Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May/June	2026	MATH1551-WE01-SP

Title:
Maths For Engineers and Scientists (2024/25 syllabus)

Time:	3 hours	
Additional Material provided:	Formula sheet	
Materials Permitted:	None	
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.

Instructions to Candidates:	Answer all questions. The indicative marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply. Write your answer in the white-covered answer booklet with barcodes. Begin your answer to each question on a new page.

Revision:	

ED01/2026

1. Consider the following system of linear equations:

$$\begin{aligned} 3x + y + z &= 9, \\ x + 3y + z &= 11, \\ x + y + kz &= d, \end{aligned}$$

where k and d are real numbers.

(a) Show that the system does not have a unique solution when $k = \frac{1}{2}$. **[3]**

(b) When $k = \frac{1}{2}$, for which value(s) of d does the system have no solutions and for which value(s) of d does it have infinitely many solutions? **[3]**

(c) In the case of infinitely many solutions, find the solutions in parametric form. **[3]**

(d) Explain why Gauss-Seidel is guaranteed to converge for any starting values when $k = 1$ and $d = 1$ and write down the Gauss-Seidel iterative scheme for this case. **[3]**

---

2. (a) Find all complex solutions to $e^{8z} - 2e^{4z} + 2 = 0$. **[4]**

(b) Show that $|z - w|^2 = |z|^2 + |w|^2 - 2Re(z\overline{w})$ and hence show that when $|z| = |w| = 1$, $\left|\dfrac{z - w}{1 - \overline{z}w}\right| = 1$. **[5]**

(c) Sketch the region satisfying $|z - 2 - 2i| \leq 2$, $\text{Re}(z) \geq 1$. **[3]**

---

3. (a) (i) For which value(s) of c does the set $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ c \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}$ form a basis for $\mathbb{R}^3$? **[2]**

(ii) Find the coordinates of $\boldsymbol{u} = \begin{pmatrix} 3 \\ 7 \\ 4 \end{pmatrix}$ with respect to the basis in part (i). **[4]**

(b) Consider the plane P: $2x - y + 2z = 6$ and the line L:

$$\boldsymbol{x} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \quad t \in \mathbb{R}.$$

(i) Find a Cartesian equation of the plane that contains L and is perpendicular to P. **[4]**

(ii) Find the shortest distance between the point $(1, 2, 1)$ and plane P. **[2]**

---

4. You are given that the eigenvalues of the matrix $A = \begin{pmatrix} 97 & -70 \\ 105 & -78 \end{pmatrix}$ are $\lambda_1 = -8$ and $\lambda_2 = 27$.

(a) Find the eigenspaces corresponding to the eigenvalues and hence find a matrix Y such that $Y^{-1}AY$ is diagonal. **[5]**

(b) Find a matrix B that satisfies $B^3 = A$. **[4]**

(c) Find a formula for A^k where k is a positive even integer. Express your answer as a single matrix. **[3]**

---

5. (a) Calculate the limit of the following sequence as $n \to \infty$.

$$s_n = n\left(\frac{1}{4n+1} - \frac{1}{n^2+4}\right).$$ **[3]**

(b) Calculate the following limits or explain why they don't exist, stating any standard results you use.

$$\lim_{x\to\infty} \frac{1}{\sqrt{x^2+9x} - \sqrt{x^2+x}}, \qquad \lim_{x\to 0}\left(\frac{1}{x^2+2x} - \frac{\sin(2x)}{(x^2+2x)^2}\right).$$ **[9]**

---

6. (a) Using the definition of the derivative as a limit, show that $f(x) = \cos x$ is differentiable for all x and find its derivative $f'(x)$.

You may use the fact that $\lim_{t\to 0} \frac{\sin t}{t} = 1$ *without proof.* **[5]**

(b) Find the degree 2 Taylor polynomial of the function $f(x) = \sqrt{3+x}$ about the point $x = 0$. Estimate the maximum error in using this polynomial to estimate $f(x)$ over the interval $0 \le x \le 0.1$. **[7]**

---

7. (a) Evaluate the divergence $\nabla \cdot \mathbf{A}$ and curl $\nabla \times \mathbf{A}$ of the vector-valued function

$$\mathbf{A}(x,y,z) = (x+y+z, xy+yz+zx, xyz)$$

at the point $\mathbf{p} = (1,1,1)$. **[5]**

(b) Determine all critical points of the function

$$f(x,y) = x^3 + 2y^3 - 6x^2 - 6y^2 + 9x$$

and classify each as a local minimum, local maximum or saddle point. **[7]**

---

8. (a) By using the substitution $u = y^{-2}$ or otherwise, find the general solution of the first order differential equation

$$2x^2 \frac{dy}{dx} - 2xy = y^3 e^x.$$ [**5**]

(b) Find the general solution of the second order differential equation

$$y'' + 4y' + 4y = 4e^{-2x}$$

and hence find the solution satisfying $y(0) = y'(0) = 1$. [**7**]