Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May/June	2020	MATH1607-WE01

Title:
Dynamics I

Time (for guidance only):	2 hours	
Additional Material provided:		
Materials Permitted:		
Calculators Permitted:	Yes	Models Permitted: There is no restriction on the model of calculator which may be used.

Instructions to Candidates:	Credit will be given for your answers to all questions. All questions carry the same marks. Please start each question on a new page. Please write your CIS username at the top of each page. Show your working and explain your reasoning.

Revision:	

ED01/2020

Q1 **1.1** A particle of mass m is moving along the x-axis, and is slowed by a frictional force $F = -\alpha\sqrt{v}$, where v is the speed of the particle and α is a constant. If the initial speed is u, how long does it take the particle to come to rest?

1.2 A particle of mass m moves on the half-line $x > 0$, and is repelled from $x = 0$ by a force $F = k/\sqrt{x}$, where k is a positive constant. Determine the position $x(t)$ of the particle, given the initial conditions $x(0) = 1$ and $\dot{x}(0) = 2\sqrt{k/m}$.

1.3 A particle of unit mass moves along a line, attached to a spring with spring constant 5, and subject to a damping force of magnitude $\alpha|v|$ where α is a positive constant and v is the velocity of the particle.

(a) Write down the equation of motion for the position $x(t)$ of the particle.

(b) Calculate $x(t)$ if $\alpha = 2$, the particle is initially at equilibrium, and it is given the initial velocity $v(0) = 1$.

(c) For which values of α is the system overdamped?

Q2 **2.1** A particle of mass m moves along the x-axis, acted on by a force arising from the potential $V(x) = me^x(x-1)^2$.

(a) Sketch the graph of V, and find the equilibrium position(s) of the particle.

(b) Calculate the period of small oscillations about the stable equilibrium.

(c) The particle starts from $x = 0$ with initial speed u. How large does u have to be in order that $x(t) \to -\infty$ as $t \to \infty$?

2.2 A particle of mass m travelling with speed v along the x-axis hits a stationary particle of mass $2m$. After the collision, both particles are moving in the same direction; particle m has speed v_1 and particle $2m$ has speed v_2, where $v_1 - v_2 = v/2$. Compute the loss of kinetic energy in the collision, in terms of m and v.

Q3 **3.1** A ball of mass $m = 1$ and electric charge $q = 1$ moves under the influence of gravity and of the Lorentz force $\boldsymbol{F} = \boldsymbol{v} \times \boldsymbol{B}$, where $\boldsymbol{v}$ is the velocity of the ball, and $\boldsymbol{B}$ is a constant magnetic field of magnitude B which points vertically upwards from the ground. The ball is thrown from ground level with speed u, at an angle α to the ground. Determine the subsequent position of the ball. Hence find a condition on the quantity $S = Bu\sin\alpha$ which ensures that the ball will fall back to the ground at exactly the position it was thrown from. [Denote by g the magnitude of the the acceleration due to gravity.]

3.2 (a) Write down the wave equation with wave speed c, and its general solution (no derivation or proof is needed).

(b) Find the deviation $u(x,t)$ of a vibrating string which satisfies the initial conditions $u(x,0) = 0$ and $\partial u(x,0)/\partial t = cx/(1+x^2)$.

Q4 **4.1** A particle of unit mass $m = 1$ moves under the action of an attractive central force $f(r) = -\alpha/r^2 - \beta/r^3$, where α and β are positive constants, and (r, θ) are polar coordinates. You may use the fact that $u = 1/r$ satisfies the equation

$$\frac{d^2u}{d\theta^2} + u = -\frac{f(u^{-1})}{L^2u^2},$$

where L is the magnitude of the angular momentum. At time $t = 0$, the particle is at $(r, \theta) = (c, 0)$, and is moving with speed $\sqrt{4\beta/(3c^2)}$ in a direction perpendicular to the line joining it to the centre.

(a) Evaluate L.

(b) Find the function $u(\theta)$, in terms of θ and the constants α, β and c.

4.2 A compound pendulum consists of a light rigid rod of length L which swings freely in a vertical plane, about a horizontal axis through one end. A mass m is attached to the other end of the rod, and a mass $2m$ is attached halfway along the rod.

(a) How far from the axis is the centre of mass of this system?

(b) Compute the moment of inertia of the system about its axis.

(c) If the rod is rotating with angular velocity ω, what is the total kinetic energy K?

(d) Compute its potential energy V in terms of the angle θ by which the rod deviates from hanging downwards. [Let g denote the acceleration due to gravity.]

(e) Hence write down the equation of motion for $\theta(t)$.