

### Hints to exercises 11.5, 11.6 (week 18)

11.5 I will make a spreadsheet page showing a small simulation.

11.6 Down-and-in comes alive when price drops below  $v$  before  $t$ . Value is  $D_i(s, t, K)$  where  $s > v$ . Suppose its exercise time is  $t = n/N$  and we take  $j$  steps per day. Let  $t_k = kt/nj$ ,  $k = 0, 1, \dots, nj$  and  $I_k = 1$  if  $S(mt/n) < v$  for any  $m \leq k/j$  and  $I_k = 0$  otherwise (so  $I_k = 1$  only if the end-of-day price no later than  $t_k$  is below the barrier). Let  $V_k(i)$  denote the expected payoff given that  $S(t_k) = u^i d^{k-i} S(0)$  (where  $u, d$  are the risk-neutral jumps). We have

$$V_k(i) = I_k(pV_{k+1}(i+1) + (1-p)V_{k+1}(i)) + (1-p)(1-I_k)I_{\{u^i d^{k+1-i} S(0) < v\}} V_{k+1}(i) .$$

Then  $V_0(0)$  is the expected return and hence the price of the option.