Durham University

EXAMINATION PAPER

| Examination Session: | Year: | Exam Code: |
|---|---|---|
| May/June | 2026 | MATH1561-WE01-SP |

**Title:**

Single Mathematics A (2024/25 syllabus)

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| Time: | 3 hours | |
| Additional Material provided: | None | |
| Materials Permitted: | None | |
| Calculators Permitted: | No | Models Permitted: Use of electronic calculators is forbidden. |

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| Instructions to Candidates: | Answer all questions.<br>The indicative marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply.<br>Write your answer in the white-covered answer booklet with barcodes.<br>Begin your answer to each question on a new page. |

| Revision: | |
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ED01/2026

**1.** (a) Find the real and imaginary part of the complex numbers

$$\frac{7-i}{2-3i} \qquad \text{and} \qquad e^{-i\pi/4}.$$

[Your answers should be simplified, not left in terms of cos or sin functions.] **[6]**

(b) Differentiate with respect to $x$ the functions

$$\ln(1-x^2) \qquad \text{and} \qquad x^{(1/x)}.$$ **[7]**

(c) Use the definitions of sinh and cosh in terms of exponentials to show that

$$\sinh(x)\cosh(y) = A\sinh(x+y) + B\sinh(x-y)$$

where $A$ and $B$ are constants which you should give. **[7]**

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**2.** (a) Use de Moivre's theorem to show that

$$\cos(3\theta) = C\cos^3(\theta) + D\cos(\theta),$$

where $C$ and $D$ are constants which you should find. **[6]**

(b) Evaluate the indefinite integral

$$\int \frac{4}{x^4-1}dx.$$ **[7]**

(c) Find all (real or complex) solutions to the equation

$$z^4 = -1 - i.$$

(You can leave your answers in polar form, but you should state how many different solutions there are.) **[7]**

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**3.** (a) Compute the limits, stating clearly any results about limits that you use:

$$\lim_{x\to 1}\frac{x^2+x-2}{x^3-1}, \qquad \lim_{x\to 0} x\cos\left(\frac{1}{x}\right), \qquad \lim_{x\to\infty}\frac{x^3+1}{x^5-1}.$$ **[9]**

(b) Use the derivative of $\sinh x$ to find $\frac{d}{dx}(\text{arcsinh}\, x)$. Your result should be simplified so it does not involve (inverse) hyperbolic functions. **[6]**

(c) Evaluate the definite integral

$$\int_0^{\pi/2} \cos x \sin^2 x \, dx \ .$$ **[5]**

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**4.** (a) Explain the difference between conditional and absolute convergence of a series $\sum_{k=0}^{\infty} a_k$. **[4]**

(b) Determine whether or not the series

$$\sum_{n=1}^{\infty}(n+1)^2 e^{-n}, \qquad \sum_{n=2}^{\infty}\frac{1}{n\cdot\ln(n)}$$

converge. **[8]**

(c) Determine the interval of convergence for the power series

$$\sum_{n=1}^{\infty}\frac{4^n}{\ln(n)}(x-2)^n.$$

Determine whether the series converges at the endpoints of the interval of convergence. **[8]**

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**5.** (a) Let $f(x) = \ln(\cos(x))$.

(i) Find the third-order Taylor polynomial $p_3(x)$ of $f(x)$ about $x = 0$.

(ii) Give the Lagrange form of the remainder and use it and the fact that $|\tan(x)| \le \frac{1}{2}$ for $x \in [0, \frac{1}{3}]$, to show the estimate

$$\left| f\left(\frac{1}{3}\right) - p_3\left(\frac{1}{3}\right)\right| \le \frac{35}{2^6\cdot 3^5}.$$ **[10]**

(b) For which values of $\lambda \in \mathbb{R}$ do the following four vectors form a basis of $\mathbb{R}^4$?

$$v_1 = \begin{pmatrix}\lambda\\1\\\lambda\\2\end{pmatrix}, \quad v_2 = \begin{pmatrix}1\\\lambda\\1\\3\end{pmatrix}, \quad v_3 = \begin{pmatrix}2\\1\\1\\\lambda\end{pmatrix}, \quad v_4 = \begin{pmatrix}1\\1\\1\\1\end{pmatrix}.$$ **[10]**

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**6.** For which values $c \in \mathbb{R}$ does the system of linear equations

$$\begin{aligned}(c^2-1)x + \phantom{2}(c-1)y - cz &= -1\\(c^2-1)x + 2(c-1)y - \phantom{c}z &= -1\\(c-1)y + \phantom{c}z &= \phantom{-}c\end{aligned}$$

have (a) no solutions, (b) a unique solution, (c) infinitely many solutions?

Find the solutions in cases (b) and (c) explicitly and, in case (c), determine whether the solution represents a line or a plane. **[20]**

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**7.** Consider the matrix

$$M := \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 3 \end{pmatrix}.$$

Find a matrix $P$ such that $P^{-1}MP$ is diagonal. Compute $P^{-1}$ and use these results to show that

$$M^{21} = \begin{pmatrix} 1 & 2^{21}-1 & 2^{21}-1 \\ 2^{21}-1 & 1 & 1-2^{21} \\ 1-2^{21} & 2^{21}-1 & 2^{22}-1 \end{pmatrix}.$$

[20]

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