91	3.82	3.75	3.69	3.63	3.58	3.53
40	0.1	1.76	1.74	1.71	1.70	1.68	1.66	1.65	1.64	1.62	1.61
	0.05	2.08	2.04	2.00	1.97	1.95	1.92	1.90	1.89	1.87	1.85
	0.025	2.39	2.33	2.29	2.25	2.21	2.18	2.15	2.13	2.11	2.09
	0.01	2.80	2.73	2.66	2.61	2.56	2.52	2.48	2.45	2.42	2.39
	0.001	3.87	3.75	3.64	3.55	3.47	3.40	3.34	3.28	3.23	3.19
50	0.1	1.73	1.70	1.68	1.66	1.64	1.63	1.61	1.60	1.59	1.58
	0.05	2.03	1.99	1.95	1.92	1.89	1.87	1.85	1.83	1.81	1.80
	0.025	2.32	2.26	2.22	2.18	2.14	2.11	2.08	2.06	2.03	2.01
	0.01	2.70	2.63	2.56	2.51	2.46	2.42	2.38	2.35	2.32	2.29
	0.001	3.67	3.55	3.44	3.35	3.27	3.20	3.14	3.09	3.04	2.99
60	0.1	1.71	1.68	1.66	1.64	1.62	1.60	1.59	1.58	1.56	1.55
	0.05	1.99	1.95	1.92	1.89	1.86	1.84	1.82	1.80	1.78	1.76
	0.025	2.27	2.22	2.17	2.13	2.09	2.06	2.03	2.01	1.98	1.96
	0.01	2.63	2.56	2.50	2.44	2.39	2.35	2.31	2.28	2.25	2.22
	0.001	3.54	3.42	3.32	3.23	3.15	3.08	3.02	2.96	2.91	2.87
100	0.1	1.66	1.64	1.61	1.59	1.57	1.56	1.54	1.53	1.52	1.50
	0.05	1.93	1.89	1.85	1.82	1.79	1.77	1.75	1.73	1.71	1.69
	0.025	2.18	2.12	2.08	2.04	2.00	1.97	1.94	1.91	1.89	1.87
	0.01	2.50	2.43	2.37	2.31	2.27	2.22	2.19	2.15	2.12	2.09
	0.001	3.30	3.18	3.07	2.99	2.91	2.84	2.78	2.73	2.68	2.63
200	0.1	1.63	1.60	1.58	1.56	1.54	1.52	1.51	1.49	1.48	1.47
	0.05	1.88	1.84	1.80	1.77	1.74	1.72	1.69	1.67	1.66	1.64
	0.025	2.11	2.06	2.01	1.97	1.93	1.90	1.87	1.84	1.82	1.80
	0.01	2.41	2.34	2.27	2.22	2.17	2.13	2.09	2.06	2.03	2.00
	0.001	3.12	3.00	2.90	2.82	2.74	2.67	2.61	2.56	2.51	2.46
1000	0.1	1.61	1.58	1.55	1.53	1.51	1.49	1.48	1.46	1.45	1.44
	0.05	1.84	1.80	1.76	1.73	1.70	1.68	1.65	1.63	1.61	1.60
	0.025	2.06	2.01	1.96	1.92	1.88	1.85	1.82	1.79	1.77	1.74
	0.01	2.34	2.27	2.20	2.15	2.10	2.06	2.02	1.98	1.95	1.92
	0.001	2.99	2.87	2.77	2.69	2.61	2.54	2.48	2.43	2.38	2.34