University of Durham

Formula sheet for **Mathematics for Engineers & Scientists (MATH1551-WE01)**.

TRIGONOMETRIC FUNCTIONS

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

$$\cos^2 A + \sin^2 A = 1$$

$$1 + \tan^2 A = \sec^2 A$$

$$1 + \cot^2 A = \mathrm{cosec}^2 A$$

$$\sin A\cos B = \frac{1}{2}\Big(\sin(A+B) + \sin(A-B)\Big)$$

$$\cos A\cos B = \frac{1}{2}\Big(\cos(A+B) + \cos(A-B)\Big)$$

$$\sin A\sin B = \frac{1}{2}\Big(\cos(A-B) - \cos(A+B)\Big)$$

$$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

HYPERBOLIC FUNCTIONS

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh^{-1} x = \ln\left(x \pm \sqrt{x^2-1}\right)$$

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$$

$$\cosh(iA) = \cos A$$

$$\sinh(iA) = i\sin A$$

$$\cosh^2 A - \sinh^2 A = 1$$

$$\cosh(A+B) = \cosh A\cosh B + \sinh A\sinh B$$

$$\sinh(A+B) = \sinh A\cosh B + \cosh A\sinh B$$

$$\tanh(A+B) = \frac{\tanh A + \tanh B}{1 + \tanh A\tanh B}$$

ELEMENTARY RULES FOR DIFFERENTIATION AND INTEGRATION

$$(u+v)' = u'+v' \qquad (uv)' = u'v+uv' \qquad \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \qquad (u(v))' = u'(v)v' \qquad \int u'v\,dx = uv - \int uv'\,dx$$

TAYLOR'S THEOREM

Taylor approximation: $f(x) \approx p_{n,a}(x) = f(a) + f'(a)(x-a) + \cdots + \frac{1}{n!}f^{(n)}(a)(x-a)^n$

and if $|f^{(n+1)}(x)| \le M$ for $c \le x \le b$ then $|f(x) - p_{n,a}(x)| \le \frac{|x-a|^{n+1}}{(n+1)!}M.$

DIFFERENTIAL OPERATORS

$$\mathrm{grad}\, f = \nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

$$\mathrm{div}\,\mathbf{A} = \nabla\cdot\mathbf{A} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\cdot(A_1, A_2, A_3) = \frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial y} + \frac{\partial A_3}{\partial z}$$

$$\mathrm{curl}\,\mathbf{A} = \nabla\times\mathbf{A} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix} = \left(\frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z}\right)\mathbf{i} + \left(\frac{\partial A_1}{\partial z} - \frac{\partial A_3}{\partial x}\right)\mathbf{j} + \left(\frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y}\right)\mathbf{k}$$

TABLE OF DERIVATIVES

$y(x)$	dy/dx
x^n	nx^{n-1}
$\ln x$	x^{-1}
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$\dfrac{-1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\dfrac{1}{1+x^2}$
$\sinh^{-1} x$	$\dfrac{1}{\sqrt{1+x^2}}$
$\cosh^{-1} x$	$\dfrac{1}{\sqrt{x^2-1}}$

TABLE OF INTEGRALS

$f(x)$	$\int f(x)\,dx$
x^n	$\dfrac{x^{n+1}}{n+1}$ $(n \neq -1)$
x^{-1}	$\ln \lvert x \rvert$
e^x	e^x
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
$\tan x$	$-\ln \lvert \cos x \rvert$
$\operatorname{cosec} x$	$-\ln \lvert \operatorname{cosec} x + \cot x \rvert$
$\sec x$	$\ln \lvert \sec x + \tan x \rvert$
$\cot x$	$\ln \lvert \sin x \rvert$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln \lvert \cosh x \rvert$
$\dfrac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1} \dfrac{x}{a}$ $(a > x)$
$\dfrac{1}{a^2+x^2}$	$\dfrac{1}{a} \tan^{-1} \left(\dfrac{x}{a}\right)$
$\dfrac{1}{\sqrt{a^2+x^2}}$	$\sinh^{-1} \left(\dfrac{x}{a}\right)$
$\dfrac{1}{\sqrt{x^2-a^2}}$	$\cosh^{-1} \left(\dfrac{x}{a}\right)$ $(x > a)$

CRITICAL POINTS

Local maximum: $\dfrac{\partial f}{\partial x} = \dfrac{\partial f}{\partial y} = 0$ and $\dfrac{\partial^2 f}{\partial x^2}\dfrac{\partial^2 f}{\partial y^2} - \left(\dfrac{\partial^2 f}{\partial y \partial x}\right)^2 > 0$ and $\dfrac{\partial^2 f}{\partial x^2} < 0$

Local minimum: $\dfrac{\partial f}{\partial x} = \dfrac{\partial f}{\partial y} = 0$ and $\dfrac{\partial^2 f}{\partial x^2}\dfrac{\partial^2 f}{\partial y^2} - \left(\dfrac{\partial^2 f}{\partial y \partial x}\right)^2 > 0$ and $\dfrac{\partial^2 f}{\partial x^2} > 0$

Saddle point: $\dfrac{\partial f}{\partial x} = \dfrac{\partial f}{\partial y} = 0$ and $\dfrac{\partial^2 f}{\partial x^2}\dfrac{\partial^2 f}{\partial y^2} - \left(\dfrac{\partial^2 f}{\partial y \partial x}\right)^2 < 0$

Inconclusive: $\dfrac{\partial f}{\partial x} = \dfrac{\partial f}{\partial y} = 0$ and $\dfrac{\partial^2 f}{\partial x^2}\dfrac{\partial^2 f}{\partial y^2} - \left(\dfrac{\partial^2 f}{\partial y \partial x}\right)^2 = 0$

ITERATION METHODS

$$A\mathbf{x} = \mathbf{b} \qquad A = D - L - U \qquad T_j = D^{-1}(L+U) \qquad T_g = (D-L)^{-1}U$$

Jacobi's Method:

$$D\mathbf{x}^{(k+1)} = \mathbf{b} + (L+U)\mathbf{x}^{(k)}, \qquad \mathbf{x}^{(k+1)} = D^{-1}\mathbf{b} + D^{-1}(L+U)\mathbf{x}^{(k)}$$

Gauss-Seidel Method:

$$D\mathbf{x}^{(k+1)} = \mathbf{b} + L\mathbf{x}^{(k+1)} + U\mathbf{x}^{(k)}, \qquad \mathbf{x}^{(k+1)} = (D-L)^{-1}\mathbf{b} + (D-L)^{-1}U\mathbf{x}^{(k)}$$

SOR Method:

$$\mathbf{x}^{(k+1)} = (1-\omega)\mathbf{x}^{(k)} + \omega D^{-1}\left(\mathbf{b} + L\mathbf{x}^{(k+1)} + U\mathbf{x}^{(k)}\right)$$

Optimal Value of ω:

$$\omega = \frac{2}{1 + \sqrt{1 - \rho(T_j)^2}}$$