Hints for exercises 8.5 and 8.6 (week 12)

8.5 Consider the following situation: Buy a European (K, t) call option and sell a European $(B + K, t)$ call option. Determine the payoff of this portfolio at time t. Show that this is equivalent to the payoff at time t of the European (K, t) capped call option. Hence, for there not to be an arbitrage, the cost of the European (K, t) capped call option should be equal to the cost of the portfolio which you know as this consists of two plain European call options and the cost of a European call option is given by the Black-Scholes formula.

8.6 Consider the following portfolio: Buy α European $((1 + \beta)s, 1)$ call options and a cash amount of $(1 + \beta)se^{-r}$. Determine the payoff of this portfolio and compare it with the payoff at time 1 of the investment given in the exercise. As they are the same, we know that for there not to be an arbitrage the cost of the portfolio and the investment should be the same. It then follows that

$$\alpha = \frac{s - (1 + \beta)se^{-r}}{C(s, 1, (1 + \beta)s, \sigma, r)}$$

where $C(s, 1, (1+\beta)s, \sigma, r)$ is the Black-Scholes formula for the cost of a call option with $K = (1 + \beta)s$ and $s = S(0)$.