



EXAMINATION PAPER

Examination Session: May/June	Year: 2026	Exam Code: MATH1031-WE01
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Title: Discrete Mathematics

Time:	2 hours	
Additional Material provided:	None	
Materials Permitted:	None	
Calculators Permitted:	Yes	Models Permitted: Casio FX83 series or FX85 series.

Instructions to Candidates:	<p>Answer all questions.</p> <p>The indicative marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply.</p> <p>Write your answer in the white-covered answer booklet with barcodes.</p> <p>Begin your answer to each question on a new page.</p>
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Revision:	
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1. (a) (i) How many arrangements are there of the letters **MATHEMATICS**? [3]
(ii) How many arrangements have the two M's adjacent? [3]
(iii) How many arrangements have no two adjacent letters the same? [7]
(b) Prove by induction that for any integer $n \geq 1$,

$$1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2.$$

Hint: First find the RHS. [12]

2. (a) Solve the recurrence relation

$$a_n = -5a_{n-1} - 6a_{n-2} + 12n - 5$$

with initial conditions $a_0 = 1$, $a_1 = 1$. [12]

- (b) A flower shop sells roses, lilies, peonies, tulips, azaleas and asters. Bouquets are made of between 2 and 7 flowers of each type and contain 22 flowers in total. In how many different ways can bouquets be constituted? [13]
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3. (a) Let

$$a_n = 0 + 1 + 2 + \cdots + n, \quad S_n = a_0 + a_1 + a_2 + \cdots + a_n, \quad (n \geq 0).$$

- (i) Find $f(x) = \sum_{n=0}^{\infty} a_n x^n$, the generating function for the sequence a_n . [5]
(ii) Find $g(x) = \sum_{n=0}^{\infty} S_n x^n$, the generating function for the sequence S_n . [5]
(iii) Find S_n (as a function of n). [5]

- (b) How many solutions $(x_1, x_2, x_3, x_4, x_5, x_6)$ are there to the equation

$$x_1 + x_2 + x_3 - x_4 - x_5 - x_6 = 25$$

where each x_i is an integer and

$$x_1 \geq 2, \quad x_2 \geq 2, \quad x_3 \geq 2; \quad x_4 \leq -3, \quad x_5 \leq -3, \quad x_6 \leq -3? \quad [10]$$

4. (a) (i) Justify whether the graphs G_1 and G_2 pictured below are isomorphic. [4]
(ii) Show that the graphs G_1 and G_3 are isomorphic by giving an explicit isomorphism between their vertex sets. [8]
(b) Show that G_2 is Hamiltonian by giving a Hamiltonian cycle. [5]
(c) Using Euler's Theorem, show that G_2 is Eulerian and explain why neither G_1 nor G_3 is Eulerian. [8]

