

## EXAMINATION PAPER

Examination Session: May/June

2020

Year:

Exam Code:

MATH1561-WE01

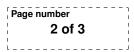
Title:

## Single Mathematics A

| Time (for guidance only):     | 3 hours |   |
|-------------------------------|---------|---|
| Additional Material provided: |         |   |
|                               |         |   |
| Materials Permitted:          |         |   |
| Calculators Permitted:        | Yes     | Models Permitted: There is no restriction on the model of calculator which may be used. |

| Instructions to Candidates: | Credit will be given for your answers to all questions.<br>All questions carry the same marks.       |  |
|-----------------------------|--|--|
|                             | Please start each question on a new page.<br>Please write your CIS username at the top of each page. |  |
|                             | Show your working and explain your reasoning.  |  |
|                             |  |  |

Revision:



- **Q1 1.1** Write down the derivatives of  $\sinh(x)$  and  $\cosh(x)$  and use these to find the derivative of  $\tanh(x)$ .
  - **1.2** Compute the derivative of  $\operatorname{arctanh}(x)$ , using your previous result and

 $\tanh(\operatorname{arctanh}(x)) = x$ .

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Simplify your answer so it is free of any hyperbolic functions (sinh, cosh, tanh) as well as of their inverses.

**1.3** Compute the following three limits

$$\lim_{x \to 0} \frac{x}{\cos(3x - \pi/2)}, \qquad \lim_{x \to 2} \frac{\ln(x - 1)}{(x - 2)}, \qquad \lim_{x \to \infty} \frac{\sin(2/x)}{x}.$$

You are allowed to use L'Hôpital's rule for only one of the three limits.

Q2 2.1 Compute the following definite integral

$$\int_0^{\sqrt{\pi/2}} x^3 \sin(3x^2) dx \, .$$

2.2 Compute the following indefinite integral

$$\int \frac{3x-1}{(x-1)(x^2-3x+2)} dx.$$

2.3 Compute the following indefinite integral

$$\int \frac{\arcsin^2(x)}{\sqrt{1-x^2}} dx \, .$$

2.4 Find the real part, the imaginary part, the modulus and the argument of

$$\exp\left((1-2i)(1/2+i)(3+i\pi)\right)$$
.

**Q3** 3.1 Find all real solutions x to the equation

$$\ln(x + 12/x) = \ln(x) + \ln(7/x).$$

**3.2** With z = x + iy, find the (possibly zero) constants a, b and c such that

$$\tanh(z)\tanh(z+i\pi/2) = a + b\tanh(z) + \frac{c}{\tanh(z)}$$

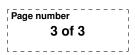
**3.3** Find all complex solutions z to the equation

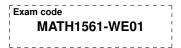
$$z^5 + (1-i)z^2 = 0.$$

You can give your answers in the polar form. State clearly the number of distinct solutions you found.

**3.4** Find all complex solutions z to the equation

$$\cos(z) = \sin(z - \pi).$$





Q4 4.1 Determine whether the series

$$\sum_{n=1}^{\infty} \frac{2}{1+n^2}, \qquad \sum_{n=1}^{\infty} \frac{1+\sqrt{n}}{1+n^2}$$

converge.

**4.2** Show that the series 
$$\sum_{n=1}^{\infty} \frac{n}{1+n}$$
 diverges.

 $\mathbf{Q5}~~\mathbf{5.1}$  Compute the radius of convergence of the following power series :

$$\sum_{k=1}^{\infty} \frac{1-k}{2} x^k, \qquad \sum_{k=1}^{\infty} (k!+5) x^k.$$

**5.2** Find the Taylor series for the function  $f(x) = 1/x^2$  about x = -1.

**Q6 6.1** Find the values of  $\lambda \in \mathbb{R}$  for which the following system of linear equations has exactly one solution:

$$\begin{cases} \lambda x + y - z &= 0\\ 2x + \lambda y + z &= 1\\ y + z &= 2. \end{cases}$$

**6.2** Solve the following system for (x, y, z):

$$\begin{cases} x-y = 0\\ x+y+z = 0\\ y-z = 2 \end{cases}$$

**Q7** 7.1 Let  $A = \begin{pmatrix} 5 & 4 \\ -\frac{3}{2} & 0 \end{pmatrix}$ . Compute  $A^{10}$ .

**7.2** Calculate the inverse of the matrix  $\mathbf{1}$ 

$$\begin{pmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -3 & 4 & -2 \end{pmatrix}.$$