Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May/June	2022	MATH1607-WE01

Title: Dynamics I

Time:	2 hours	
Additional Material provided:		
Materials Permitted:		
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.

Instructions to Candidates:	Credit will be given for your answers to each question. All questions carry the same marks. Students must use the mathematics specific answer book.

Revision:	

ED01/2022

Q1 (a) A particle of unit mass moves along a line, acted on by a force $F = -2v+e^{-t}$, where v denotes its velocity and t denotes time. At time $t = 0$ the velocity is zero. For what value of t is the speed a maximum?

(b) A particle of mass $m = 1$ on the x-axis is attached to a spring with spring constant $k = 4$, and is subject to an additional force $F = 8\sin(pt)\cos(pt)$, where p is a positive constant. Find its position $x(t)$ if p is tuned to the value which makes the system resonant, and the initial conditions are $x(0) = 0$, $\dot{x}(0) = 0$.

(c) Determine for which value(s) of the constants n and m the force

$$\mathbf{F} = yz(nx + y)\mathbf{i} + xz(x + my)\mathbf{j} + xy(x + y)\mathbf{k}$$

is conservative; and for those values find the potential $V(x, y, z)$.

Q2 (a) A particle of mass $2m$ and speed v moves along the x-axis, and collides head-on with a particle of mass m at rest. They continue on the x-axis, with the lighter particle moving at speed $v/3$. What fraction β of the initial kinetic energy is lost in the collision?

(b) A particle of unit mass and unit charge moves in a magnetic field $\mathbf{B} = B\mathbf{k}$, where B is a positive constant. In addition to the Lorentz force, it feels a resistive force, pointing in the opposite direction to its velocity $\mathbf{v}(t)$, and of magnitude $\alpha|\mathbf{v}|$ where α is a constant. Given that $\mathbf{v}(0) = w\mathbf{i}$, find $\mathbf{v}(t)$. *[Hint: you could use an integrating factor* $\exp(\alpha t)$*.]*

Q3 (a) A particle of unit mass moves along the positive x-axis in a potential

$$V(x) = -2\log x - \tfrac{1}{2}x^2 + 3x.$$

(i) Determine the equilibrium positions of the particle.
(ii) Calculate the period of small oscillations about the stable equilibrium.
(iii) If the particle starts at $x = 1$ with initial velocity u, for what values of u does it escape to infinity?

(b) A mass m_1 is travelling in the positive x-direction. A mass m_2 is travelling in a direction which makes an angle $\pi/3$ with the x-axis, and which has negative x-component. The two masses collide elastically. Afterwards, m_1 is stationary, while the velocity of m_2 makes an angle $\pi/6$ with the x-axis. Compute the ratio $R = m_1/m_2$.

Q4 (a) An object of mass m slides in a straight line along a horizontal surface, starting with initial speed w. The forces acting on it are: gravity with magnitude mg; a normal force $\mathbf{N}$ acting vertically upwards; and a frictional force of magnitude $\mu|\mathbf{N}|$ in the direction opposite to its motion. Calculate the time T that it takes to come to rest, in terms of w, g and μ.

(b) A unit-mass particle moves in an attractive central force of magnitude $f(r) = k/r^3$, where (r, θ) are polar coordinates and k is a constant. At time $t = 0$ the particle is at the point $(r, \theta) = (R, 0)$ and its velocity is $s\mathbf{e}_\theta$.

(i) Assuming that $0 < k < R^2 s^2$, find an expression for r in terms of θ, k, R and s. You may use, without proof, the orbit equation

$$\frac{d^2u}{d\theta^2} + u = (Lu)^{-2} f(u^{-1}),$$

where $u = 1/r$ and L is the magnitude of angular momentum.

(ii) Compute the energy E of the trajectory, with the potential energy satisfying $V(r) \to 0$ as $r \to \infty$. Is the trajectory bounded? (Justify your answer.)

Q5 (a) Find the solution $u(x, t)$ of the wave equation $\partial^2 u/\partial t^2 = 4\partial^2 u/\partial x^2$ satisfying the initial conditions $u(x, 0) = \sin^2(2x)$ and $\partial u(x, 0)/\partial t = 8 \sin(2x) \cos(2x)$.

(b) A light rigid rod of length L is pivoted at one end, and can swing freely in a vertical plane. Four equal masses are attached to the rod, with equal spacing: so the masses are at distances $L/4$, $L/2$, $3L/4$ and L from the pivot. The total mass is M, and gravity acts downwards with acceleration g.

(i) Compute the distance D from the pivot to the centre of mass of the system.

(ii) Compute the moment of inertia I of the system about its pivot.

(iii) Let θ denote the angle by which the rod deviates from its stable equilibrium. Write an expression for the total energy E in terms of L, M, g, θ and $\dot{\theta}$.

(iv) If the pendulum begins at its stable equilibrium position, with initial angular speed $\dot{\theta}(0) = q$, what is the maximum value of q for which the system will not reach its unstable equilibrium?