EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May/June	2022	MATH2657-WE01

Title:
Special Relativity Electro II

Time:	2 hours	
Additional Material provided:		
Materials Permitted:		
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.

Instructions to Candidates:	Answer all questions. Section A is worth 40% and Section B is worth 60%. Within each section, all questions carry equal marks. Students must use the mathematics specific answer book.

Revision:	

ED01/2022

SECTION A

Q1 **1.1** The Duke and the Baron are on separate ships, travelling from Caladan to Arrakis, when the Duke monitors a treacherous radio transmission (with wavelength 6 metres) being sent from his own ship. The Duke orders a search of his ship but is dismayed to observe that the spy flees in an escape pod travelling at a relative speed $c/2$ in the direction of Caladan. The Baron receives the radio message (with wavelength 9 metres) from his spy and soon after observes an escape pod approaching his ship. Calculate the relative speed between the Baron's ship and the escape pod, as a fraction of the speed of light c.

1.2 Two particles, each of rest mass $10m$, collide and fuse to form a single particle with rest mass $52m$. Find the speeds, as a fraction of the speed of light c, of the two initial particles in the rest frame of the final particle.

Q2 **2.1** In an electrostatics problem, in a region of space the electric field has the form

$$\mathbf{E} = (ay^4z^2, vxy^3z^2, 14az^6 + wxy^4z),$$

where a, v, w are constants. Determine v and w in terms of a.

Find the electrostatic scalar potential ϕ and the electric charge density ρ, in terms of the spatial coordinates, the constant a and the electric constant ε_0.

2.2 Let $R = \sqrt{x^2 + y^2}$ be the radius in cylindrical polar coordinates.

Show that, for $R > 0$, any magnetic field of the form $\mathbf{B} = (-yf, xf, 0)$ satisfies $\boldsymbol{\nabla} \cdot \mathbf{B} = 0$, where f is a differentiable function that depends only on R.

An infinitely long straight wire lies along the z-axis and carries a current I_1. Assume the form given above for $\mathbf{B}$ and use the integral form of Ampère's law

$$\oint_C \mathbf{B} \cdot d\mathbf{r} = \mu_0 I,$$

to calculate the magnitude of the magnetic field at a distance D from the wire, in terms of D, I_1 and μ_0.

SECTION B

Q3 **3.1** Derive conditions on the constants p, a, u, ℓ, so that $x'^{\mu} = L^{\mu}{}_{\nu} x^{\nu}$ is a Lorentz transformation, where

$$L^{\mu}{}_{\nu} = \begin{pmatrix} p & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & u & 0 \\ 0 & 0 & 0 & \ell \end{pmatrix}.$$

State the number of solutions of these conditions.

List the solutions that correspond to orthochronous proper Lorentz transformations, giving a physical interpretation of each.

Let $\varepsilon^{\mu\nu\alpha\beta}$ be the totally antisymmetric tensor with $\varepsilon^{0123} = 1$.

Derive an expression for $\varepsilon'^{\mu\nu\alpha\beta}$, obtained by applying a Lorentz transformation of the form given above.

Use your result to suggest the type of Lorentz transformations under which $\varepsilon^{\mu\nu\alpha\beta}$ is invariant.

3.2 In a frame $\mathcal{R}$ there is no electric field but there is a magnetic field given by $\mathbf{B} = (b, 0, -b)$, where b is a constant.

Calculate, in terms of b and the speed of light c, the electric field in the frame $\mathcal{R}'$ that moves along the positive x-axis with a speed $4c/5$.

Q4 **4.1** Electric charge is contained within a ball of radius R that is centred at the origin. The electric charge density vanishes outside the ball but inside it is given by

$$\rho = \frac{qR^3}{(k + r^3)^2},$$

where q and k are positive constants and r is the distance to the origin.

Calculate the electric field due to the ball.

Find k_0 in terms of R, so that for $k < k_0$ the magnitude of the electric field takes its maximum value inside the ball.

In the case that $k = k_0$, calculate the magnitude of the force on a point charge q that is a distance $2R$ from the origin.

4.2 In a region of space the magnetic vector potential is given by

$$\mathbf{A} = p\,(2xyz + 6x^2z, x^2z - 2yz^2, x^2y - 2y^2z + 2x^3),$$

where p is a positive constant.

Calculate this magnetic vector potential in Coulomb gauge.