

MATH 3421: Formula Sheet

Gamma $\text{Gamma}(\alpha, \beta)$ distribution

If $X|\alpha, \beta \sim \text{Gamma}(\alpha, \beta)$, then it has probability density function

$$f(x|\alpha, \beta) = \frac{\beta^\alpha x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)}, \quad x > 0, \quad \alpha > 0, \beta > 0.$$

Also, $E(X|\alpha, \beta) = \alpha/\beta$ and $\text{Var}(X|\alpha, \beta) = \alpha/\beta^2$.

Inverse gamma $\text{InvGamma}(\alpha, \beta)$ distribution

If $X|\alpha, \beta \sim \text{InvGamma}(\alpha, \beta)$, then it has probability density function

$$f(x|\alpha, \beta) = \frac{\beta^\alpha x^{-\alpha-1} e^{-\beta/x}}{\Gamma(\alpha)}, \quad x > 0, \quad \alpha > 0, \beta > 0.$$

Also, $E(X|\alpha, \beta) = \beta/(\alpha - 1)$ for $\alpha > 1$ and $\text{Var}(X|\alpha, \beta) = \beta^2/\{(\alpha - 1)^2(\alpha - 2)\}$ for $\alpha > 2$.

Log-normal $\text{logN}(\mu, \sigma^2)$ distribution

If $X|\mu, \sigma \sim \text{logN}(\mu, \sigma^2)$, then it has probability density function

$$f(x|\mu, \sigma) = \frac{1}{x\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(\log x - \mu)^2}{2\sigma^2}\right\}, \quad x > 0, \quad -\infty < \mu < \infty, \sigma > 0.$$

Also, $E(X|\mu, \sigma) = \exp(\mu + \sigma^2/2)$ and $\text{Var}(X|\mu, \sigma) = \{\exp(\sigma^2) - 1\} \exp(2\mu + \sigma^2)$.

Normal $\text{N}(\mu, \sigma^2)$ distribution

If $X|\mu, \sigma \sim \text{N}(\mu, \sigma^2)$, then it has probability density function

$$f(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x - \mu)^2}{2\sigma^2}\right\}, \quad -\infty < x < \infty, \quad -\infty < \mu < \infty, \sigma > 0.$$

Also, $E(X|\mu, \sigma) = \mu$ and $\text{Var}(X|\mu, \sigma) = \sigma^2$.

Uniform $\text{U}(a, b)$ distribution

If $X|a, b \sim \text{U}(a, b)$, then it has probability density function

$$f(x|a, b) = \frac{1}{b - a}, \quad a < x < b, \quad -\infty < a < b < \infty.$$

Also, $E(X|a, b) = (a + b)/2$ and $\text{Var}(X|a, b) = (b - a)^2/12$.