



EXAMINATION PAPER

Examination Session: May/June	Year: 2026	Exam Code: MATH2581-WE01
---	----------------------	------------------------------------

Title: Algebra II (2024/25 syllabus)
--

Time:	3 hours	
Additional Material provided:	None	
Materials Permitted:	None	
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.

Instructions to Candidates:	<p>Answer all questions.</p> <p>The indicative marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply.</p> <p>Write your answer in the white-covered answer booklet with barcodes.</p> <p>Begin your answer to each question on a new page.</p>
-----------------------------	--

Revision:	
------------------	--

SECTION A

1. Decide whether the following polynomials are irreducible in the corresponding rings. Justify your answers.

- (a) $x^3 - x^2 + \bar{1}$ in $(\mathbb{Z}/3)[x]$. [2]
(b) $x^5 - 2x^4 - 2x^2 + 9x - 10$ in $\mathbb{Q}[x]$. [2]
(c) $x^9 - 14x^8 + 42x^5 + 35x^2 + 70$ in $\mathbb{Q}[x]$. [2]
(d) $x^4 + x^3 + x^2 + x + \bar{1}$ in $(\mathbb{Z}/2)[x]$. [2]
(e) $7x^2 + 7x + 7$ in $\mathbb{Z}[x]$. [2]
-

2. Let $\phi : \mathbb{Z}[i] \rightarrow \mathbb{Z}/13$ be given by $\phi(a + bi) = \overline{a + 5b}$.

- (a) Show that ϕ is a ring homomorphism. [5]
(b) Show that $\text{Ker } \phi = (-5 + i, 13)$, the ideal in $\mathbb{Z}[i]$ generated by $-5 + i$ and 13. [5]
-

3. (a) Find the rank and torsion coefficients of the group

$$\mathbb{Z}^2 \times \mathbb{Z}/12 \times \mathbb{Z}^4 \times \mathbb{Z}/8 \times \mathbb{Z} \times \mathbb{Z}/18. \quad [5]$$

- (b) Let G , H , and K be groups. Prove or give a counterexample to the following statement: If G is normal in H and H is normal in K , then G is also normal in K . [5]
-

4. (a) Give all the groups of order 302, up to isomorphism. [3]

- (b) Find representatives for the conjugacy classes of A_5 and determine the size of each conjugacy class. [7]
-

SECTION B

5. (a) Show that $1 + 3\sqrt{-5}$ is irreducible in $\mathbb{Z}[\sqrt{-5}]$. [5]
(b) Write $x^3 + \bar{6} \in (\mathbb{Z}/7)[x]$ as a product of irreducible polynomials in $(\mathbb{Z}/7)[x]$. [5]
(c) Let F be a field. Prove that if $f(x) \in F[x]$ is such that $(f(x))$ is a maximal ideal of $F[x]$, then $f(x)$ is irreducible in $F[x]$. [5]
-
6. Let R be the ring $(\mathbb{Z}/3)[x]/(x^2 + \bar{1})$.
(a) Prove that R is isomorphic to $\mathbb{Z}[x]/(3, x^2 + 1)$. [5]
(b) Find all the elements of R . You must show that your list contains all the elements and that the elements in your list are distinct. [5]
(c) Show that R is a field and find the multiplicative inverse of the element $x^7 + \bar{2}x + \bar{1} + I \in R$. Here I denotes the ideal in $(\mathbb{Z}/3)[x]$ generated by the element $x^2 + \bar{1}$. [5]
-
7. Let G be a group. The *commutator subgroup* G' is the subgroup of G generated by all elements of the form $xyx^{-1}y^{-1}$, $x, y \in G$.
(a) Show that $G' = \{e\}$ if and only if G is abelian. [3]
(b) Show that $S'_3 = A_3$. Deduce from this that $S'_n = A_n$ for every $n \geq 3$. [3]
(c) Show that for any surjective homomorphism $\varphi : G \rightarrow H$ one has $\varphi(G') = H'$. [3]
(d) Deduce from (c) that G' is normal in G . [3]
(e) Show that G/G' is abelian. Moreover, let N be a normal subgroup of G such that G/N is abelian. Show that $G' \subseteq N$. (In other words, G' is the *minimal* normal subgroup such that the quotient is abelian.) [3]
-
8. (a) Find all abelian groups (up to isomorphism) of order 80. [3]
(b) Give two different non-abelian groups of order 8. [3]
(c) Find a generator of the group $(\mathbb{Z}/23)^\times$, the multiplicative group of units of $\mathbb{Z}/23$. (You may assume the group is cyclic.) [3]
(d) Show that $(\mathbb{Z}/16)^\times$ is not cyclic. [3]
(e) Show that $(\mathbb{Z}/16)^\times$ can be generated by two elements. [3]
-