Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May/June	2023	MATH1607-WE01

Title:
Dynamics I

Time:	2 hours	
Additional Material provided:		
Materials Permitted:		
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.

Instructions to Candidates:	Credit will be given for your answers to each question. All questions carry the same marks. Students must use the mathematics specific answer book.

Revision:	

ED01/2023

Q1 **1.1** A particle of unit mass moves on the x-axis, acted on by a force $F = 2v/t$, where $t \geq 1$ denotes time, and v denotes velocity. The initial conditions are $v(1) = 1$ and $x(1) = 0$. Find $x(t)$ for $t > 1$.

1.2 A unit-mass particle moves along the positive x-axis, subject to a force $F = v^2/x$, where v is its velocity. Given the initial conditions $v = x = 1$ at time $t = 0$, find $v(x)$ and hence $x(t)$.

1.3 A particle of mass $m = 1$ on the x-axis is attached to a spring with spring constant $k = 4$, and experiences a damping force of magnitude $4|\dot{x}|$. Find its position $x(t)$, given the initial conditions $x(0) = 1$ and $\dot{x}(0) = -1$.

Q2 **2.1** A particle of mass m, moving along a line with speed v, collides elastically with stationary particle of mass $3m$. The lighter particle then rebounds with speed u, while the heavier particle moves along the line with speed $w > 0$. Find the ratios u/v and w/v.

2.2 A particle of unit mass and unit charge moves in a magnetic field $\mathbf{B} = \mathbf{k}$ and an electric field $\mathbf{E} = 2\mathbf{k}$. Its position and velocity at time $t = 0$ are $\mathbf{r}(0) = \mathbf{0}$ and $\mathbf{v}(0) = \mathbf{i} + \mathbf{j}$ respectively. Determine its position $\mathbf{r}(t)$ for $t > 0$.

Q3 **3.1** A particle of mass $m = 1$ moves along the x-axis, subject to a force derived from the potential $V(x) = (x^2 + x - 1)\exp(-x)$.

(i) Find the equilibrium positions of the particle.

(ii) Calculate the period P of small oscillations about the stable equilibrium.

(iii) How large does the speed u of the particle at $x = 0$ need to be, for it to escape to $x = \infty$?

3.2 Particles of masses m and $2m$ move in opposite directions at the same speed along the x-axis. After colliding, the lighter particle moves at right angles to the x-axis, while the heavier one moves at an angle $\pi/6$ to the x-axis. Calculate the loss of energy in the collision, as a fraction β of the initial energy.

Q4 **4.1** Determine for which values of the constants α and β, if any, the force

$$\mathbf{F} = y\cosh(xy)\mathbf{i} + [\alpha z\sinh(yz) - \beta x\cosh(xy)]\mathbf{j} + y\sinh(yz)\mathbf{k}$$

is conservative. For those values, calculate the corresponding potential $V(x, y, z)$.

4.2 A unit-mass particle moves in an attractive central force of magnitude $5/r^2$, where (r, θ) are polar coordinates based at the centre of force. Initially its position and velocity are given by $\mathbf{r} = 2\mathbf{e}_r$ and $\mathbf{v} = \gamma\mathbf{e}_r + \mathbf{e}_\theta$ respectively, where γ is a positive constant.

(i) Using conservation of angular momentum and energy, determine for which values of γ the trajectory of the particle is bounded.

(ii) For which value of γ is the furthest distance of the particle from the centre of force equal to nine times the shortest distance?

Q5 **5.1** Find the solution $u(x,t)$ of the wave equation $4\,\partial^2 u/\partial t^2 = \partial^2 u/\partial x^2$, subject to the initial conditions $u(x,0) = 1/\cosh(2x)$ and $\partial u(x,0)/\partial t = \sinh(2x)/\cosh^2(2x)$.

5.2 A light rigid square plate of side-length L is pivoted at its centre so that it hangs vertically and can swing freely in the plane of the plate. Masses m are attached to three of its corners. Gravity acts downwards with acceleration g.

(i) Compute the moment of inertia I of the system about its pivot.

(ii) Obtain an expression for the energy E of the system, in terms of the angle $\theta(t)$ by which the plate deviates from its stable equilibrium.

(iii) The system starts from its stable equilibrium position with each mass having initial speed w. How large does w^2 have to be for the plate to rotate all the way round, passing its unstable equilibrium?