EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May/June	2023	MATH2627-WE01

Title:
Geometric Topology II

Time:	2 hours	
Additional Material provided:		
Materials Permitted:		
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.

Instructions to Candidates:	Answer all questions. Section A is worth 40% and Section B is worth 60%. Within each section, all questions carry equal marks. Students must use the mathematics specific answer book.

Revision:	

ED01/2023

SECTION A

Q1 In this question, L will always be an oriented 2-component link.

(a) Given a diagram D of L, explain how to compute the linking number $\mathrm{lk}(L)$ from D.

(b) Show that the linking number $\mathrm{lk}(L)$ thus computed does not depend on the given diagram D.

(c) Show that making fewer than $|\mathrm{lk}(L)|$ crossing changes to D (by crossing change we mean changing an undercrossing to an overcrossing) cannot result in a diagram of the unlink.

(d) Give an example of a link L with $\mathrm{lk}(L) = 2$ such that no diagram of L can be transformed into a diagram of the unlink by making 2 crossing changes.

Q2 (a) Draw a vector field on the open unit disc whose only singularity is at the origin and of index -1.

(b) Determine the winding number of the map

$$\gamma \colon S^1 \to S^1$$

given by

$$\gamma \colon z \mapsto \frac{z^3 + 4z}{|z^3 + 4z|}$$

where $S^1 \subset \mathbb{C}$ is the unit circle centered at the origin.

SECTION B

Q3 This question is about the Alexander-Conway polynomial $\nabla(z)$. In Figure 1 on the next page we define the link $K(p,q)$ where $p,q \in \mathbb{Z}$ are integers.

(a) Give the defining relations for the Alexander-Conway polynomial.

(b) Compute the Alexander-Conway polynomial $\nabla_{K(2,2)}(z)$ of the knot $K(2,2)$.

(c) Compute the Alexander-Conway polynomial $\nabla_{K(2022,2024)}(z)$ of the knot $K(2022,2024)$.

Q4 (a) We know that the trefoil has crossing number 3 and genus 1, while the Figure eight knot has crossing number 4 and genus 1. By considering the knot $K(2,q)$ for $q \in \mathbb{Z}$ as defined in Figure 1 on the next page, or otherwise, show that for every number $c \geq 3$ there exists a knot with crossing number c and genus 1.

(b) For each $n \geq 1$ you may assume that the knot $K(-1,2n)$ has crossing number $2n+1$ and genus n. By using connect sums of knots or otherwise, show that for every pair of positive integers $c,g > 0$ satisfying

$$2g + 1 \leq c$$

there exists a knot with crossing number c and genus g.

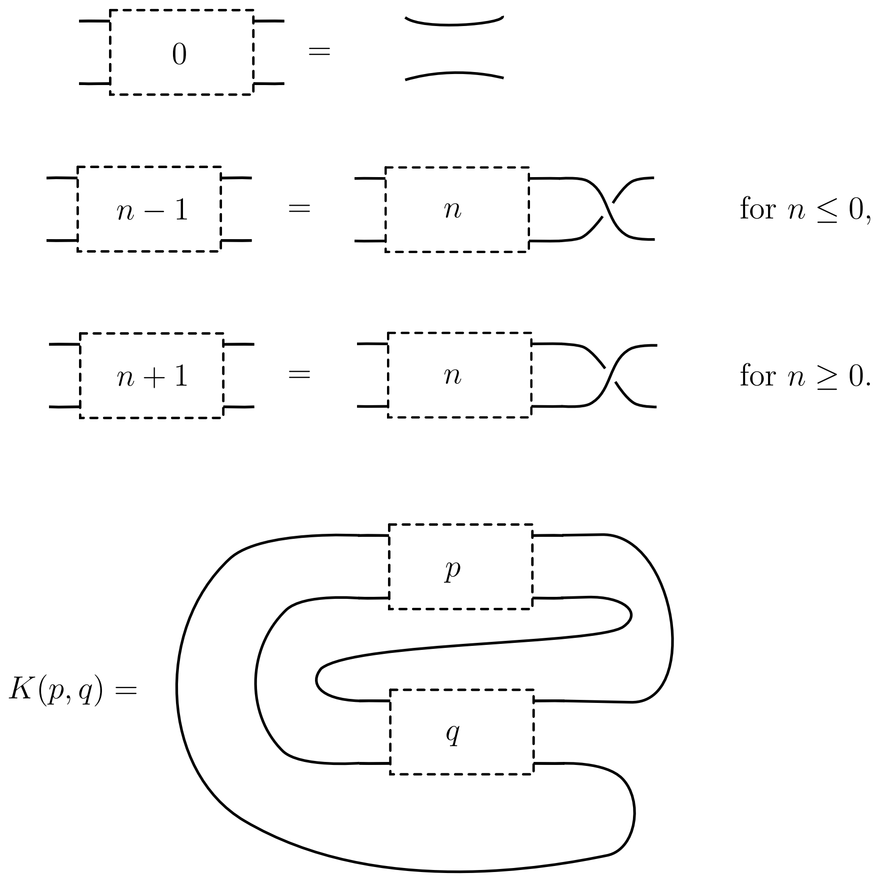

Figure 1: In this figure we define the link $K(p, q)$ for each pair of integers $p, q \in \mathbb{Z}$. We see that a box labelled n consists of n crossings, so that the diagram given of $K(p, q)$ has $p + q$ crossings in total.