

Homework 3

1. Denote by $S(t)$ the price per share of a certain security at time t . Suppose the initial price of a share is £50 and the price after one period (time 1) is assumed to be £40, £50 or £55. At a cost of £ c per share, we can purchase at time 0 a European option to buy the stock at time 1 for the price of £52. The one-period risk-free interest rate is 5%. Let $p_{40} = P(S(1) = 40)$, $p_{50} = P(S(1) = 50)$, and $p_{55} = P(S(1) = 55)$.
 - (a) Determine the expected present value return from purchasing one share of the stock in terms of p_{40} and p_{55} .
 - (b) Determine the risk-neutral price of the call option in terms of p_{55} .
 - (c) Use the arbitrage theorem to find an interval for which there is no arbitrage if c lies in that interval by determining the minimal and maximal possible value of p_{55} .
 - (d) Show that there is a weak arbitrage possible if the cost of the option is equal to the smallest endpoint of the interval determined.
2. Assume that the price of a certain security follows a geometric Brownian motion with volatility parameter σ . Consider a European call option to buy a single unit of this security at time t for the price K . Suppose further that the initial price of the security is S_0 and that the nominal yearly interest rate, compounded continuously, is r .
 - (a) Determine the risk-neutral valuation of this European call option when $K = £45$, $S_0 = £46.75$, $t = 1/2$, $\sigma = 0.26$ and $r = 0.06$.
 - (b) Give a 95% confidence interval for the price of this security at time t .
 - (c) What is the present value of the payoff from this European call option when the volatility parameter approaches zero. Show that this is consistent with the risk-neutral valuation of such a call.
 - (d) Consider now a European asset-or-nothing call option that pays its holder a fixed amount F if the price of this security at expiration time is larger than K and pays 0 otherwise. Assume the same values of the parameters as in part a. Determine the value of F such that the risk-neutral valuation of this asset-or-nothing call is the same as the risk-neutral valuation of the normal European call option (which was calculated in part a).