Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May/June	2023	MATH1031-WE01

Title:
Discrete Mathematics

Time:	2 hours	
Additional Material provided:		
Materials Permitted:		
Calculators Permitted:	Yes	Models Permitted: Casio FX83 series or FX85 series.

Instructions to Candidates:	Credit will be given for your answers to each question. All questions carry the same marks. Students must use the mathematics specific answer book.

Revision:	

ED01/2023

Q1 (a) Consider the letters DERANGEMENT.

(i) How many arrangements of these letters are there?

(ii) How many arrangements contain the word RANGE as a contiguous string?

(iii) How many arrangements contain neither of the words RANGE, GEM?

(b) Prove by induction that for any integer $n \in \mathbb{Z}_+$

$$\sum_{i=1}^{n} (-1)^i i^2 = (-1)^n \frac{n(n+1)}{2}.$$

Q2 (a) (i) A strange pixie can climb stairs two, three or five steps at a time. Find a recurrence relation for the number a_n of distinct ways the pixie can climb a flight of n stairs. Also, calculate $a_0, a_1, a_2, a_3, a_4, a_5$.

(ii) Use the recurrence relation to derive a closed form for the generating function of a_n. [Please do not find the coefficients of the generating function.]

(b) Solve the recurrence relation

$$a_n = -a_{n-1} + 6a_{n-2} - 7 + 4n$$

with initial conditions $a_0 = 4$ and $a_1 = -2$.

Q3 (a) (i) Find the generating function for the sequence $(a_n, n \in \{0, 1, 2, \ldots\})$ where $a_n := (n+1)\binom{n+3}{n}$.

(ii) Evaluate $\sum_{n=0}^{\infty} (n+1)\binom{n+3}{n} 2^{-n}$.

(b) How many solutions $(x_1, x_2, x_3, x_4, x_5)$ are there to the equation

$$x_1 - x_2 - x_3 + x_4 + x_5 = 3$$

where each x_i is an integer and

$$x_1 \le 1,\ x_2 \ge 2,\ x_3 \ge 1, \quad \text{and} \qquad x_i \le -1 + 4i \text{ for } i \in \{4, 5\}\,?$$

Q4 A connected simple planar graph has every vertex of degree 3 and every face with either 4 or 6 edges. There are exactly 2 faces with 6 edges. How many faces with 4 edges are there in this graph? Draw an example of a planar graph with these properties. [Hint: Use the handshaking lemmas and Euler's formula for simple connected planar graphs.]