Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May/June	2020	MATH1031-WE01

Title:

Discrete Mathematics

Time (for guidance only):	2 hours	
Additional Material provided:		
Materials Permitted:		
Calculators Permitted:	Yes	Models Permitted: There is no restriction on the model of calculator which may be used.

Instructions to Candidates:	Credit will be given for your answers to all questions. All questions carry the same marks. Please start each question on a new page. Please write your CIS username at the top of each page. Show your working and explain your reasoning.

Revision:	

ED01/2020

Q1 **1.1** Consider the letters CONTRIBUTION.

(i) How many arrangements of these letters are there?

(ii) How many arrangements contain the word TONIC as a contiguous string?

(iii) How many arrangements contain neither of the words BRITON, TONIC?

1.2 A **ternary** number is a number written in base 3, that is, using the digits 0, 1 and 2.

Define the graph G as follows: Each vertex is labelled with a 2-digit ternary string, and there is an edge between two vertices if and only if their ternary representations differ in exactly one place (i.e. 00 is adjacent to 20 and to 01 but not to 12).

(i) How many vertices does G have? How many edges?
Make a (large, clear) drawing of G, labelling the vertices.

(ii) Is G bipartite? Does G have a Euler circuit? Does G have a Hamilton cycle?
(Make sure you fully justify your answers.)

(iii) Explain what is meant by a *subdivision* of a graph. Show that G is not planar by finding a subdivision of $K_{3,3}$ in G (that is, draw your subdivision of $K_{3,3}$ and label which vertices and edges from G you have used).

Q2 **2.1** (i) A pixie can climb stairs one, two or three steps at a time. Find a recurrence relation for the number of distinct ways the pixie can climb a flight of n stairs.

(ii) Find a closed form for the generating function for this recurrence relation [please do **not** find the coefficients of the generating function].

2.2 Write down the definition of a *bipartite* graph. Write down the definition of a *complete bipartite* graph.

2.3 A graph G is **regular** if every vertex of G has the same degree. A regular graph where every vertex has degree d is called d-**regular**.

(i) Which of the complete bipartite graphs $K_{m,n}$ are regular?

(ii) Draw a simple 3-regular graph on 6 vertices which is not isomorphic to the complete bipartite graph $K_{3,3}$.

(iii) Are there any other simple 3-regular graphs on 6 vertices? Fully justify your answer.

Q3 **3.1** (i) Solve the recurrence relation

$$a_n = -2a_{n-1} + 8a_{n-2}, \quad (n \geq 2),$$

with initial conditions $a_0 = 1$ and $a_1 = 0$.

(ii) Solve the recurrence relation

$$a_n = -2a_{n-1} + 8a_{n-2} - 5n + 4, \quad (n \geq 2),$$

with initial conditions $a_0 = 3$ and $a_1 = 1$.

3.2 Kate chooses fruit for the party. She can choose from cherries, oranges and apples. Cherries are identical, and so are oranges, while apples can be of 4 different varieties. She will choose at least 5 cherries, 2 oranges and between 1 and 6 apples of each variety.

Let d_n denote the number of ways in which Kate can select the n fruit.

(i) Write down a generating function for d_n and express it as compactly as possible.

(ii) Use your generating function to find d_{26}.

Q4 **4.1** Let c_n denote the number of integer solutions $(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)$ to

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 = n,$$

with each $x_i \geq 0$.

(i) Derive a formula for c_n via a combinatorial argument.

(ii) Write down a generating function for c_n, and express it as compactly as possible.

(iii) Evaluate $\sum_{n=0}^{\infty} \binom{n+7}{n} 3^{-n}$.

4.2 (i) State the pigeon-hole principle.

(ii) A sweet shop sells bags of assorted sweets. Each bag contains a selection of 20 sweets chosen from 3 possible types (drumstick lollies, liquorice strings, and mint humbugs) with the restriction that there is at least two sweets of each type in every bag. Each student out of a class of 121 buys a bag of sweets.

Show that there are (at least) two students who end up with the same selection of sweets.

END