Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May/June	2025	MATH1551-WE01

Title:

Maths For Engineers and Scientists

Time:	3 hours	
Additional Material provided:	Formula sheet	
Materials Permitted:		
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.

Instructions to Candidates:	Credit will be given for your answers to each question. All questions carry the same marks. Write your answer in the white-covered answer booklet with barcodes. Begin your answer to each question on a new page.

Revision:	

ED01/2025

Q1 Consider the matrix

$$A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 3 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

(a) Determine whether A is strictly diagonally dominant and positive definite. Justify your answer.

(b) Consider the system $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} = \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix}$. Are the Jacobi and the Gauss-Seidel iterative methods guaranteed to converge for any starting values, $\mathbf{x}^{(0)}$? Justify your answer.

(c) Write down the Jacobi and the Gauss-Seidel iteration schemes for the system in part (b).

(d) Find the LU decomposition of A, and hence solve the system in part (b).

Q2 (a) Consider the complex numbers $z = \sqrt{3} + i$ and $w = -1 - i$. Calculate the modulus and the principal argument (in terms of π) of zw.

(b) Find all complex solutions of the equation $\sin z = 4i \cos z$, expressing your answers in the form $z = x + iy$ where $x, y \in \mathbb{R}$.

(c) Sketch the set of numbers z in the complex plane satisfying $|z - 2i| \leq \operatorname{Im}(z)$. Justify your answer.

Q3 (a) Find a vector $\mathbf{n}$ normal to both vectors $\mathbf{u} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$.

(b) Find a Cartesian equation of the plane passing through the point $\mathbf{a} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and being perpendicular to $\mathbf{n}$.

(c) Calculate the shortest distance between the plane found in part (b) and the parallel plane $x - 3y + z = 24$.

Q4 (a) Calculate the eigenvalues and corresponding eigenspaces of the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$

(b) From your answer to part (a), do you expect A to be diagonalisable? Explain your reasoning.

(c) Using your answer to part (a), find a formula for A^k where k is a positive integer. Express your answer as a single matrix.

Q5 (a) Calculate the limit of the sequence $s_n = n^2\left(\frac{1}{n} - \frac{2}{2n+3}\right)$ as $n \to \infty$.

(b) Use l'Hôpital's Rule and the standard result $\lim_{x\to 0} \frac{\sin x}{x} = 1$ to evaluate the limit

$$\lim_{x\to 0} \frac{\cos x - \cos(2x)}{\sin(x^2)}.$$

(c) *Without* using l'Hôpital's Rule, evaluate the limits

$$\lim_{x\to -1} \left(\frac{1+x^3}{1-x^2}\right) \qquad \text{and} \qquad \lim_{x\to\infty} \left(x\sqrt{x^2+3} - x^2\right).$$

Q6 Consider the function $f(x) = x^{1/3}$.

(a) Find the degree 2 Taylor polynomial $p(x)$ of $f(x)$ about the point $x = 1$.

(b) Use your result to evaluate the limit

$$\lim_{x\to 1} \frac{3f(x) - x - 2}{(x-1)^2}.$$

(c) Use the bound on the remainder term to show that

$$|f(x) - p(x)| \le \frac{1}{16200} \qquad \text{for all } 1 \le x \le 1.1.$$

Q7 (a) Calculate the divergence $\nabla \cdot \mathbf{A}$ and curl $\nabla \times \mathbf{A}$ of the vector-valued function

$$\mathbf{A}(x,y,z) = \big(\sin(xy), \sin(yz), \sin(zx)\big).$$

(b) Consider the surface in 3-dimensional space $\mathbb{R}^3$ defined by the equation

$$x^2 + y^2 - 2yz = 1$$

and let $\mathbf{p}$ be the point $(1,0,1)$ on the surface.

(i) Find Cartesian equations for the tangent plane of the surface at $\mathbf{p}$ and for the normal line to the surface at $\mathbf{p}$.

(ii) This normal line intersects the surface at $\mathbf{p}$ and one other point. Determine the coordinates of this other point.

Q8 (a) Find the solution $y(x)$ of the first order differential equation

$$y' + 2xy = e^{-x^2} \sin x \qquad \text{with initial condition } y(0) = 2.$$

(b) Find the solution $y(x)$ of the second order differential equation

$$y'' + 4y = 4\sin(2x) \qquad \text{with initial conditions } y(0) = y'(0) = 0.$$