Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May/June	2020	MATH1561-WE01

Title: Single Mathematics A

Time (for guidance only):	3 hours	
Additional Material provided:		
Materials Permitted:		
Calculators Permitted:	Yes	Models Permitted: There is no restriction on the model of calculator which may be used.

Instructions to Candidates:	Credit will be given for your answers to all questions. All questions carry the same marks. Please start each question on a new page. Please write your CIS username at the top of each page. Show your working and explain your reasoning.

Revision:	

ED01/2020

Q1 **1.1** Write down the derivatives of $\sinh(x)$ and $\cosh(x)$ and use these to find the derivative of $\tanh(x)$.

1.2 Compute the derivative of $\operatorname{arctanh}(x)$, using your previous result and

$$\tanh(\operatorname{arctanh}(x)) = x\,.$$

Simplify your answer so it is free of any hyperbolic functions (sinh, cosh, tanh) as well as of their inverses.

1.3 Compute the following three limits

$$\lim_{x\to 0}\frac{x}{\cos(3x-\pi/2)}\,,\qquad \lim_{x\to 2}\frac{\ln(x-1)}{(x-2)}\,,\qquad \lim_{x\to\infty}\frac{\sin(2/x)}{x}\,.$$

You are allowed to use L'Hôpital's rule *for only one of the three limits.*

Q2 **2.1** Compute the following definite integral

$$\int_0^{\sqrt{\pi/2}} x^3\sin(3x^2)dx\,.$$

2.2 Compute the following indefinite integral

$$\int\frac{3x-1}{(x-1)(x^2-3x+2)}dx\,.$$

2.3 Compute the following indefinite integral

$$\int\frac{\arcsin^2(x)}{\sqrt{1-x^2}}dx\,.$$

2.4 Find the real part, the imaginary part, the modulus and the argument of

$$\exp\left((1-2i)(1/2+i)(3+i\pi)\right)\,.$$

Q3 **3.1** Find all real solutions x to the equation

$$\ln(x+12/x) = \ln(x)+\ln(7/x)\,.$$

3.2 With $z = x+iy$, find the (possibly zero) constants a, b and c such that

$$\tanh(z)\tanh(z+i\pi/2) = a + b\tanh(z) + \frac{c}{\tanh(z)}\,.$$

3.3 Find all complex solutions z to the equation

$$z^5 + (1-i)z^2 = 0\,.$$

You can give your answers in the polar form. State clearly the number of distinct solutions you found.

3.4 Find all complex solutions z to the equation

$$\cos(z) = \sin(z-\pi)\,.$$

Q4 **4.1** Determine whether the series

$$\sum_{n=1}^{\infty} \frac{2}{1+n^2}\,, \qquad \sum_{n=1}^{\infty} \frac{1+\sqrt{n}}{1+n^2}$$

converge.

4.2 Show that the series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{1+n}$ diverges.

Q5 **5.1** Compute the radius of convergence of the following power series :

$$\sum_{k=1}^{\infty} \frac{1-k}{2} x^k\,, \qquad \sum_{k=1}^{\infty} (k!+5) x^k\,.$$

5.2 Find the Taylor series for the function $f(x) = 1/x^2$ about $x = -1$.

Q6 **6.1** Find the values of $\lambda \in \mathbb{R}$ for which the following system of linear equations has exactly one solution:

$$\begin{cases} \lambda x + y - z &= 0 \\ 2x + \lambda y + z &= 1 \\ y + z &= 2. \end{cases}$$

6.2 Solve the following system for (x, y, z):

$$\begin{cases} x - y &= 0 \\ x + y + z &= 0 \\ y - z &= 2. \end{cases}$$

Q7 **7.1** Let $A = \begin{pmatrix} 5 & 4 \\ -\frac{3}{2} & 0 \end{pmatrix}$. Compute A^{10}.

7.2 Calculate the inverse of the matrix

$$\begin{pmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -3 & 4 & -2 \end{pmatrix}.$$