EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May/June	2026	MATH1561-WE01

Title:
Single Mathematics A

Time:	2 hours	
Additional Material provided:	None	
Materials Permitted:	None	
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.

Instructions to Candidates:	Answer all questions. The indicative marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply. Write your answer in the white-covered answer booklet with barcodes. Begin your answer to each question on a new page.

Revision:	

ED01/2026

1. (a) Find the real and imaginary part of the complex numbers

$$\frac{7-i}{2-3i} \qquad \text{and} \qquad e^{-i\pi/4}.$$

[Your answers should be simplified, not left in terms of cos or sin functions.] **[5]**

(b) Differentiate with respect to x the functions

$$\ln(1-x^2) \qquad \text{and} \qquad x^{(1/x)}.$$ **[7]**

(c) Use the definitions of sinh and cosh in terms of exponentials to show that

$$\sinh(x)\cosh(y) = A\sinh(x+y) + B\sinh(x-y)$$

where A and B are constants which you should give. **[7]**

(d) Use de Moivre's theorem to show that

$$\cos(3\theta) = C\cos^3(\theta) + D\cos(\theta),$$

where C and D are constants which you should find. **[6]**

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2. (a) Evaluate the indefinite integral

$$\int \frac{4}{x^4 - 1} dx.$$

[**7**]

(b) Find all (real or complex) solutions to the equation

$$z^4 = -1 - i.$$

(You can leave your answers in polar form, but you should state how many different solutions there are.)

[**6**]

(c) Compute the limits, stating clearly any results about limits that you use:

$$\lim_{x \to 1} \frac{x^2 + x - 2}{x^3 - 1}, \qquad \lim_{x \to 0} x \cos\left(\frac{1}{x}\right).$$

[**6**]

(d) Let $G = \{e, a, b\}$ with the operation $\bullet$ given by

$\bullet$	e	a	b
e	e	a	b
a	a	e	a
b	b	b	e

Find the inverse of each element. Show that $(G, \bullet)$ is *not* a group. [**6**]

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3. (a) Find the interval of convergence of the power series

$$\sum_{k=0}^{\infty} \frac{1}{1 + e^k} x^k,$$

and decide about convergence and divergence of this power series at the boundary values of this interval. [**7**]

(b) Let $f(x) = e^x \cos(x)$. Calculate its Taylor polynomial $P_3(x)$ of degree 3 about $x = 0$ and show that, in the interval $x \in (-\frac{1}{2}, \frac{1}{2})$,

$$|f(x) - P_3(x)| \leq \frac{\sqrt{3}}{96}.$$

[**10**]

(c) Consider the following system of linear equations:

$$\begin{cases} x_1 + 2x_2 + 3x_3 - 3x_4 = a, \\ 7x_1 + x_2 + 8x_3 + 5x_4 = b, \\ 2x_1 - 5x_2 - 3x_3 + 12x_4 = c. \end{cases}$$

Show that this system has a solution if and only if $37a - 9b + 13c = 0$. [**8**]

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4. (a) Let

$$A = \begin{pmatrix} 2 & 3 & -1 \\ 0 & -1 & -1 \\ 2 & 1 & 2 \end{pmatrix}.$$

Using Gaussian elimination, calculate the inverse A^{-1} of this matrix. **[10]**

(b) Show that the characteristic polynomial of

$$M = \begin{pmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{pmatrix}$$

is $(5-\lambda)(3-\lambda)^2$. Verify that two eigenvectors of M associated to the eigenvalue $\lambda = 3$ are

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \quad \text{and} \quad v_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$$

Find an eigenvector associated to the eigenvalue $\lambda = 5$ and give a matrix P such that $P^{-1}MP$ is diagonal. **[9]**

(c) Decide about the validity of each of the following statements. Give a proof for the statements which are true, and provide a counterexample for the statements which are false.

(i) If A and B are invertible matrices of the same size, then AB is also invertible.

(ii) If a matrix A is orthogonal, then $-A$ is also orthogonal.

(iii) The set of all real symmetric 3×3 matrices is a group under matrix multiplication. **[6]**
