DURHAM UNIVERSITY

Formula sheet given to candidates taking **Mathematics for Engineers & Scientists (MATH1551)**

TRIGONOMETRIC FUNCTIONS

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

$$\cos^2 A + \sin^2 A = 1$$

$$1 + \tan^2 A = \sec^2 A$$

$$1 + \cot^2 A = \text{cosec}^2 A$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

HYPERBOLIC FUNCTIONS

$$\cosh x = \frac{1}{2}\left(e^x + e^{-x}\right)$$

$$\sinh x = \frac{1}{2}\left(e^x - e^{-x}\right)$$

$$\cosh^{-1} x = \ln\left(x \pm \sqrt{x^2-1}\right)$$

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$$

$$\cosh(iA) = \cos A$$

$$\sinh(iA) = i\sin A$$

$$\cosh^2 A - \sinh^2 A = 1$$

$$\cosh(A+B) = \cosh A\cosh B + \sinh A\sinh B$$

$$\sinh(A+B) = \sinh A\cosh B + \cosh A\sinh B$$

$$\tanh(A+B) = \frac{\tanh A + \tanh B}{1 + \tanh A\tanh B}$$

ELEMENTARY RULES FOR DIFFERENTIATION AND INTEGRATION

$$(u+v)' = u'+v' \qquad (uv)' = u'v+uv' \qquad \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \qquad (u(v))' = u'(v)v' \qquad \int u'v\,dx = uv - \int uv'\,dx$$

TAYLOR'S THEOREM

Taylor approximation: $\quad f(x) \approx p_{n,a}(x) = f(a) + f'(a)(x-a) + \cdots + \frac{1}{n!}f^{(n)}(a)(x-a)^n$

and if $\;|f^{(n+1)}(x)| \le M$ for $c \le x \le b$ then $|f(x) - p_{n,a}(x)| \le \frac{|x-a|^{n+1}}{(n+1)!}M.$

MULTIVARIABLE DIFFERENTIAL OPERATORS

For scalar field $f(\mathbf{x}) = f(x,y,z)$ and vector field $\mathbf{A}(\mathbf{x}) = (A_1(x,y,z), A_2(x,y,z), A_3(x,y,z))$,

$$\textbf{grad}\, f = \nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

$$\textbf{div}\,\mathbf{A} = \nabla \bullet \mathbf{A} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \bullet (A_1, A_2, A_3) = \frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial y} + \frac{\partial A_3}{\partial z}$$

$$\textbf{curl}\,\mathbf{A} = \nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix} = \left(\frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z}\right)\mathbf{i} + \left(\frac{\partial A_1}{\partial z} - \frac{\partial A_3}{\partial x}\right)\mathbf{j} + \left(\frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y}\right)\mathbf{k}$$

TABLE OF DERIVATIVES / TABLE OF INTEGRALS

$y(x)$	dy/dx	$f(x)$	$\int f(x)\,dx$
x^n	nx^{n-1}	x^n	$\frac{x^{n+1}}{n+1},\ (n \neq -1)$
$\ln x$	x^{-1}	x^{-1}	$\ln\lvert x\rvert$
e^x	e^x	e^x	e^x
$\sin x$	$\cos x$	$\sin x$	$-\cos x$
$\cos x$	$-\sin x$	$\cos x$	$\sin x$
$\tan x$	$\sec^2 x$	$\tan x$	$-\ln\cos x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x\ \cot x$	$\operatorname{cosec} x$	$-\ln(\operatorname{cosec} x + \cot x)$
$\sec x$	$\sec x \tan x$	$\sec x$	$\ln(\sec x + \tan x)$
$\cot x$	$-\operatorname{cosec}^2 x$	$\cot x$	$\ln\sin x$
$\sinh x$	$\cosh x$	$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$	$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$	$\tanh x$	$\ln\cosh x$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right),\ (a > x)$
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$		
$\tan^{-1} x$	$\frac{1}{1+x^2}$	$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$	$\frac{1}{\sqrt{a^2+x^2}}$	$\sinh^{-1}\left(\frac{x}{a}\right)$
$\cosh^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$	$\frac{1}{\sqrt{x^2-a^2}}$	$\cosh^{-1}\left(\frac{x}{a}\right),\ (x > a)$

CRITICAL POINTS

For a function $f(x, y)$,

Local maximum: $f_x = f_y = 0$, $f_{xx}f_{yy} - f_{xy}^2 > 0$, and $f_{xx} < 0$.

Local minimum: $f_x = f_y = 0$, $f_{xx}f_{yy} - f_{xy}^2 > 0$, and $f_{xx} > 0$.

Saddle point: $f_x = f_y = 0$ and $f_{xx}f_{yy} - f_{xy}^2 < 0$.

Inconclusive: $f_x = f_y = 0$ and $f_{xx}f_{yy} - f_{xy}^2 = 0$.

ITERATION METHODS

To solve $A\mathbf{x} = \mathbf{b}$, write $A = D - L - U$, $T_j = D^{-1}(L + U)$, $T_g = (D - L)^{-1}U$.

Jacobi's Method:

$$D\mathbf{x}^{(k+1)} = \mathbf{b} + (L + U)\mathbf{x}^{(k)}, \qquad \mathbf{x}^{(k+1)} = D^{-1}\mathbf{b} + T_j\mathbf{x}^{(k)}$$

Gauss-Seidel Method:

$$D\mathbf{x}^{(k+1)} = \mathbf{b} + L\mathbf{x}^{(k+1)} + U\mathbf{x}^{(k)}, \qquad \mathbf{x}^{(k+1)} = (D - L)^{-1}\mathbf{b} + T_g\mathbf{x}^{(k)}$$

SOR Method:

$$\mathbf{x}^{(k+1)} = (1 - \omega)\mathbf{x}^{(k)} + \omega D^{-1}\left(\mathbf{b} + L\mathbf{x}^{(k+1)} + U\mathbf{x}^{(k)}\right)$$