Durham University

EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May/June	2026	MATH2657-WE01

Title: Special Relativity and Electromagnetism II

Time:	2 hours	
Additional Material provided:	None	
Materials Permitted:	None	
Calculators Permitted:	No	Models Permitted: Use of electronic calculators is forbidden.

Instructions to Candidates:	Answer all questions. The indicative marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply. Write your answer in the white-covered answer booklet with barcodes. Begin your answer to each question on a new page.

Revision:	

SECTION A

1. (a) A particle has 4-momentum $p^\mu = (26ac, 0, -24ac, 0)$, where a is a positive constant and c denotes the speed of light. Calculate, in terms of a and c, the rest mass of the particle and its kinetic energy. **[5]**

(b) In this question take the speed of light c to be 3×10^8 metres per second.

An unstable particle has a lifetime of 2.8×10^{-8} seconds, as measured in its rest frame. The particle is moving, with respect to the laboratory, with a speed 1.8×10^8 metres per second. Calculate the lifetime of the particle and the distance it travels before it decays, as measured in the laboratory. **[5]**

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2. (a) In an electrostatics problem, in a region of space the electric field has the form

$$\mathbf{E} = (\, 3\beta x^2y^5z^2 + 12ax^5y^5,\ 5\beta x^3y^4z^2 + 5\alpha x^6y^4,\ 2bx^3y^5z\,),$$

where α, a, β, b are non-zero constants. Calculate the electric charge density, in terms of the spatial coordinates, the constants a, b and the electric constant ε_0. **[5]**

(b) In a magnetostatics problem, in a region of space the magnetic field is

$$\mathbf{B} = \left(3axy^2z^2 - 2ax^2y^2z,\ 3bxy^2z^2 - by^3z^2,\ 6dxy^2z^2 - 6dxyz^3\right),$$

where a, b, d are constants. Determine b and d in terms of a and hence calculate the electric current density $\mathbf{J}$, in terms of the spatial coordinates, the constant a and the magnetic constant μ_0. **[5]**

SECTION B

3. (a) In a frame $\mathcal{R}$ a contravariant vector is given by $V^\mu = (4\ell, 8\ell, 5\ell, 3\ell)$, where ℓ is an invariant positive constant.

In a frame $\mathcal{R}'$ it is found that $V'^\mu = (-\ell, m, 5\ell, 3\ell)$, where m is an invariant positive constant.

Determine m in terms of ℓ and find the speed, as a fraction of the speed of light c, of the Lorentz boost that relates the frames $\mathcal{R}$ and $\mathcal{R}'$. [**7**]

(b) Two particles, each with rest mass m, have the same speed v but their velocities are perpendicular. The particles collide and fuse to form a new particle with rest mass $4m$ and speed $\tilde{v}$.

Calculate v and $\tilde{v}$, as a fraction of the speed of light c. [**8**]

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4. (a) State the source-free version of Maxwell's equations and use these to show that the electric field satisfies the wave equation. [**6**]

(b) Electric charge is contained within a ball of radius R that is centred at the origin. The electric charge density vanishes outside the ball but inside it is given by

$$\rho = qb^3 \cosh(br),$$

where q and b are positive constants, with $r = |\mathbf{r}|$ the distance to the origin.

Calculate the electric field, both inside and outside the ball, at position $\mathbf{r}$ due to the ball, in terms of q, b, R, ε_0. [**9**]

END