

Cylindrical coordinates (r, θ, x)

- Continuity equation (for an incompressible fluid)

$$\frac{1}{r} \frac{\partial (ru_r)}{\partial r} + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_x}{\partial x} = 0$$

- Momentum equation (for an incompressible fluid)

– r -component

$$\begin{aligned} \rho \left(\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + u_x \frac{\partial u_r}{\partial x} \right) \\ = -\frac{\partial p}{\partial r} + \left(\frac{1}{r} \frac{\partial}{\partial r} (r\tau_{rr}) + \frac{1}{r} \frac{\partial \tau_{\theta r}}{\partial \theta} - \frac{\tau_{\theta\theta}}{r} + \frac{\partial \tau_{xr}}{\partial x} \right) + \rho g_r \end{aligned}$$

– θ -component

$$\begin{aligned} \rho \left(\frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} + u_x \frac{\partial u_\theta}{\partial x} \right) \\ = -\frac{1}{r} \frac{\partial p}{\partial \theta} + \left(\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \tau_{r\theta}) + \frac{1}{r} \frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{\theta x}}{\partial x} + \frac{\tau_{\theta r} - \tau_{r\theta}}{r} \right) + \rho g_\theta \end{aligned}$$

– x -component

$$\begin{aligned} \rho \left(\frac{\partial u_x}{\partial t} + u_r \frac{\partial u_x}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_x}{\partial \theta} + u_x \frac{\partial u_x}{\partial x} \right) \\ = -\frac{\partial p}{\partial x} + \left(\frac{1}{r} \frac{\partial}{\partial r} (r\tau_{rx}) + \frac{1}{r} \frac{\partial \tau_{\theta x}}{\partial \theta} + \frac{\partial \tau_{xx}}{\partial x} \right) + \rho g_x \end{aligned}$$

In the above, ρ denotes the fluid density, $\vec{u} = (u_r, u_\theta, u_x)$ is the fluid velocity, t is time, p represents the pressure, τ_{ij} denotes the ij -component of the stress tensor, $\vec{g} = (g_r, g_\theta, g_x)$ represent external force densities.