Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May/June	2026	MATH1551-WE01

Title:
Maths For Engineers and Scientists

Time:	2 hours	
Additional Material provided:	Formula sheet	
Materials Permitted:	None	
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.

Instructions to Candidates:	Answer all questions. The indicative marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply. Write your answer in the white-covered answer booklet with barcodes. Begin your answer to each question on a new page.

Revision:	

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1. (a) Consider the following system of linear equations:

$$
\begin{aligned}
3x + y + z &= 9, \\
x + 3y + z &= 11, \\
x + y + kz &= d,
\end{aligned}
$$

where k and d are real numbers.

(i) Show that the system does not have a unique solution when $k = \frac{1}{2}$. **[4]**

(ii) When $k = \frac{1}{2}$, for which value(s) of d does the system have no solutions and for which value(s) of d does it have infinitely many solutions? **[4]**

(iii) In the case of infinitely many solutions, find the solutions in parametric form. **[4]**

(b) Find all complex solutions to $e^{8z} - 2e^{4z} + 2 = 0$. **[6]**

(c) Show that $|z - w|^2 = |z|^2 + |w|^2 - 2Re(z\overline{w})$ for $z, w \in \mathbb{C}$. Hence show that when $|z| = |w| = 1$, $\left|\dfrac{z - w}{1 - \overline{z}w}\right| = 1$. **[7]**

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2. (a) (i) For which value(s) of $c \in \mathbb{R}$ does the set $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ c \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}$ form a basis for $\mathbb{R}^3$? **[3]**

(ii) Find the coordinates of $\boldsymbol{u} = \begin{pmatrix} 3 \\ 7 \\ 4 \end{pmatrix}$ with respect to the basis in part (i). **[4]**

(b) Consider the plane P: $2x - y + 2z = 6$ and the line L:

$$\boldsymbol{x} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \quad t \in \mathbb{R}.$$

Find a Cartesian equation of the plane that contains L and is perpendicular to P. **[6]**

(c) You are given that the eigenvalues of the matrix $A = \begin{pmatrix} 97 & -70 \\ 105 & -78 \end{pmatrix}$ are $\lambda_1 = -8$ and $\lambda_2 = 27$.

(i) Find the eigenspaces corresponding to the eigenvalues and hence find a matrix Y such that $Y^{-1}AY$ is diagonal. **[7]**

(ii) Find a matrix B that satisfies $B^3 = A$. **[5]**

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3. (a) Calculate the following limits or explain why they don't exist, stating any standard results you use.

$$\lim_{x\to\infty} \frac{1}{\sqrt{x^2+9x}-\sqrt{x^2+x}}, \qquad \lim_{x\to 0}\left(\frac{1}{x^2+2x}-\frac{\sin(2x)}{(x^2+2x)^2}\right).$$ [8]

(b) Using the definition of the derivative as a limit, show that $f(x) = \cos x$ is differentiable for all x and find its derivative $f'(x)$.

You may use the fact that $\lim_{t\to 0}\frac{\sin t}{t} = 1$ *without proof.* [8]

(c) Find the degree 2 Taylor polynomial of the function $f(x) = \sqrt{3+x}$ about the point $x = 0$. Estimate the maximum error in using this polynomial to estimate $f(x)$ over the interval $0 \le x \le 0.1$. [9]

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4. (a) Determine all critical points of the function

$$f(x,y) = x^3 + 2y^3 - 6x^2 - 6y^2 + 9x$$

and classify each as a local minimum, local maximum or saddle point. [8]

(b) By using the substitution $u = y^{-2}$ or otherwise, find the general solution of the first order differential equation

$$2x^2\frac{dy}{dx} - 2xy = y^3 e^x.$$ [8]

(c) Find the general solution of the second order differential equation

$$y'' + 4y' + 4y = 4e^{-2x}$$

and hence find the solution satisfying $y(0) = y'(0) = 1$. [9]

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