EXAMINATION PAPER

Examination Session:	Year:	Exam Code:
May/June	2026	MATH3141-WE01

Title:
Operations Research III

Time:	3 hours	
Additional Material provided:	None	
Materials Permitted:	None	
Calculators Permitted:	Yes	Models Permitted: Casio FX83 series or FX85 series.

Instructions to Candidates:	Answer all questions. The indicative marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply. Write your answer in the white-covered answer booklet with barcodes. Begin your answer to each question on a new page.

Revision:	

ED01/2026

SECTION A

1. Which of the following matrices correspond to the transition matrix of a unichain? Briefly justify each answer.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{A}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{B}$$

$$\begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{C}$$

$$\begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix} \tag{D}$$

$$\begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.2 & 0.3 & 0.5 \\ 0.5 & 0 & 0.5 \end{bmatrix} \tag{E}$$

[**10**]

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2. A remotely operated subsea vehicle, Squidward, is deployed to stabilize a leaking subsea wellhead during a storm. Each control cycle, the operations center issues one of two commands to Squidward: scan the site (action S), or attempt a repair action (action R). The visibility near the leak, critical for success, can be either poor (state 0), marginal (state 1), or good (state 2). The state evolves according to silt plumes, currents, and lighting adjustments.

The rewards are 0, 1, or 2, depending on the amount of progress made toward stopping the leak. More progress can be made under better visibility. Although scanning yields no direct progress, it may improve Squidward's situational awareness and occasionally improve visibility.

The operations center collected the following 6 episodes under their current policy A, using exploring starts, where x_t, a_t, and r_t denote the state, action, and reward at time t, respectively:

t	x_t	a_t	r_t
0	0	R	0
1	0	S	0
2	1	R	1
3	0	S	0
4	0	S	0
5	1	R	1

t	x_t	a_t	r_t
0	1	S	0
1	2	R	2
2	1	R	1
3	0	S	0
4	2	R	1
5	0	S	0

t	x_t	a_t	r_t
0	2	S	0
1	2	R	2
2	1	R	1
3	0	S	0
4	1	R	1
5	0	S	0

t	x_t	a_t	r_t
0	0	R	0
1	0	S	0
2	0	S	0
3	1	R	1
4	2	R	2
5	0	S	0

t	x_t	a_t	r_t
0	1	S	0
1	1	R	1
2	1	R	1
3	0	S	0
4	0	S	0
5	2	R	1

t	x_t	a_t	r_t
0	2	S	0
1	2	R	1
2	1	R	1
3	0	S	0
4	1	R	1
5	0	S	0

(a) What is the current policy? Briefly justify your answer. **[2]**

(b) Use the collected data to determine whether the current policy from state 1, i.e. $A(1)$, can be improved. Use a discount rate of 0.5. **[8]**

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3. Solve the following linear programming problem using the two-phase method, and find all feasible values of x_1, x_2 and x_3 for which the optimum value is attained.

$$\begin{array}{lrcrcrcl}
\text{max} & x_1 & + & x_2 & & & & \\
\text{subject to} & x_1 & + & x_2 & + & 2x_3 & \leq & 6 \\
 & & & x_2 & - & x_3 & \geq & 5 \\
 & x_1 & & & & & \geq & 0 \\
 & & & x_2 & & & \geq & 0 \\
 & & & & & x_3 & \geq & 0.
\end{array}$$

[10]

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4. (a) Starting from the north-west initial assignment, find the optimal transportation scheme and optimal value for the transportation problem with costs c_{ij}, supplies a_i and demands b_j given below:

$$\begin{bmatrix} c_{ij} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 3 & 1 & 2 & 4 \\ 5 & 4 & 2 & 5 \end{bmatrix}, \qquad \begin{bmatrix} a_i \end{bmatrix} = \begin{bmatrix} 10 \\ 30 \\ 40 \end{bmatrix}, \qquad \begin{bmatrix} b_j \end{bmatrix} = \begin{bmatrix} 15 & 10 & 20 & 35 \end{bmatrix}.$$

[7]

(b) Now suppose that Supplier 3 imposes a fixed levy increasing all costs by 10, so that

$$\begin{bmatrix} c_{ij} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 3 & 1 & 2 & 4 \\ 15 & 14 & 12 & 15 \end{bmatrix}.$$

Briefly explain why this has no impact on the reduced costs, and hence determine the new optimal transportation scheme and optimal value. **[3]**

SECTION B

5. A neighbourhood café located on a busy shopping street sells fresh croissants over a three-day holiday weekend, from Saturday to Monday. Unsold croissants at the end of the day have zero value as they must be discarded.

There are three possible demand regimes, representing the level of foot traffic: low ($i = 1$), medium ($i = 2$), and high ($i = 3$). The number of croissants demanded is assumed to be $20(i + 1)$, i.e.

i	croissants demanded
1	40
2	60
3	80

The predicted demand distribution for Saturday is $P(1) = 0.3$, $P(2) = 0.4$, and $P(3) = 0.3$. For subsequent days, demand evolves as a Markov chain with the following transition matrix:

$$\begin{bmatrix} 0.6 & 0.4 & 0 \\ 0.2 & 0.6 & 0.2 \\ 0 & 0.4 & 0.6 \end{bmatrix}$$

The selling price per croissant is 3. The production cost per croissant is 1.

The café wants to figure out how many croissants to produce, prior to opening, on Saturday, Sunday and Monday.

(a) State a full mathematical model for the above problem. **[6]**

(b) Solve the problem. **[9]**

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6. A manufacturer currently replaces its generator either after five years of operation or upon failure, whichever happens first. In either case, the cost of replacement is 40. Yearly profit $t(i)$ and failure probability $q(i)$ as a function of the age i of the generator are as follows:

i	$t(i)$	$q(i)$
0	130	0.05
1	130	0.05
2	120	0.05
3	120	0.10
4	120	0.15
5	110	0.20
6	110	0.25
7	100	0.50
8	90	0.75
9	80	1

For instance, a new generator will fail during its first year of operation with probability $q(0)$. More generally, if the generator is still working after i years of operation, it will fail during its $(i+1)$th year of operation with probability $q(i)$.

If the generator fails, the company has an insurance contract to keep production running for the remainder of the year (with no impact on operational cost or profit during this time), however the generator must be replaced for the next year.

Determine whether or not the current replacement strategy maximizes the manufacturer's expected long-run average net income. **[15]**

7. The linear programming problem

$$\begin{array}{lrcrcrcrcl} \text{max} & 2x_1 & + & 3x_2 & + & 4x_3 & & & & \\ \text{subject to} & 2x_1 & + & x_2 & + & x_3 & + & 2x_4 & \leq & 3 \\ & 3x_1 & + & x_2 & + & 2x_3 & + & 2x_4 & \leq & 4 \\ & x_1 & + & 2x_2 & + & 4x_3 & + & x_4 & \leq & 9 \end{array}$$

and to $x_i \geq 0$ for all $i = 1, 2, 3, 4$, is solved using the simplex algorithm, beginning with initial table:

T_0	x_1	x_2	x_3	x_4	s_1	s_2	s_3	
z	-2	-3	-4	0	0	0	0	0
s_1	2	1	1	2	1	0	0	3
s_2	3	1	2	2	0	1	0	4
s_3	1	2	4	1	0	0	1	9

and terminating at the final table:

T_*	x_1	x_2	x_3	x_4	s_1	s_2	s_3	
z	5	0	0	6	2	1	0	10
x_2	1	1	0	2	2	-1	0	2
x_3	1	0	1	0	-1	1	0	1
s_3	-5	0	0	-3	0	-2	1	1

Consider the modified linear programming problem below, with modifications indicated in bold:

$$\begin{array}{lrcrcrcrcl} \text{max} & 2x_1 & + & 3x_2 & + & 4x_3 & & & & \\ \text{subject to} & 2x_1 & + & x_2 & + & x_3 & + & 2x_4 & \leq & 3 \boldsymbol{+ 4\alpha} \\ & 3x_1 & + & x_2 & + & 2x_3 & + & 2x_4 & \leq & 4 \\ & x_1 & + & 2x_2 & + & 4x_3 & + & x_4 & \leq & 9 \boldsymbol{- \alpha} \end{array}$$

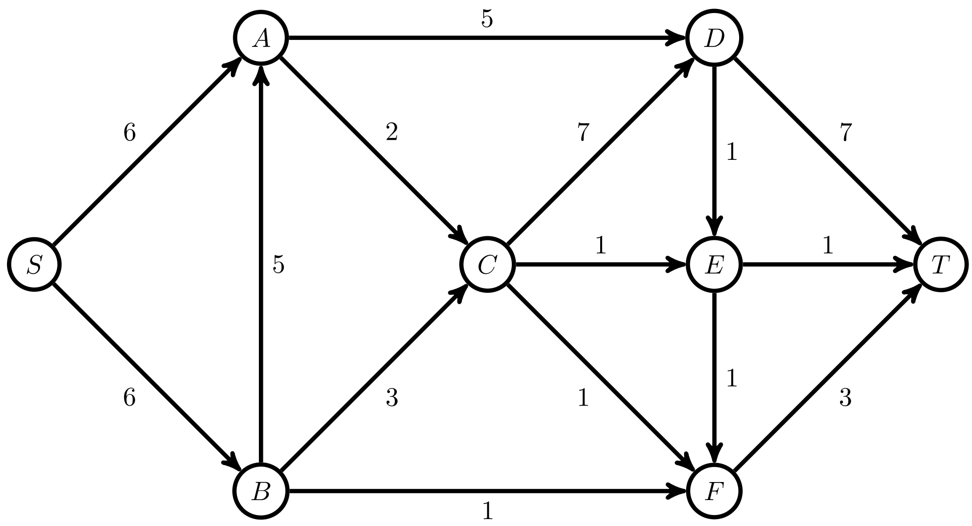
and to $x_i \geq 0$ for all $i = 1, 2, 3, 4$. Use post-optimal analysis to answer the following questions about this modified problem.

(a) For what values of $\alpha \in \mathbb{R}$ is the basis $\{x_2, x_3, s_3\}$ feasible? **[5]**

(b) For what values of $\alpha \in \mathbb{R}$ is the basis $\{x_2, x_3, s_3\}$ feasible **and optimal**? Find the optimal value of the problem for these values of α. **[5]**

(c) Find the optimal solution when $\alpha = 1$. **[5]**

8. Consider a transportation system that can be represented by the following flow network, with capacities as labelled:



(a) Apply the Ford–Fulkerson algorithm to find the maximum flow through the network from source node S to terminus node T. **[10]**

(b) Is it possible to increase the maximum flow from S to T by increasing the capacity along just one arc? If yes, identify such an arc and find the new flow. If no, provide a justification using any theorems from the course (which you may use without proof). **[5]**