Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May/June	2021	MATH1607-WE01

Title: Dynamics I

Time (for guidance only):	2 hours	
Additional Material provided:		
Materials Permitted:		
Calculators Permitted:	Yes	Models Permitted: There is no restriction on the model of calculator which may be used.

Instructions to Candidates:	Credit will be given for your answers to all questions. All questions carry the same marks. Please start each question on a new page. Please write your CIS username at the top of each page. To receive credit, your answers must show your working and explain your reasoning.

Revision:	

ED01/2021

Q1 **1.1** A particle of unit mass moves on the x-axis. It starts at $x = 0$ with positive velocity u, and is slowed by a force of magnitude $2vx$, where v denotes its speed. What is the maximum value x_m of x that it reaches?

1.2 A particle of constant mass m is moving along the z-axis, starting from rest at $z = z_0 > 0$. Acting on it are a force of magnitude km/z^2 directed away from $z = 0$, and a force of magnitude mh directed towards $z = 0$, where k and h are constants. What does h need to be, in order that the particle has zero speed when it reaches $z = z_1$? Here $0 < z_1 < z_0$.

1.3 A water drop falls vertically from rest, with the only force on it being its weight mg. Since vapour condenses on it as it falls, its mass m increases, in such a way that $m(t) = m_0 \exp(kt)$ where m_0 and k are positive constants. What is its speed w when its mass has doubled?

Q2 **2.1** A mass m at position x is attached to a spring with spring constant k. There is no damping, but there is an external force $F(t) = 4m\cos^3(\omega t)$ where ω is a positive constant.

(i) Determine for which values of ω, if any, resonance occurs.

(ii) For all other values of ω, find $x(t)$ given the initial conditions $x(0) = 0$ and $\dot{x}(0) = 0$.

2.2 A car of mass m travelling with speed $3v$, collides with a second car of mass $2m$ travelling in the same direction with speed v along a straight road, and they stick together. Calculate the loss δK of kinetic energy in the collision.

2.3 Vibrations $u(x,t)$ of a string satisfy the equation $4\,\partial^2 u/\partial t^2 = 25\,\partial^2 u/\partial x^2$. Find u if the initial conditions are $u(x,0) = 0$ and $\partial u(x,0)/\partial t = 20\sin^2 x$.

Q3 **3.1** A particle of unit mass and charge moves in an electric field $\mathbf{E} = E\mathbf{j}$ and a magnetic field $\mathbf{B} = B\mathbf{k}$, where E and B are nonzero constants. Its position and velocity at time $t = 0$ are $\mathbf{0}$ and $u\mathbf{i}$ respectively. Find the path $\mathbf{r}(t)$ of the particle.

3.2 A particle of mass m_1 is moving along the x-axis, and collides elastically with a particle of mass m_2 stationary at the origin. Afterwards, the m_1-particle is moving along the y-axis. Find the angle θ between the x-axis and the direction of the m_2-particle after the collision. You may assume that $m_2 > m_1$.

Q4 **4.1** A particle of unit mass moves in an attractive central force of magnitude c/r^2 where c is a constant. Let $\mathbf{L}$ denote its angular momentum. Prove that the vector $\mathbf{w} = \dot{\mathbf{r}} \times \mathbf{L} + h(r)\,\mathbf{r}$ is constant in time, for some function $h(r)$ which you should determine.

4.2 A compound pendulum consists of a light rigid rod of length L, pivoted at one end, and free to rotate in a vertical plane with gravity (acceleration g) acting downwards. A mass m is attached at the other end, and a mass $3m$ is attached a distance $L/3$ from the pivot.

(i) At what distance D from the pivot is the centre of mass of the system?

(ii) What is the moment of inertia I of the system about its pivot?

(iii) The pendulum begins in its stable equilibrium position, and the mass at the end is given an initial velocity u. What value of u causes the rod to reach a horizontal position, but no higher?

(iv) What is the period P of small oscillations about the stable equilibrium position?

Q5 **5.1** A unit-mass particle moves along the x-axis, and is acted on by a force derived from the potential energy $V(x) = ax^2(b - x)$, where a and b are positive constants.

(i) Determine the angular frequency ω of small oscillations about the stable equilibrium of the particle.

(ii) If the particle is released from rest at $x = -b$, compute its speed v as it passes $x = 0$.

(iii) If the particle starts at $x = 0$ with initial speed u, how large does u have to be in order for the particle to escape to $x = \infty$?

5.2 A mass m is attached to a light string which is wrapped around the rim of a wheel of radius R and does not slip. Gravity pulls the mass down with force mg, and this causes the wheel to rotate about the fixed horizontal axis through its centre. The mass starts at rest, and then falls a distance D in time T. Compute the moment of inertia I of the wheel, in terms of m, R, D, T and g.

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