Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May/June	2024	MATH1607-WE01

Title: Dynamics I

Time:	2 hours	
Additional Material provided:		
Materials Permitted:		
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.

Instructions to Candidates:	Credit will be given for your answers to each question. All questions carry the same marks. Students must use the mathematics specific answer book.

Revision:	

Q1 **1.1** A particle of mass $m = 2$ moves along the positive x-axis, acted on by a force $F = v^2/x + v$, where v is its velocity. The initial conditions are $v = x = 1$ at time $t = 0$. Determine $v(x)$ and hence $x(t)$.

1.2 A particle of unit mass and unit charge moves in a magnetic field $\mathbf{B} = 2\mathbf{k}$ and an electric field $\mathbf{E} = -E\mathbf{k}$, where E is a positive constant. Its position and velocity at time $t = 0$ are $\mathbf{r}(0) = \mathbf{k}$ and $\mathbf{v}(0) = \mathbf{i}$ respectively. For what values of E, if any, does the particle reach $\mathbf{r} = \mathbf{0}$?

Q2 **2.1** A unit-mass particle on the x-axis is attached to a spring with spring constant k, and is subject to an additional force $F(t) = \cos^2(t) - \sin^2(t)$. Determine the value of k for which the system resonates, and for that value calculate the position $x(t)$ of the particle. The initial conditions are $x(0) = 1$ and $\dot{x}(0) = 0$.

2.2 A particle of mass m, moving along the x-axis with speed v, collides with a particle of mass M moving with speed $2v$ in the opposite direction. They coalesce and continue moving along the x axis with $1/10$ of the original total energy. Find two possible values for the ratio $R = m/M$.

Q3 **3.1** A unit-mass particle moves along the positive x-axis in a potential $V(x) = 2/x - 8/(x + 2)$.

(i) Calculate the period P of small oscillations about the equilibrium position of the particle.

(ii) If the particle is released from rest at $x = 1$, what is its speed u as it passes the equilibrium position?

3.2 A particle of mass M moving along the x-axis collides elastically with two particles, each of mass m, stationary at $x = 0$. After the collision, the M-particle is at rest, while each m-particle is moving at an angle ϕ to the x-axis. Find an expression for the ratio $R = M/m$ in terms of ϕ.

Q4 **4.1** Determine for which values of the constants a, b and c, if any, the force

$$\mathbf{F} = [y\mathbf{i} + x(b + ayz)\mathbf{j} + cxy^2\mathbf{k}] \exp(ayz)$$

is conservative. For those values, calculate the corresponding potential $V(x, y, z)$.

4.2 A unit-mass particle moves in an attractive central force of magnitude $\lambda^2 r$, where λ is a positive constant. Its initial position and velocity are $\mathbf{r}(0) = \mathbf{e}_r$ and $\mathbf{v}(0) = u\mathbf{e}_\theta$ respectively, where u is a constant with $0 < u < \lambda$.

(i) Write the equation of energy conservation in the form $\dot{r}^2 = W(r)$, where $W(r)$ is a function which you should determine and sketch.

(ii) Find the maximum and minimum values of r in the orbit of the particle, as functions of u and λ.

Q5 **5.1** Find the solution $u(x,t)$ of the wave equation

$$9\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2},$$

subject to the initial conditions

$$u|_{t=0} = \frac{3}{1+x^4}\,, \qquad \left.\frac{\partial u}{\partial t}\right|_{t=0} = \frac{4x^3}{(1+x^4)^2}.$$

5.2 A uniform straight rod of mass M and length L is pivoted at one end, and can swing freely in a vertical plane. A mass $2M$ is at attached at the other end, and a mass $3M$ is attached in the middle. Gravity acts downwards with acceleration g.

(i) What is the distance D from the pivot to the centre of mass of this system? (For purposes of computing D, you may regard the rod itself as a single particle.)

(ii) Compute the moment of inertia I of the system about its pivot.

(iii) Obtain an expression for the energy E, in terms of the angle $\theta(t)$ by which the system deviates from its stable equilibrium.

(iv) If the rod is released from rest in a horizontal position, what is the speed u of its free end when it is vertical?