



EXAMINATION PAPER

Examination Session: May/June	Year: 2026	Exam Code: MATH1627-WE01
---	----------------------	------------------------------------

Title: Dynamics and Relativity I
--

Time:	2 hours	
Additional Material provided:	None	
Materials Permitted:	None	
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.

Instructions to Candidates:	<p>Answer all questions.</p> <p>The indicative marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply.</p> <p>Write your answer in the white-covered answer booklet with barcodes.</p> <p>Begin your answer to each question on a new page.</p>
-----------------------------	--

Revision:	
------------------	--

1. (a) A unit-mass particle moves on the x -axis, subject to a force $F = 1 - v$, where v is its velocity. Given the initial conditions $x(0) = v(0) = 0$, find its position $x(t)$. [7]
- (b) A light rigid rod of length $2L$ is pivoted at one end, and swings freely in a vertical plane, without friction. Gravity acts downwards with acceleration g . A mass m is attached to the other end of the rod, and a mass $2m$ is attached halfway down it. Let θ denote the angle between the rod and the downward vertical.
- (i) Obtain an expression for the energy E of this system, in terms of θ and $\dot{\theta}$. You may think of two simple pendulums glued together, and use results for the simple pendulum without proof.
- (ii) Hence compute the period P of small oscillations about the stable equilibrium.
- (iii) If the rod is released from rest in a horizontal position, what is the speed u of the mass m as it passes the equilibrium? [13]
-
2. (a) Particles of mass m and $2m$ respectively travel along the x -axis at the same speed, and they collide head-on. Afterwards, the total kinetic energy is $5/6$ of what it was before, the $2m$ -particle is moving along the y -axis, and the m -particle is moving at an angle ϕ to the x -axis. Determine ϕ . [10]
- (b) A particle of unit mass and unit charge moves in a magnetic field $\mathbf{B} = 2\mathbf{e}_3$. Given the initial conditions $\mathbf{r}(0) = \mathbf{0}$ and $\dot{\mathbf{r}}(0) = 2\mathbf{e}_1 + \mathbf{e}_3$, find $\mathbf{r}(t)$. [10]
-
3. (a) A unit-mass particle on the x -axis is attached to a spring with spring constant $k = 4$. An additional force $F(t) = 4 \cos(pt)$, where p is a positive constant, acts on the particle. The initial conditions are $x(0) = 0$ and $\dot{x}(0) = 0$. Find $x(t)$ if p is tuned to the value for which the system is resonant. [10]
- (b) A particle of mass $m = 1$ moves in an attractive central force of magnitude $|\mathbf{F}| = 5/(2r^5)$. Initially it is at $r = 1/2$, and its initial velocity \mathbf{v}_0 is orthogonal to \mathbf{r} with $|\mathbf{v}_0| = 5$. Using conservation of energy, find the maximum and the minimum values of $r(t)$. [10]
-

4. (a) Alice is at rest on the x -axis of an inertial frame, and Bob is moving along the x -axis, away from Alice, with constant speed v .
- (i) Bob's watch ticks once per second in its rest-frame. Alice observes the time between ticks of his watch to be $3/\sqrt{5}$ seconds. What is their relative speed v , as a fraction of the speed of light c ?
 - (ii) Bob fires a projectile with speed $v/2$ relative to him, directly away from Alice. What is the speed u of the projectile relative to Alice?
 - (iii) If the rest-length of the projectile is $3\sqrt{2}$ metres, what is its length L as measured by Bob?

In parts (ii) and (iii), use the value of v obtained in part (i). [12]

- (b) Two particles, each with mass m , travel in opposite directions with the same speed v , and collide to form a single particle of mass $5m/2$. Determine the speed v , as a fraction of the speed of light c . [8]
-

5. (a) For each of the 4-vectors $P = (1, 1, 1, 0)$ and $Q = (2, 1, 1, \sqrt{2})$, state whether it is timelike, lightlike, or spacelike. [8]
- (b) A particle has 4-momentum $P = c(b, 1, 2, -2)$, where b is a constant. For which value of b does the kinetic energy of the particle equal c^2 ? [12]
-