

EXAMINATION PAPER

Examination Session: May/June Year: 2023

Exam Code:

MATH1551-WE01

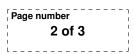
Title:

Maths For Engineers and Scientists

Time:	3 hours	
Additional Material provided:	Formula sheet.	
Materials Permitted:		
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.

Instructions to Candidates:	Credit will be given for your answers to each question. All questions carry the same marks. Students must use the mathematics specific answer book.

Revision:



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Q1 Consider the system of equations

$$3x + 2y - z = 12$$
$$2x + 4y - z = 8$$
$$-x - y + 2z = 9.$$

- (a) (i) Is the Jacobi method guaranteed to converge for this system for any given initial values? State your reasoning.
 - (ii) Is the Gauss-Seidel method guaranteed to converge for this system for any given initial values? State your reasoning.
- (b) Write down the Gauss-Seidel iteration scheme for this system and perform one iteration starting from initial values $(x^{(0)}, y^{(0)}, z^{(0)}) = (1, 1, 2)$.
- (c) Use Gaussian elimination to find the exact solution of the system of equations.
- Q2 (a) Find the modulus and principal argument of all possible values of $(1-i)^{2i}$.
 - (b) Find all complex solutions of the equation $\tan(2z) = 3i$, expressing your answers in the form z = x + iy where $x, y \in \mathbb{R}$.
 - (c) (i) Show that if $z = e^{i\theta}$, then

$$z^n + \frac{1}{z^n} = 2\cos(n\theta).$$

(ii) Given $z + \frac{1}{z} = \sqrt{2}$, use the result in part (i) to determine the value of $z^8 + \frac{1}{z^8}$.

Q3 (a) Consider the following set of vectors in \mathbb{R}^3 : $S = \left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\}.$

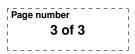
- (i) Is S a basis for \mathbb{R}^3 ? Justify your answer.
- (ii) Suppose each of the vectors in S gives the position of a corner of a triangle.Find a Cartesian equation for the plane that this triangle lies in.
- (iii) Which point on this plane is the closest to the origin?
- (b) (i) Find Cartesian equations of the line that has parametric equation

$$\mathbf{x} = \begin{pmatrix} 0\\1\\2 \end{pmatrix} + t \begin{pmatrix} 1\\4\\-1 \end{pmatrix}, t \in \mathbb{R}.$$

(ii) What angle does this line make with the line that has Cartesian equations

$$\frac{x-1}{4} = \frac{y-2}{4} = \frac{z+1}{2}?$$

Give your answer in terms of π .



Q4 (a) Calculate the eigenvalues and corresponding eigenspaces of the matrix

$$A = \begin{pmatrix} -1 & -5 & -4 \\ 0 & -3 & 0 \\ -2 & 5 & 1 \end{pmatrix}$$

- (b) From your answer to part (a), do you expect A to be diagonalisable? Explain your reasoning.
- Q5 (a) Find each of the following limits or argue why they don't exist:

(i)
$$\lim_{x \to 0} (x+1)^{2023}$$
, (ii) $\lim_{x \to 0} \frac{5x^3 + 8x^2}{3x^4 - 16x^2}$, (iii) $\lim_{x \to 0} \frac{\cosh(x) - 1}{x^2}$.

- (b) Without using l'Hôpital's Rule, use the limit definition of differentiation to find f'(1) for $f(x) = \frac{1}{\sqrt{3x-2}}$.
- **Q6** (a) Find the degree 3 Taylor polynomial of tan(x) ln(1+x) about x = 0.
 - (b) Consider the equation $e^{-x} = \sin x$.
 - (i) Use an appropriate theorem to show that there is at least one solution x_* to this equation in the interval $0 < x < \frac{\pi}{2}$.
 - (ii) Write out a Newton-Raphson iteration formula (for this equation) that would converge to x_* for a close enough initial guess x_0 .
- Q7 The elevation of a surface is specified by the function

$$f(x,y) = 2x^3 + 2y^3 - 9x^2 + 3y^2 - 12y.$$

- (a) Determine the critical points of f(x, y) and identify each as a local minimum, local maximum or saddle point.
- (b) Find the directional derivative of f(x, y) in the direction $\mathbf{i} 2\mathbf{j}$ at the point (0, 0).
- (c) An ant can walk on the surface, and is standing at the point (0,0). In which direction should the ant walk in order for their elevation to decrease most rapidly?
- $\mathbf{Q8}$ (a) Find the general solution of the first-order differential equation

$$2x\frac{\mathrm{d}y}{\mathrm{d}x} + y = 2x^{5/2}.$$

(b) Find the specific solution of the second-order differential equation

$$y'' + 16y = 16\cos(4x)$$

satisfying the initial conditions y(0) = 1 and y'(0) = 1.