Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May/June	2024	MATH1031-WE01

Title:
Discrete Mathematics

Time:	2 hours	
Additional Material provided:		
Materials Permitted:		
Calculators Permitted:	Yes	Models Permitted: Casio FX83 series or FX85 series.

Instructions to Candidates:	Credit will be given for your answers to each question. All questions carry the same marks. Students must use the mathematics specific answer book.

Revision:	

ED01/2024

Q1 (a) Recall that a derangement of a word is an arrangement of the letters in which no letter is in the correct position. Use the inclusion-exclusion principle to find the number of derangements of GAMMA.

(b) How many solutions $(x_1, x_2, x_3, x_4, x_5)$ are there to the equation

$$x_1 + x_2 - x_3 - x_4 + x_5 = 2$$

where each x_i is an integer and

$$x_1 \leq 2, \quad x_2 \leq 3, \quad x_3 \geq -3, \quad x_4 \geq 1, \quad x_5 \leq 5?$$

Q2 (a) Kate is arranging her building blocks end-to-end in a line. The blocks are of three types: one of length 1 cm, the second of length 3 cm, and the other of length 4 cm. Assuming Kate has an unlimited supply of all types of block, let a_n denote the number of ways in which she can make a line of length n in centimetres.

(i) Evaluate directly a_1, a_2, a_3, and a_4.

(ii) Write down a recurrence relation for a_n, $n > 4$.

(iii) Let $g(x)$ denote the generating function $\sum_{n=0}^{\infty} a_n x^n$. Use your recurrence relation from part (ii) to derive an expression for $g(x)$ as the reciprocal of a polynomial.

(b) Solve the recurrence relation

$$a_n = -4a_{n-1} - 3a_{n-2} - 18 + 8n, \quad n \geq 2,$$

with initial conditions $a_0 = +1$ and $a_1 = 0$.

Q3 (a) Kate chooses fruit for the party. She can choose from plums, cherries and apples. All plums are the same, as are all cherries, and there are 4 types of apple. She will choose at least 3 plums, at least 5 cherries and between 2 and 6 apples of each type.

Let d_n denote the number of ways in which Kate can select the n fruit.

(i) Write down a generating function for d_n and express it as compactly as possible.

(ii) Use your generating function to find d_{28}.

(b) For any integer $n \geq 0$ prove by induction

$$(2n)! > 3^{n-1}(n!)^2.$$

Q4 Let K_n be the complete graph on $n \geq 3$ vertices. Justify your answers to the following questions.

(a) For which values of n is K_n Hamiltonian?

(b) For which values of n is K_n Eulerian?

(c) For which values of n is K_n planar?

(d) Show that K_{2n} is a union of n edge-disjoint walks in K_{2n}.