

EXAMINATION PAPER

Examination Session: May/June

Year: 2022

Exam Code:

MATH2657-WE01

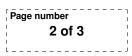
Title:

Special Relativity Electro II

Time:	2 hours	
Additional Material provided:		
Matariala Darmittadu		
Materials Permitted:		
Calculators Permitted:	No	Models Permitted: Use of electronic calculators
		is forbidden.

Instructions to Candidates:	Answer all questions.Section A is worth 40% and Section B is worth 60%. Within each section, all questions carry equal marks.Students must use the mathematics specific answer book.		

Revision:



SECTION A

- **Q1 1.1** The Duke and the Baron are on separate ships, travelling from Caladan to Arrakis, when the Duke monitors a treacherous radio transmission (with wavelength 6 metres) being sent from his own ship. The Duke orders a search of his ship but is dismayed to observe that the spy flees in an escape pod travelling at a relative speed c/2 in the direction of Caladan. The Baron receives the radio message (with wavelength 9 metres) from his spy and soon after observes an escape pod approaching his ship. Calculate the relative speed between the Baron's ship and the escape pod, as a fraction of the speed of light c.
 - 1.2 Two particles, each of rest mass 10m, collide and fuse to form a single particle with rest mass 52m. Find the speeds, as a fraction of the speed of light c, of the two initial particles in the rest frame of the final particle.
- Q2 2.1 In an electrostatics problem, in a region of space the electric field has the form

$$\mathbf{E} = (ay^4 z^2, vxy^3 z^2, 14az^6 + wxy^4 z),$$

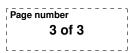
where a, v, w are constants. Determine v and w in terms of a.

Find the electrostatic scalar potential ϕ and the electric charge density ρ , in terms of the spatial coordinates, the constant a and the electric constant ε_0 .

2.2 Let $R = \sqrt{x^2 + y^2}$ be the radius in cylindrical polar coordinates. Show that, for R > 0, any magnetic field of the form $\mathbf{B} = (-yf, xf, 0)$ satisfies $\nabla \cdot \mathbf{B} = 0$, where f is a differentiable function that depends only on R. An infinitely long straight wire lies along the z-axis and carries a current I_1 . Assume the form given above for \mathbf{B} and use the integral form of Ampère's law

$$\oint_C \mathbf{B} \cdot d\mathbf{r} = \mu_0 I,$$

to calculate the magnitude of the magnetic field at a distance D from the wire, in terms of D, I_1 and μ_0 .



Exam code	
MATH2657-WE)1
i l	i
L	

SECTION B

Q3 3.1 Derive conditions on the constants p, a, u, ℓ , so that $x'^{\mu} = L^{\mu}_{\ \nu} x^{\nu}$ is a Lorentz transformation, where

$$L^{\mu}_{\ \nu} = \begin{pmatrix} p & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & u & 0 \\ 0 & 0 & 0 & \ell \end{pmatrix}.$$

State the number of solutions of these conditions.

List the solutions that correspond to orthochronous proper Lorentz transformations, giving a physical interpretation of each.

Let $\varepsilon^{\mu\nu\alpha\beta}$ be the totally antisymmetric tensor with $\varepsilon^{0123} = 1$.

Derive an expression for $\varepsilon'^{\mu\nu\alpha\beta}$, obtained by applying a Lorentz transformation of the form given above.

Use your result to suggest the type of Lorentz transformations under which $\varepsilon^{\mu\nu\alpha\beta}$ is invariant.

3.2 In a frame \mathcal{R} there is no electric field but there is a magnetic field given by $\mathbf{B} = (b, 0, -b)$, where b is a constant.

Calculate, in terms of b and the speed of light c, the electric field in the frame \mathcal{R}' that moves along the positive x-axis with a speed 4c/5.

Q4 4.1 Electric charge is contained within a ball of radius R that is centred at the origin. The electric charge density vanishes outside the ball but inside it is given by

$$\rho = \frac{qR^3}{(k+r^3)^2},$$

where q and k are positive constants and r is the distance to the origin.

Calculate the electric field due to the ball.

Find k_0 in terms of R, so that for $k < k_0$ the magnitude of the electric field takes its maximum value inside the ball.

In the case that $k = k_0$, calculate the magnitude of the force on a point charge q that is a distance 2R from the origin.

4.2 In a region of space the magnetic vector potential is given by

$$\mathbf{A} = p \left(2xyz + 6x^2z, x^2z - 2yz^2, x^2y - 2y^2z + 2x^3 \right),$$

where p is a positive constant.

Calculate this magnetic vector potential in Coulomb gauge.