Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May/June	2026	MATH1607-WE01

Title:
Dynamics I

Time:	2 hours	
Additional Material provided:	None	
Materials Permitted:	None	
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.

Instructions to Candidates:	Answer all questions. The indicative marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply. Write your answer in the white-covered answer booklet with barcodes. Begin your answer to each question on a new page.

Revision:	

ED01/2026

1. (a) A unit-mass particle moves on the x-axis, subject to a force $F = 1 - v$, where v is its velocity. Given the initial conditions $x(0) = v(0) = 0$, find its position $x(t)$. **[7]**

(b) A light rigid rod of length $2L$ is pivoted at one end, and swings freely in a vertical plane, without friction. Gravity acts downwards with acceleration g. A mass m is attached to the other end of the rod, and a mass $2m$ is attached halfway down it. Let θ denote the angle between the rod and the downward vertical.

(i) Obtain an expression for the energy E of this system, in terms of θ and $\dot{\theta}$. You may think of two simple pendulums glued together, and use results for the simple pendulum without proof.

(ii) Hence compute the period P of small oscillations about the stable equilibrium.

(iii) If the rod is released from rest in a horizontal position, what is the speed u of the mass m as it passes the equilibrium? **[13]**

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2. (a) Particles of mass m and $2m$ respectively travel along the x-axis at the same speed, and they collide head-on. Afterwards, the total kinetic energy is $5/6$ of what it was before, the $2m$-particle is moving along the y-axis, and the m-particle is moving at an angle ϕ to the x-axis. Determine ϕ. **[10]**

(b) A particle of unit mass and unit charge moves in a magnetic field $\boldsymbol{B} = 2\boldsymbol{e}_3$. Given the initial conditions $\boldsymbol{r}(0) = \boldsymbol{0}$ and $\dot{\boldsymbol{r}}(0) = 2\boldsymbol{e}_1 + \boldsymbol{e}_3$, find $\boldsymbol{r}(t)$. **[10]**

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3. (a) A unit-mass particle on the x-axis is attached to a spring with spring constant $k = 4$. An additional force $F(t) = 4\cos(pt)$, where p is a positive constant, acts on the particle. The initial conditions are $x(0) = 0$ and $\dot{x}(0) = 0$. Find $x(t)$ if p is tuned to the value for which the system is resonant. **[10]**

(b) A particle of mass $m = 1$ moves in an attractive central force of magnitude $|\boldsymbol{F}| = 5/(2r^5)$. Initially it is at $r = 1/2$, and its initial velocity $\boldsymbol{v}_0$ is orthogonal to $\boldsymbol{r}$ with $|\boldsymbol{v}_0| = 5$. Using conservation of energy, find the maximum and the minimum values of $r(t)$. **[10]**

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4. (a) Find the solution $u(x,t)$ of the wave equation $\partial^2 u/\partial t^2 = 4\,\partial^2 u/\partial x^2$, satisfying the initial conditions

$$u\big|_{t=0} = \frac{1}{\cosh(x)}\,, \qquad \left.\frac{\partial u}{\partial t}\right|_{t=0} = -2\,\frac{\sinh(x)}{\cosh^2(x)}.$$ **[10]**

(b) Two particles, of masses M and m, move along the x-axis towards each other, both with speed v. They collide elastically. Afterwards, the M-particle has speed $v/2$, and is moving in the same direction as it was before. Determine the ratio $R = M/m$. **[10]**

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5. (a) Show that for certain values of the constants a and b (which you should determine) the following force can be derived from a potential $V(x, y, z)$:

$$\boldsymbol{F} = az\cos(y)\boldsymbol{e}_1 + bxz\sin(y)\boldsymbol{e}_2 - x\cos(y)\boldsymbol{e}_3.$$ **[7]**

(b) Two uniform straight rods, each of length L, and of mass M and m respectively, are joined end-to-end in a straight line to form a composite pendulum of length $2L$. The composite rod is pivoted at the other end of the M-rod, and can swing freely in a vertical plane, without friction. Gravity acts downwards with acceleration g.

(i) Find the distance D from the pivot to the centre of mass of the system.

(ii) Compute the moment of inertia I of the system about its pivot.

(iii) Hence obtain an expression for the energy E, in terms of the angle $\theta(t)$ by which the system deviates from its stable equilibrium. **[13]**
