Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May/June	2026	MATH1597-WE01

Title:
Probability I

Time:	2 hours	
Additional Material provided:	None	
Materials Permitted:	None	
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.

Instructions to Candidates:	Answer all questions. The indicative marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply. Write your answer in the white-covered answer booklet with barcodes. Begin your answer to each question on a new page.

Revision:	

ED01/2026

1. There are 8 mathematicians and 7 natural scientists in a tutorial. We choose a particular table in the room, around which 5 students are seated.

(a) Find the probability that the group consists of 2 mathematicians and 3 natural scientists. Clearly state any additional assumptions you make. **[6]**

(b) What is the probability that the group contains at least one natural scientist? **[6]**

(c) What is the probability that the group contains more mathematicians than natural scientists? **[8]**

Full credit for **Q1** will be awarded if your answers are given in terms of binomial coefficients.

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2. Let X and Y be random variables with joint probability mass function given in the following table. Here $\alpha \in [0, \frac{1}{12}]$.

$p_{X,Y}(x, y)$	$x = 0$	$x = 1$	$x = 2$	$x = 3$
$y = 0$	0	$\frac{1}{6}$	$\frac{1}{6}$	2α
$y = 1$	$\frac{1}{6} - \alpha$	0	α	0
$y = 2$	α	$\frac{1}{6}$	$\frac{1}{6} - \alpha$	$\frac{1}{6} - 2\alpha$

(a) Calculate $\mathbb{E}[X]$, $\mathbb{E}[Y]$, and $\mathrm{Cov}(X, Y)$. Your answers may depend on α. **[6]**

(b) For which value(s) of α, if any, do we have $\mathrm{Cov}(X, Y) = 0$? For which value(s) of α, if any, are X and Y independent? **[4]**

(c) Find $\mathbb{E}[Y|X = x]$ for $x = 0, 1, 2, 3$. Verify that $\mathbb{E}[Y] = \mathbb{E}[\mathbb{E}[Y|X]]$. **[10]**

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3. Consider random variables X and Y, with joint probability density function given by

$$f_{X,Y}(x, y) = \begin{cases} 6(xy - x^2) & 0 \le x \le y \le \alpha, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the value of α. **[6]**

(b) Find the marginal probability density functions f_X and f_Y of X and Y. Use them to calculate $\mathbb{E}[X]$ and $\mathbb{E}[Y]$. **[8]**

(c) Are X and Y independent? **[2]**

(d) Calculate the cumulative distribution function of Y. **[4]**

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4. During the football World Cup, in each match 0, 1, or 2 yellow cards are given, with probabilities $\frac{13}{16}$, $\frac{1}{8}$, and $\frac{1}{16}$, respectively. The numbers of yellow cards given in different matches are independent.

(a) Find the probability that, in the first four matches of the tournament,

(i) there are exactly three games in which at least one yellow card is given; **[4]**

(ii) a total of two yellow cards are given. **[4]**

(b) Let Y be the total number of yellow cards given across all 104 matches in the tournament.

(i) Find $\mathbb{E}[Y]$ and $\text{Var}(Y)$. **[2]**

(ii) State Markov's inequality and use it to obtain an upper bound for $\mathbb{P}(Y \geq 30)$. **[4]**

(iii) State Chebyshev's inequality and use it to obtain an upper bound for $\mathbb{P}(Y \geq 30)$. What do you notice about your answer? **[6]**

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5. Suppose that random variables X_1, X_2, X_3, X_4 are independent and identically distributed, following an exponential distribution with parameter $\lambda > 0$, so that each of them has probability density function f, given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

We will consider $S = X_1 + X_2 + X_3 + X_4$. In the rest of this question, carefully state any properties of moment generating functions you use. Think carefully about the domain of each of your functions.

(a) Compute the moment generating function $M_{X_1}(t) = \mathbb{E}[e^{tX_1}]$ for X_1. **[6]**

(b) Hence or otherwise find the moment generating function $M_S(t)$ for S. **[6]**

(c) Using your answer to part (b), find $\mathbb{E}[S]$ and $\text{Var}(S)$. **[8]**
