

Continuity equation in cylindrical coordinates

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \rho u_r) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho u_\theta) + \frac{\partial}{\partial x} (\rho u_x) = 0$$

Momentum conservation law in cylindrical coordinates

r -component

$$\rho \left(\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + u_x \frac{\partial u_r}{\partial x} \right) = -\frac{\partial p}{\partial r} + \rho g_r + \left(\frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rr}) + \frac{1}{r} \frac{\partial \tau_{\theta r}}{\partial \theta} - \frac{\tau_{\theta\theta}}{r} + \frac{\partial \tau_{xr}}{\partial x} \right)$$

θ -component

$$\rho \left(\frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} + u_x \frac{\partial u_\theta}{\partial x} \right) = -\frac{1}{r} \frac{\partial p}{\partial \theta} + \rho g_\theta + \left(\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \tau_{r\theta}) + \frac{1}{r} \frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{x\theta}}{\partial x} \right)$$

x -component

$$\rho \left(\frac{\partial u_x}{\partial t} + u_r \frac{\partial u_x}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_x}{\partial \theta} + u_x \frac{\partial u_x}{\partial x} \right) = -\frac{\partial p}{\partial x} + \rho g_x + \left(\frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rx}) + \frac{1}{r} \frac{\partial \tau_{\theta x}}{\partial \theta} + \frac{\partial \tau_{xx}}{\partial x} \right)$$

Strain rate tensor for incompressible fluids in cylindrical coordinates

$$\dot{\gamma}_{rr} = 2 \frac{\partial u_r}{\partial r}$$

$$\dot{\gamma}_{\theta\theta} = 2 \left(\frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r} \right)$$

$$\dot{\gamma}_{xx} = 2 \frac{\partial u_x}{\partial x}$$

$$\dot{\gamma}_{r\theta} = \dot{\gamma}_{\theta r} = r \frac{\partial}{\partial r} \left(\frac{u_\theta}{r} \right) + \frac{1}{r} \frac{\partial u_r}{\partial \theta}$$

$$\dot{\gamma}_{\theta x} = \dot{\gamma}_{x\theta} = \frac{\partial u_\theta}{\partial x} + \frac{1}{r} \frac{\partial u_x}{\partial \theta}$$

$$\dot{\gamma}_{xr} = \dot{\gamma}_{rx} = \frac{\partial u_x}{\partial r} + \frac{\partial u_r}{\partial x}$$

Navier-Stokes equation in cylindrical coordinates

r -component

$$\rho \left(\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + u_x \frac{\partial u_r}{\partial x} \right) = -\frac{\partial p}{\partial r} + \rho g_r$$

$$+ \mu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r u_r) \right) + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_r}{\partial x^2} \right]$$

θ -component

$$\rho \left(\frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} + u_x \frac{\partial u_\theta}{\partial x} \right) = -\frac{1}{r} \frac{\partial p}{\partial \theta} + \rho g_\theta$$

$$+ \mu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r u_\theta) \right) + \frac{1}{r^2} \frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} + \frac{\partial^2 u_\theta}{\partial x^2} \right]$$

x -component

$$\rho \left(\frac{\partial u_x}{\partial t} + u_r \frac{\partial u_x}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_x}{\partial \theta} + u_x \frac{\partial u_x}{\partial x} \right) = -\frac{\partial p}{\partial x} + \rho g_x$$

$$+ \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_x}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_x}{\partial \theta^2} + \frac{\partial^2 u_x}{\partial x^2} \right]$$