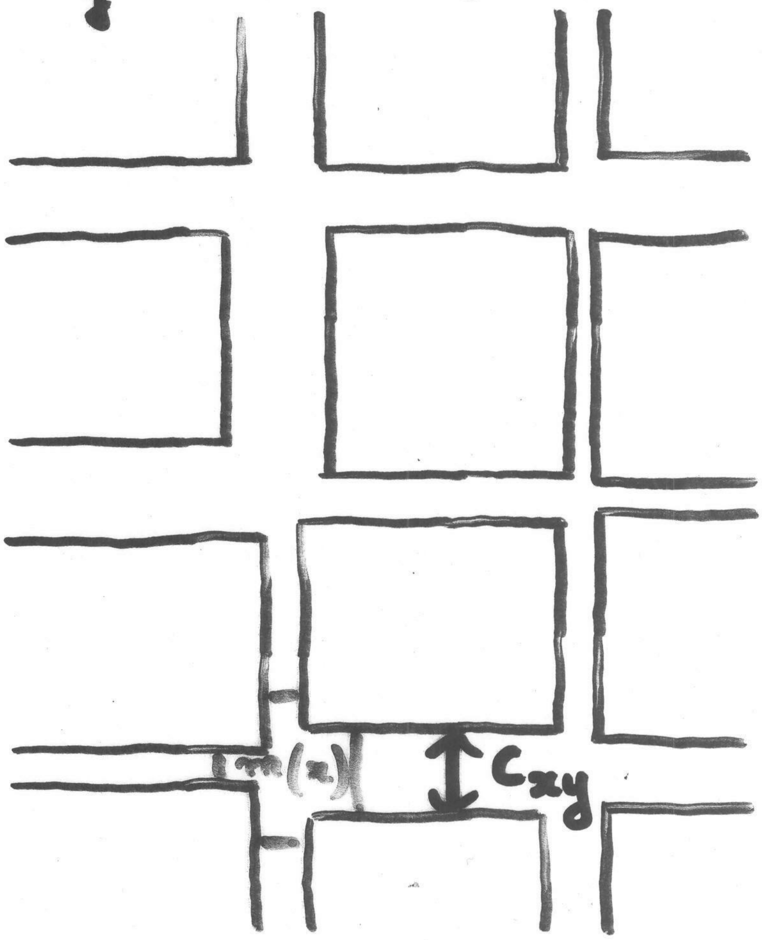
# REVERSIBLE RANDOM WALKS IN A RANDOM ENVIRONMENT OF ELLIPTIC CONDUCTANCES

Thierry DELMOTTE
Université Paul Sabatier
Toulouse (France)

Joint work with J.-D. DEUSCHEL

Conductance between $x, y \in \mathbb{Z}^d$
$\|x-y\| = 1$

$$0 < C_1 \leq C_{xy} = C_{yx} \leq C_2$$




$$m(x) = \sum_{y \sim x} C_{xy}$$

$$p(x,y) = \frac{C_{xy}}{m(x)}$$
$$\parallel$$
$$\ddot{P}(X_{n+1} = y \mid X_n = x)$$

m reversible

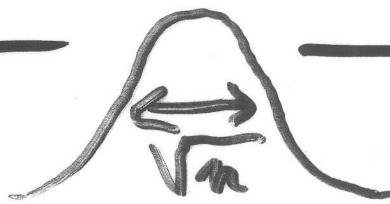
$$\left[\begin{aligned} &m(x)\, p(x,y) \\ &= m(y)\, p(y,x) \\ &= C_{xy} \end{aligned}\right]$$

$\exists\, c, C\,(d, C_1, C_2)$

for $\|x-y\|_1 \leq n$ and $\|x-y\|_1 \equiv n\ [2]$,

$$\frac{c}{n^{d/2}} e^{-C\frac{\|x-y\|^2}{n}} \leq p_n(x,y) \leq \frac{C}{n^{d/2}} e^{-c\frac{\|x-y\|^2}{n}}$$

---




and $\|y-z\|_1 = 2$,

$$\left|p_n^*(x,y) - p_n^*(x,z)\right| \leq \frac{C}{n^{d/2+1/2}} e^{-c\frac{\|x-y\|^2}{n}}$$

SRW $p^* \leftrightarrow C^*_{\cdot\cdot} \equiv 1$

---

Only $\exists\, \alpha(d, C_1, C_2) > 0$

$$\left|\frac{p_n(x,y)}{m(y)} - \frac{p_n(x,z)}{m(z)}\right| \leq \frac{C}{n^{d/2+\alpha}} e^{-c\frac{\|x-y\|^2}{n}}$$

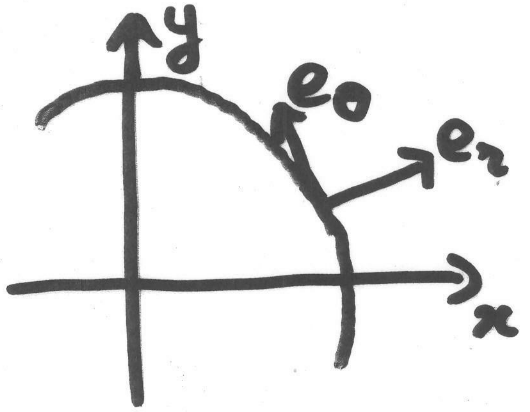
---

If $C_{\cdot\cdot}$ random stationary,

$$E\left(\left|\frac{p_n(x,y)}{m(y)} - \frac{p_n(x,z)}{m(z)}\right|\right) \leq \frac{C}{n^{d/2+1/2}} e^{-c\frac{\|x-y\|^2}{n}}$$

On $\mathbb{R}^2$ $u(r,\theta) = r^\eta \cos\theta$
satisfies $Lu = 0$

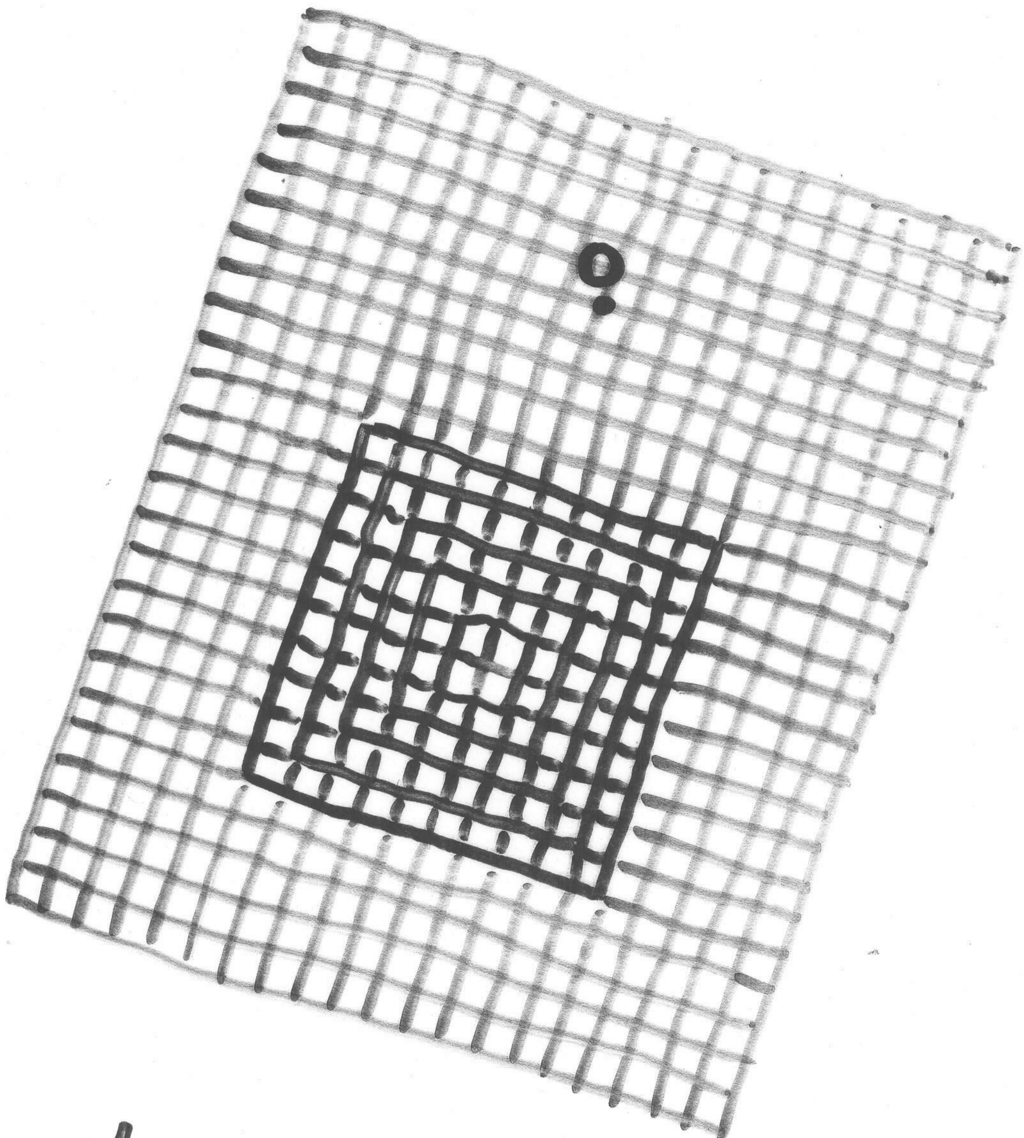
for $L = \operatorname{div}(A\nabla)$ $\quad A(x,y) \left| \begin{array}{l} e_r \longmapsto e_r \\ e_\theta \longmapsto \eta^2 e_\theta \end{array} \right.$





$$A\nabla u = \underbrace{\eta r^{\eta-1}\cos\theta}_{E_r} \cdot e_r + \underbrace{\eta^2 \frac{1}{r} r^\eta(-\sin\theta)}_{E_\theta} \cdot e_\theta$$

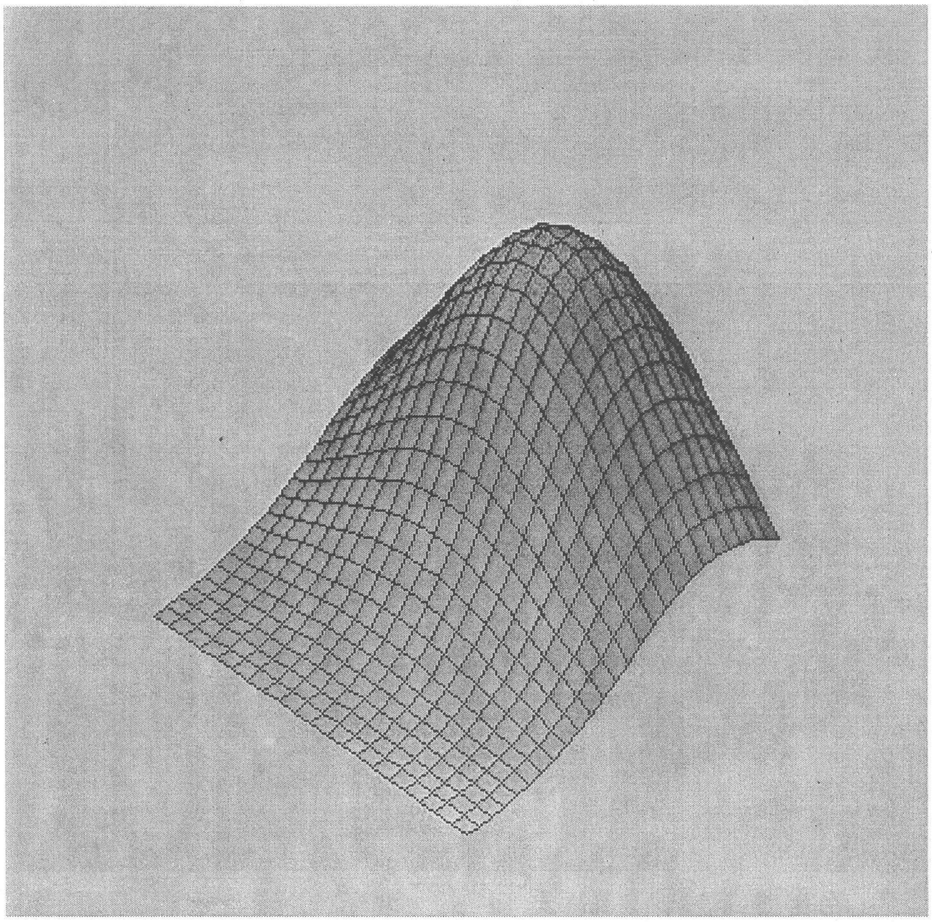
$$\begin{aligned} \operatorname{div}(A\nabla u) &= \frac{\partial E_r}{\partial r} + \frac{E_r}{r} + \frac{\partial E_\theta}{r\,\partial\theta} \\ &= \eta(\eta - \cancel{1})\, r^{\eta-2}\cos\theta \\ &\quad \cancel{+\eta\, r^{\eta-2}\cos\theta} - \frac{1}{r^2}\eta^2 r^\eta \cos\theta = 0 \end{aligned}$$

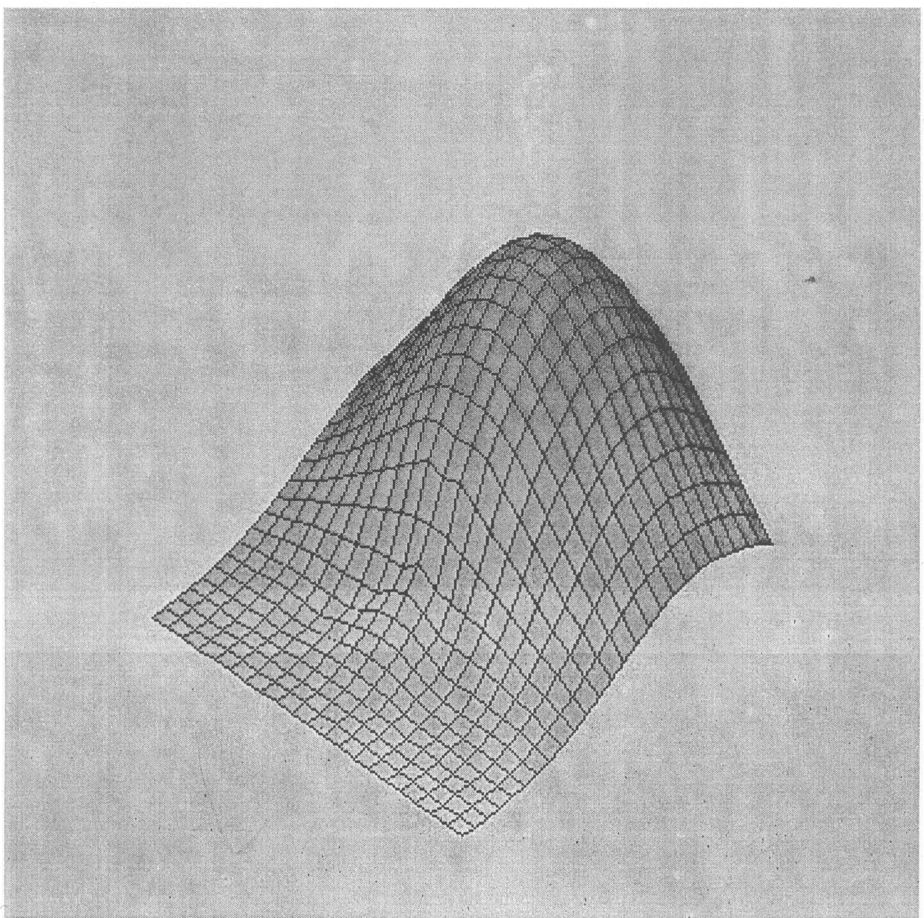


C.. = 1

C.. = 4

$$p^*_{89}(0,\cdot)+p^*_{90}(0,\cdot)$$





$$\frac{1}{m(\cdot)}\left[p_{89}(0,\cdot)+p_{90}(0,\cdot)\right]$$

lliptic reversible kernel estimates

De Giorgi Nash Moser $\mathbb{R}^d$ $\mathrm{div}(A\nabla)$
'50s '60s

Cheng - Yau '75 Manifold
Li - Yau '86 Ricc $\geq 0$

Grigor'yan '92 Manifold
Saloff-Coste '92 | Poincaré
Harnack inequality | Volume regularity
Equivalence of estimates with

Stroock-Zheng '97 $\mathbb{Z}^d$ Random
Delmotte '99 Walks

Delmotte-Deuschel '05

Derivatives in stationary environment

Fractal-like graphs
Barlow, Bass,
Grigor'yan, Telcs, ...