MATH 3421: Formula Sheet

Exponential Exp($\boldsymbol{\lambda}$) distribution

If $X|\lambda \sim \text{Exp}(\lambda)$, then it has probability density function

$$f(x|\lambda) = \lambda e^{-\lambda x}\,, \quad x > 0, \quad \lambda > 0.$$

Also, $\text{E}(X|\lambda) = 1/\lambda$ and $\text{Var}(X|\lambda) = 1/\lambda^2$.

Gamma Gamma($\boldsymbol{\alpha, \beta}$) distribution

If $X|\alpha, \beta \sim \text{Gamma}(\alpha, \beta)$, then it has probability density function

$$f(x|\alpha,\beta) = \frac{\beta^\alpha x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)}\,, \quad x > 0, \quad \alpha > 0, \beta > 0.$$

Also, $\text{E}(X|\alpha,\beta) = \alpha/\beta$ and $\text{Var}(X|\alpha,\beta) = \alpha/\beta^2$.

Log-normal logN($\boldsymbol{\mu, \sigma^2}$) distribution

If $X|\mu, \sigma \sim \text{logN}(\mu, \sigma^2)$, then it has probability density function

$$f(x|\mu,\sigma) = \frac{1}{x\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(\log x - \mu)^2}{2\sigma^2}\right\}, \quad x > 0, \quad -\infty < \mu < \infty, \sigma > 0.$$

Also, $\text{E}(X|\mu,\sigma) = \exp(\mu + \sigma^2/2)$ and $\text{Var}(X|\mu,\sigma) = (\exp(\sigma^2) - 1)\exp(2\mu + \sigma^2)$.

Normal N($\boldsymbol{\mu, \sigma^2}$) distribution

If $X|\mu, \sigma \sim \text{N}(\mu, \sigma^2)$, then it has probability density function

$$f(x|\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x - \mu)^2}{2\sigma^2}\right\}, \quad -\infty < x < \infty, \quad -\infty < \mu < \infty, \sigma > 0.$$

Also, $\text{E}(X|\mu,\sigma) = \mu$ and $\text{Var}(X|\mu,\sigma) = \sigma^2$.

Poisson Pois($\boldsymbol{\lambda}$) distribution

If $X|\theta \sim \text{Pois}(\lambda)$, then it has probability mass function

$$\Pr(X = x|\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, \ldots, \quad \lambda > 0.$$

Also, $\text{E}(X|\lambda) = \lambda$ and $\text{Var}(X|\lambda) = \lambda$.

Uniform $\mathrm{U}(a, b)$ distribution

If $X|a, b \sim \mathrm{U}(a, b)$, then it has probability density function

$$f(x|a, b) = \frac{1}{b-a}, \quad a < x < b, \quad -\infty < a < b < \infty.$$

Also, $\mathrm{E}(X|a, b) = (a+b)/2$ and $\mathrm{Var}(X|a, b) = (b-a)^2/12$.