Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May/June	2026	MATH1571-WE01

Title: Single Mathematics B

Time:	2 hours	
Additional Material provided:	None	
Materials Permitted:	None	
Calculators Permitted:	Yes	Models Permitted: Casio FX83 series or FX85 series.

Instructions to Candidates:	Answer all questions. The indicative marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply. Write your answer in the white-covered answer booklet with barcodes. Begin your answer to each question on a new page.

Revision:	

ED01/2026

1. (a) The polar coordinates for a motion of a particle are given by

$$r = t^4, \qquad \theta = \mathrm{e}^{2t}.$$

Find the radial and transverse components of the velocity and acceleration vectors of this particle. In other words, express these vectors as a linear combination of the polar basis $\mathbf{e}_r$ and $\mathbf{e}_\theta$. You may use known formulas for derivatives of the basis vectors $\mathbf{e}_r$ and $\mathbf{e}_\theta$ **[10]**

(b) Let f be a 2π-periodic function defined by $f(x) = 1+|x|$ on $(-\pi, \pi]$. Compute the Fourier series for f. **[10]**

(c) By evaluating the above Fourier series at a suitable point, prove the following infinite sum formula:

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots . \quad \textbf{[5]}$$

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2. Solve the following set of ODEs. Give all possible solutions for each of these ODEs and show your work.

(a) $y\frac{dy}{dx} - \cos(x) = 0.$ **[5]**

(b) $\frac{dy}{dx} + 2y = xy^2.$ **[10]**

(c) $y'' - 6y' + 9y = e^{2x}$, $y(0) = 2$, $y'(0) = 2.$ **[10]**

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3. (a) Find and classify the critical points of the function

$$f(x, y) = x^3 - x^2y + xy^2 - 9x. \quad \textbf{[10]}$$

(b) Using the method of separation of variables, solve the following partial differential equation for $u(x, y)$. Give the form of the general solution, as well as the particular solution when $u(x, 1) = 2x^4 + 3x^2$.

$$x\frac{\partial u}{\partial x} - 2y\frac{\partial u}{\partial y} = 0. \quad \textbf{[15]}$$

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4. (a) Suppose the words `SOME`, `DOGS`, `ARE`, and `BROWN` are each written on a card. Clare selects one of these cards at random, and then randomly chooses one of the letters on it.

(i) Let X denote the length of the word on the chosen card. Find the probability mass function of X, its expectation, and its variance. **[6]**

(ii) What is the probability that Clare chooses the letter `R`? **[4]**

(b) Now consider two continuous random variables: X and Y. If X and Y have joint probability density function

$$f(x,y) = \begin{cases} \alpha(x+2y) & 0 \leq x, \quad 0 \leq y, \quad x+y \leq 2 \\ 0 & \text{otherwise,} \end{cases}$$

then:

(i) Find α. **[4]**

(ii) Find $\mathbb{P}(X \geq 2Y)$. **[8]**

(iii) Find $\mathbb{P}(X > 1, Y > 1)$. **[3]**
