Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May/June	2025	MATH2657-WE01

Title: Special Relativity and Electromagnetism II

Time:	2 hours	
Additional Material provided:		
Materials Permitted:		
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.

Instructions to Candidates:	Answer all questions. Section A is worth 40% and Section B is worth 60%. Within each section, all questions carry equal marks. Write your answer in the white-covered answer booklet with barcodes. Begin your answer to each question on a new page.

Revision:	

ED01/2025

SECTION A

Q1 (a) The engines of the Millennium Falcon emit blue light (with a wavelength of 450 nanometres). Leia is on the planet Endor and watches Han leave the planet in the Millennium Falcon, flying directly away from her with its engines glowing red (wavelength 750 nanometres). Leia notices that Luke is also leaving the planet Endor, following the same route as the Millennium Falcon, but travelling in his X-wing fighter with a speed that is one third of the speed of light c.

Calculate the relative speed between Luke and Han, as a fraction of c.

(b) A particle has 4-momentum $p^\mu = (10ac, 0, 0, 8ac)$, where a is a positive constant and c denotes the speed of light. Calculate, in terms of a and c, the rest mass of the particle and its kinetic energy.

Q2 (a) Let ℓ be a positive constant. A point charge q is placed at the origin.

Find, in terms of q and the electric constant ε_0, the electric flux through the surface S,

$$\Psi = \int_S \mathbf{E} \cdot \mathbf{dS},$$

for the following two examples of S:

(i) S is the sphere of radius ℓ centred at the origin.

(ii) S is the square with vertices $(\pm\ell, -\ell, \ell)$ and $(\pm\ell, \ell, \ell)$.

(b) In an electrostatics problem, in a region of space the electric field is

$$\mathbf{E} = \lambda(\, y^2 + 3x^2,\, kxy,\, 3z^2\,),$$

where λ and k are positive constants.

Determine the value of k.

Calculate the electrostatic scalar potential, in terms of the spatial coordinates and the constant λ.

SECTION B

Q3 (a) The reference frame $\mathcal{R}'$ is obtained from the reference frame $\mathcal{R}$ by applying the Lorentz transformation

$$L = \frac{1}{16}\begin{bmatrix} 25 & -a & 15 & 0 \\ -15 & 20 & -9 & 0 \\ a & 0 & 20 & 0 \\ 0 & 0 & 0 & 16 \end{bmatrix}.$$

Determine the value of the constant a.

(b) Recall that in terms of the electric field $\mathbf{E} = (E_1, E_2, E_3)$ and the magnetic field $\mathbf{B} = (B_1, B_2, B_3)$, the components of the electromagnetic field tensor $F^{\mu\nu}$ are given by the entries of the matrix

$$F = \begin{pmatrix} 0 & -E_1/c & -E_2/c & -E_3/c \\ E_1/c & 0 & -B_3 & B_2 \\ E_2/c & B_3 & 0 & -B_1 \\ E_3/c & -B_2 & B_1 & 0 \end{pmatrix},$$

where c denotes the speed of light.

In the reference frame $\mathcal{R}$ there is a magnetic field $\mathbf{B} = (0, b, 0)$ but no electric field. Calculate, in terms of b and c, the electric and magnetic fields $\mathbf{E}'$ and $\mathbf{B}'$ in the reference frame $\mathcal{R}'$ given in the previous part of the question.

(c) Express the quantity $F^{\mu\nu}F_{\mu\nu}$ in terms of c and the magnitudes of the electric and magnetic fields. Verify that your result to the previous part of the question is consistent with the transformation properties of this quantity under the Lorentz transformation from the reference frame $\mathcal{R}$ to $\mathcal{R}'$.

Q4 (a) Electric charge is contained within a ball of radius R that is centred at the origin. The electric charge density vanishes outside the ball but inside it is given by

$$\rho = \frac{qb^2}{r}\sinh(br),$$

where q and b are positive constants, with $r = |\mathbf{r}|$ the distance to the origin.

Calculate the electric field at position $\mathbf{r}$ due to the ball, in terms of q, b, R, ε_0.

(b) Let f be a differentiable function of the distance to the origin r.

Calculate an expression, in terms of r, ε_0, f and its derivative f', for the electric charge density ρ that produces the radial electric field $\mathbf{E} = \mathbf{r}f$.