

## EXAMINATION PAPER

Examination Session: May/June Year: 2025

Exam Code:

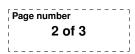
MATH1551-WE01

Title:

## Maths For Engineers and Scientists

Time:	3 hours	
Additional Material provided:	Formula sheet	
Materials Permitted:		
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.

**Revision:** 



 $\mathbf{Q1}$  Consider the matrix

$$A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 3 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

- (a) Determine whether A is strictly diagonally dominant and positive definite. Justify your answer.
- (b) Consider the system  $A\mathbf{x} = \mathbf{b}$  where  $\mathbf{b} = \begin{pmatrix} 3\\1\\3 \end{pmatrix}$ . Are the Jacobi and the Gauss-

Seidel iterative methods guaranteed to converge for any starting values,  $\mathbf{x}^{(0)}$ ? Justify your answer.

- (c) Write down the Jacobi and the Gauss-Seidel iteration schemes for the system in part (b).
- (d) Find the LU decomposition of A, and hence solve the system in part (b).
- **Q2** (a) Consider the complex numbers  $z = \sqrt{3} + i$  and w = -1 i. Calculate the modulus and the principal argument (in terms of  $\pi$ ) of zw.
  - (b) Find all complex solutions of the equation  $\sin z = 4i \cos z$ , expressing your answers in the form z = x + iy where  $x, y \in \mathbb{R}$ .
  - (c) Sketch the set of numbers z in the complex plane satisfying  $|z 2i| \leq \text{Im}(z)$ . Justify your answer.

**Q3** (a) Find a vector **n** normal to both vectors  $\mathbf{u} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$  and  $\mathbf{v} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ .

- (b) Find a Cartesian equation of the plane passing through the point  $\mathbf{a} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$  and being perpendicular to  $\mathbf{n}$ .
- (c) Calculate the shortest distance between the plane found in part (b) and the parallel plane x 3y + z = 24.
- Q4 (a) Calculate the eigenvalues and corresponding eigenspaces of the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$

- (b) From your answer to part (a), do you expect A to be diagonalisable? Explain your reasoning.
- (c) Using your answer to part (a), find a formula for  $A^k$  where k is a positive integer. Express your answer as a single matrix.

- **Q5** (a) Calculate the limit of the sequence  $s_n = n^2 \left(\frac{1}{n} \frac{2}{2n+3}\right)$  as  $n \to \infty$ .
  - (b) Use l'Hôpital's Rule and the standard result  $\lim_{x \to 0} \frac{\sin x}{x} = 1$  to evaluate the limit  $\cos x = \cos(2x)$

$$\lim_{x \to 0} \frac{\cos x - \cos(2x)}{\sin(x^2)}.$$

(c) Without using l'Hôpital's Rule, evaluate the limits

$$\lim_{x \to -1} \left( \frac{1+x^3}{1-x^2} \right) \quad \text{and} \quad \lim_{x \to \infty} \left( x\sqrt{x^2+3} - x^2 \right).$$

- **Q6** Consider the function  $f(x) = x^{1/3}$ .
  - (a) Find the degree 2 Taylor polynomial p(x) of f(x) about the point x = 1.
  - (b) Use your result to evaluate the limit

$$\lim_{x \to 1} \frac{3f(x) - x - 2}{(x - 1)^2}.$$

(c) Use the bound on the remainder term to show that

$$|f(x) - p(x)| \le \frac{1}{16200}$$
 for all  $1 \le x \le 1.1$ .

**Q7** (a) Calculate the divergence  $\nabla \cdot \mathbf{A}$  and curl  $\nabla \times \mathbf{A}$  of the vector-valued function

$$\mathbf{A}(x, y, z) = \left(\sin(xy), \sin(yz), \sin(zx)\right).$$

(b) Consider the surface in 3-dimensional space  $\mathbb{R}^3$  defined by the equation

$$x^2 + y^2 - 2yz = 1$$

and let  $\mathbf{p}$  be the point (1, 0, 1) on the surface.

- (i) Find Cartesian equations for the tangent plane of the surface at **p** and for the normal line to the surface at **p**.
- (ii) This normal line intersects the surface at p and one other point. Determine the coordinates of this other point.
- **Q8** (a) Find the solution y(x) of the first order differential equation

 $y' + 2xy = e^{-x^2} \sin x$  with initial condition y(0) = 2.

(b) Find the solution y(x) of the second order differential equation

 $y'' + 4y = 4\sin(2x)$  with initial conditions y(0) = y'(0) = 0.