

EXAMINATION PAPER

Examination Session: May/June

2025

Year:

Exam Code:

MATH1081-WE01

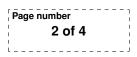
Title:

Calculus I (Maths Hons)

Time:	3 hours	
Additional Material provided:		
Materials Permitted:		
Calculators Permitted:	No	Models Permitted: Use of electronic calculators
		is forbidden.

Instructions to Candidates:	Credit will be given for your answers to each question. All questions carry the same marks.		
	Write your answer in the white-covered answer booklet with barcodes.		
	Begin your answer to each question on a new page.		

Revision:





Q1 (a) Calculate

$$\lim_{x \to 1} \frac{x^2 - 1}{x^2 - x}.$$

(b) Use the limit definition of the derivative to calculate the derivative of $f(x) = \sqrt{x}$.

 $\mathbf{Q2} \ \, \mathrm{Let}$

$$f(x) = \begin{cases} 2 - x^2 + |2x - x^2| & \text{for } |x| \le 3, \\ x & \text{for } 3 < |x| \le 5. \end{cases}$$

Find the global extremal values of f(x) on the interval [-5,5].

Q3 Let D be the intersection of the disk of radius 2 centered on the origin and the upper half-plane with $y \ge 0$. Calculate

$$\iint_D x(x+y) \ dA$$

Q4 (a) Find the general solution y(x) to the ordinary differential equation

$$y'' + y = 2e^{-x}.$$

(b) Find the general solution y(x) to the ordinary differential equation

$$y'' + y = \frac{1}{\sin\left(x\right)}.$$

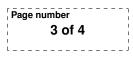
 $\mathbf{Q5}$ (a) Use integration by parts to calculate

$$I_n = \int_{-\pi}^{\pi} \cosh(x) \cos(nx) \, dx$$

for each integer $n \ge 0$.

(b) By evaluating the Fourier series of $f(x) = \cosh(x)$ at x = 0, show that

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{(n^2+1)} = \frac{\pi}{2\sinh(\pi)}$$





- **Q6** (a) A curve C in the x, y plane is given parametrically as $(x, y) = (t^3 t, t^2 + t)$. Given that at the point (x, y) = (0, 2), the function f(x, y) is such that $f_x = 2$, $f_y = 3$, use the Chain Rule to calculate $\frac{df}{dt}$ when t = 1.
 - (b) Find and classify the stationary points of the function

$$f(x,y) = 2\ln(x) + x^{2} + xy + \frac{y^{2}}{2} + x + 4y.$$

 $\mathbf{Q7}$ (a) A two-dimensional surface is given by the equation

$$f(x, y, z) \equiv x^3 + 3xy + 2yz + z^2 = -2.$$

Find the equation of the tangent plane to this surface at the point (1, -1, 2).

(b) Find the Taylor expansion of the function

$$g(x,y) = ye^{\cos\left(\frac{x}{2}\right)}$$

about the point $(\pi, 1)$ to quadratic order.

Q8 A function y(x) satisfies the differential equation

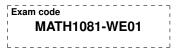
$$(x^2+1)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = \lambda y$$

where λ is a constant.

- (a) Explain briefly why you would expect that it is possible to write the solution in the form of a power series $y(x) = \sum_{n=0}^{\infty} a_n x^n$.
- (b) Find a recurrence relation satisfied by the coefficients a_n in the power series for y(x).
- (c) Use the recurrence relation from part (b) to find the value of λ for which the differential equation admits a solution in the form of a fifth order polynomial,

$$y(x) = x + \alpha x^3 + \beta x^5.$$

Here α and $\beta \neq 0$ are constants whose values you should determine to find the solution y(x).



Q9 A function u(x,t) satisfies the partial differential equation

$$u_t + u = 4u_{xx}$$

and the boundary conditions $u(0,t) = u(\pi,t) = 0$.

(a) For each integer $n \ge 0$, find the value of μ_n for which

$$u_n(x,t) = \sin(nx)e^{\mu_n t}$$

is a solution both to the partial differential equation and the boundary conditions given above.

(b) Find the solution to the partial differential equation satisfying the boundary conditions as a linear combination of the solutions from part (a) $u(x,t) = \sum_{n=1}^{\infty} A_n u_n(x,t)$ by determining the coefficients A_n if at time t = 0

$$u(x,0) = \begin{cases} \sin(\alpha x) & \text{ for } 0 < x < \frac{\pi}{\alpha} \\ 0 & \text{ for } \frac{\pi}{\alpha} < x < \pi \end{cases}$$

where α is a constant greater than one and not an integer. It may help to use the trigonometric identity

$$\sin(\alpha x)\sin(\beta x) = \frac{1}{2}\cos((\alpha - \beta)x) - \frac{1}{2}\cos((\alpha + \beta)x).$$

- **Q10** (a) Write down the definition of the Fourier transform $\tilde{f}(p)$ of the function f(x), and also the equation which gives f(x) in terms of its Fourier transform $\tilde{f}(p)$.
 - (b) You are given that the Fourier transform of $f(x) = \frac{1}{\cosh(x)}$ is

$$\tilde{f}(p) = \frac{\pi}{\cosh\left(\frac{\pi p}{2}\right)}$$

By using the Scaling theorem, find a function g(x) such that $\tilde{g}(p) = \Lambda g(p)$ where you should explicitly find the constant Λ .

(c) By considering the Fourier transform of $\frac{d^2f}{dx^2}$ where f(x) is the function given in part (b), find the Fourier transform of $(\cosh(x))^{-3}$.