Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May/June	2025	MATH1607-WE01

Title: Dynamics I

Time:	2 hours	
Additional Material provided:		
Materials Permitted:		
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.

Instructions to Candidates:	Credit will be given for your answers to each question. All questions carry the same marks. Write your answer in the white-covered answer booklet with barcodes. Begin your answer to each question on a new page.

Revision:	

ED01/2025

Q1 (a) A unit-mass particle on the x-axis is acted on by a force $F = (t+1)/v$, where v denotes its velocity. The initial conditions are $x = 0$ and $v = 1$ at time $t = 0$. Solve the equation of motion to find $x(t)$.

(b) A particle of mass $m = 2$ on the x-axis is attached to a spring with spring constant k, and also experiences a damping force of magnitude $4|\dot{x}|$. Determine the value of k which gives critical damping, and find $x(t)$ for that value. The initial conditions are $x(0) = 1$ and $\dot{x}(0) = 0$.

Q2 (a) A particle of mass $m = 2$ moves on the x-axis in a potential $V(x) = x^2 \mathrm{e}^x$. Calculate the period P of small oscillations about its stable equilibrium.

(b) A particle with unit mass and unit charge moves in a magnetic field $\mathbf{B} = \mathbf{e}_3$ and an electric field $\mathbf{E} = E\mathbf{e}_3$, where E is constant. Its position and velocity at time $t = 0$ are $\mathbf{r}(0) = \mathbf{0}$ and $\mathbf{v}(0) = -\alpha\mathbf{E} + \mathbf{e}_1$ respectively, where α is a positive constant. Find its position $\mathbf{r}(t)$, and then determine for which values of α we get $\mathbf{r} = \mathbf{0}$ for some positive t.

Q3 (a) A unit-mass particle moves along the positive x-axis in a force $F = 2x^3$. By using conservation of energy, or otherwise, find its position $x(t)$. The initial conditions are $x(0) = 1$ and $\dot{x}(0) = 1$.

(b) A unit mass moving along the x-axis strikes a stationary mass M. After the collision, the unit mass is moving perpendicular to the x-axis, while the M-mass is moving at an angle $\pi/6$ to the x-axis. Given that half of the original kinetic energy was lost in the collision, determine M.

Q4 (a) Find a potential $V(x, y, z)$ corresponding to the force

$$\mathbf{F} = yz(y - 2x)\mathbf{e}_1 + xz(2y - x)\mathbf{e}_2 + xy(y - x)\mathbf{e}_3.$$

(b) A unit-mass particle moves in an attractive central force of magnitude $1/r^2$. Its initial position and velocity are $\mathbf{r}(0) = 2\mathbf{e}_r$ and $\mathbf{v}(0) = \mathbf{e}_r + \mathbf{e}_\theta$ respectively. Using conservation of energy, determine whether or not the trajectory is bounded, and find the minimum value r_0 of $r(t)$.

Q5 (a) Find the solution $u(x, t)$ of the wave equation $\partial^2 u/\partial t^2 = 4\,\partial^2 u/\partial x^2$, subject to the initial conditions $u(x, 0) = 2\,\mathrm{sech}(x)$ and $\partial u/\partial t|_{t=0} = 0$.

(b) A light rigid rod of length L is pivoted at one end, and is free to swing in a vertical plane. A mass m is attached to the other end, as well as another mass $4m$ halfway along the rod. Gravity acts downwards with acceleration g.

(i) Find the distance D from the pivot to the centre of mass of this system.

(ii) Compute the moment of inertia I of the system about its pivot.

(iii) Derive an expression for the total energy E, in terms of the angle $\theta(t)$ by which the rod deviates from its stable equilibrium.

(iv) If the rod is released from rest at $\theta = \pi/3$, what is the speed u of the free end of the rod when it passes its equilibrium position?