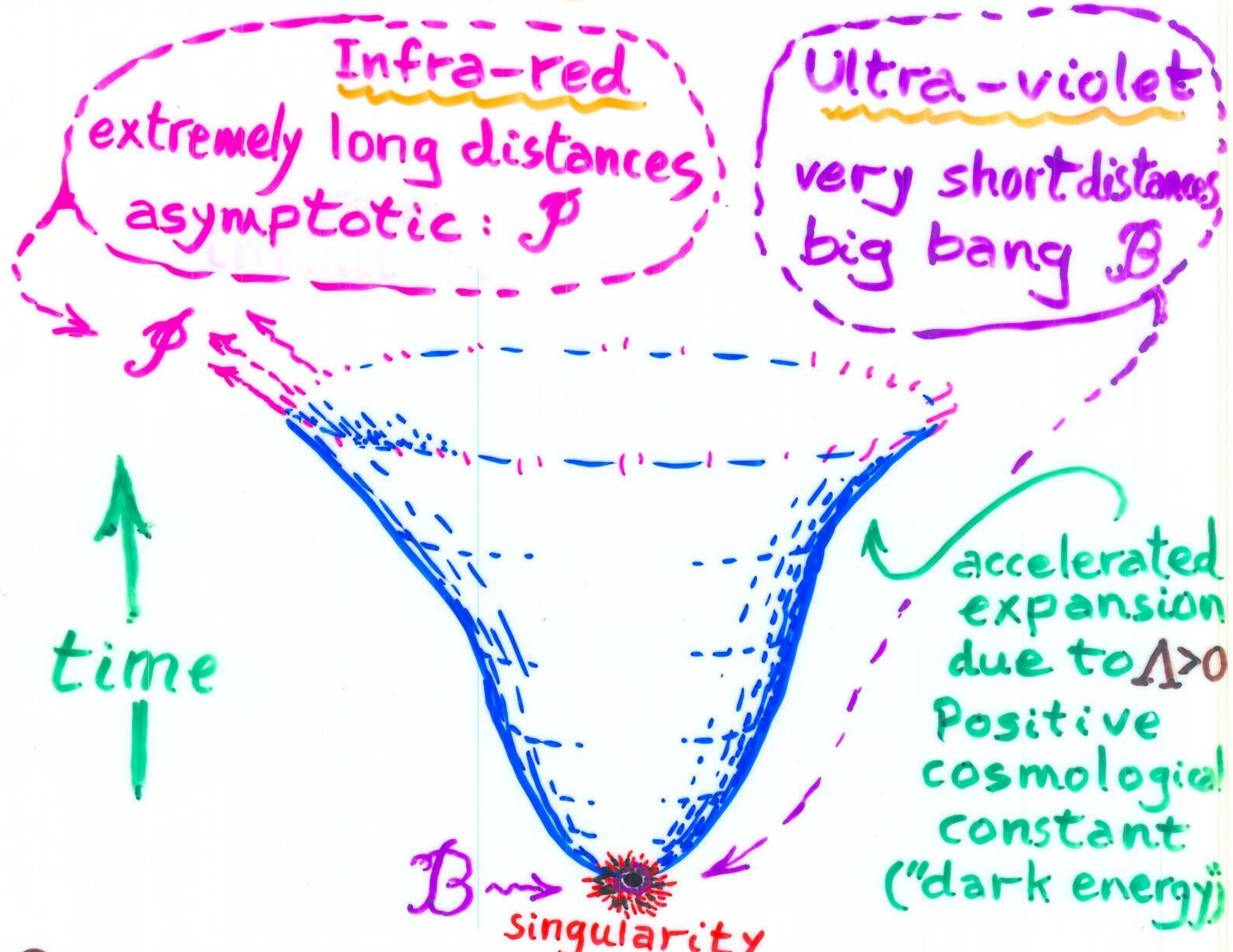
Cosmological Twistors

Relevance to Scattering Amplitudes:

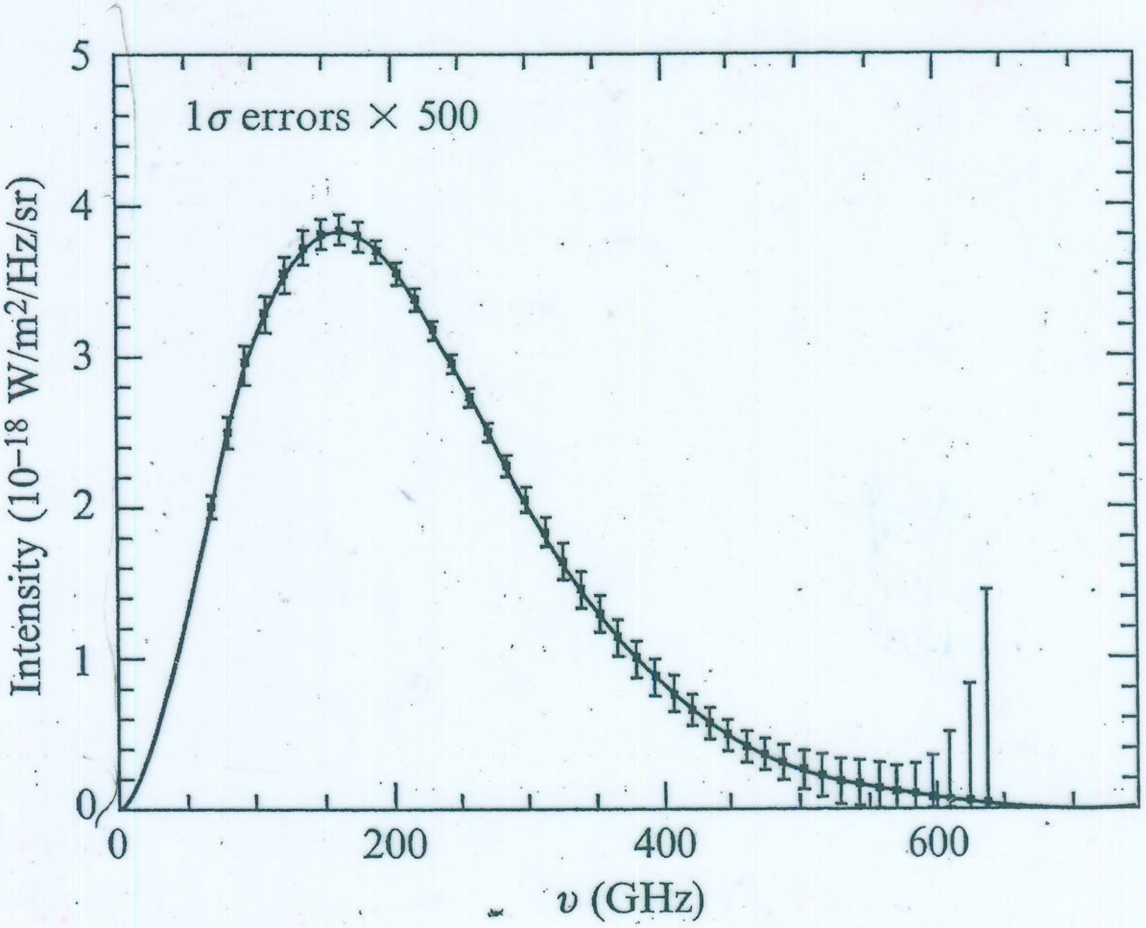
Divergences



Current standard picture of the expanding universe

- inflation?
- spatial curvature close to flat; could be positive, negative or zero.

Spectrum of the Microwave Background



Note: error bars are exaggerated by a factor of 500.

The solid curve displays the Planck black body spectrum of *thermal equilibrium*.

2nd Law of Thermodynamics

Entropy increases with time

= "disorder" (roughly speaking)

Gas in a box

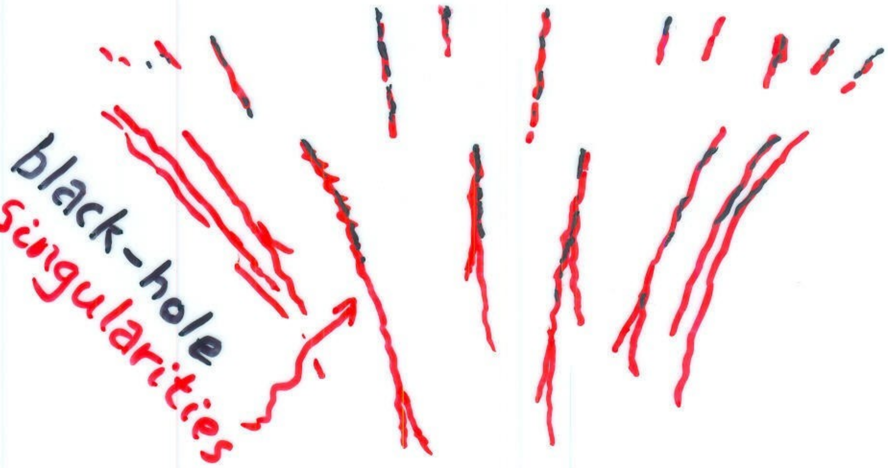
time increases

entropy increases

Gravitating bodies

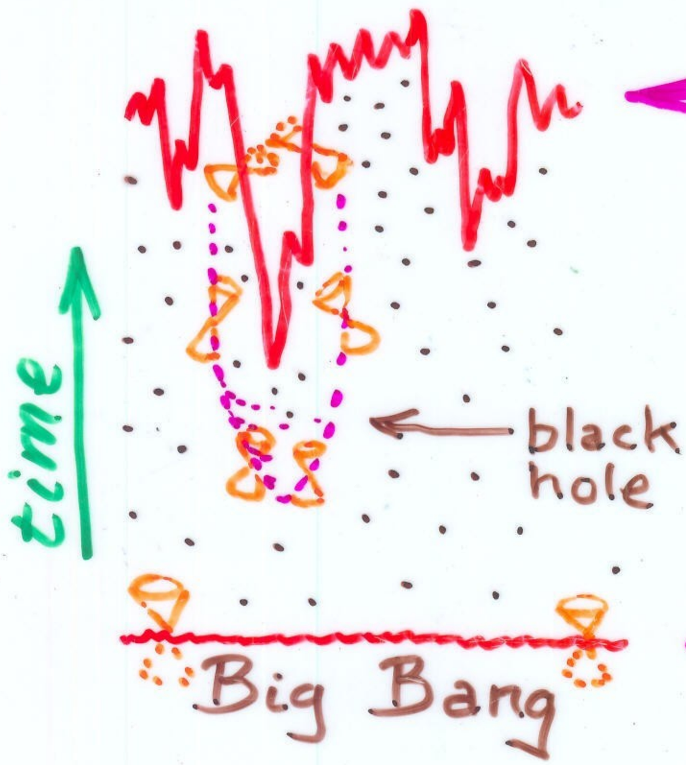
Maximum entropy:

BLACK HOLE



with irregularities

Fundamental Asymmetry in space-time singularity structure:



future singularity
Weyl $\rightsquigarrow \infty$

past singularity
Weyl $\rightsquigarrow 0$

$\boxed{\text{Weyl curvature hypothesis}}$

Implies
- ~Uniformity of microwave background
- 2^{nd} Law of Thermodynamics

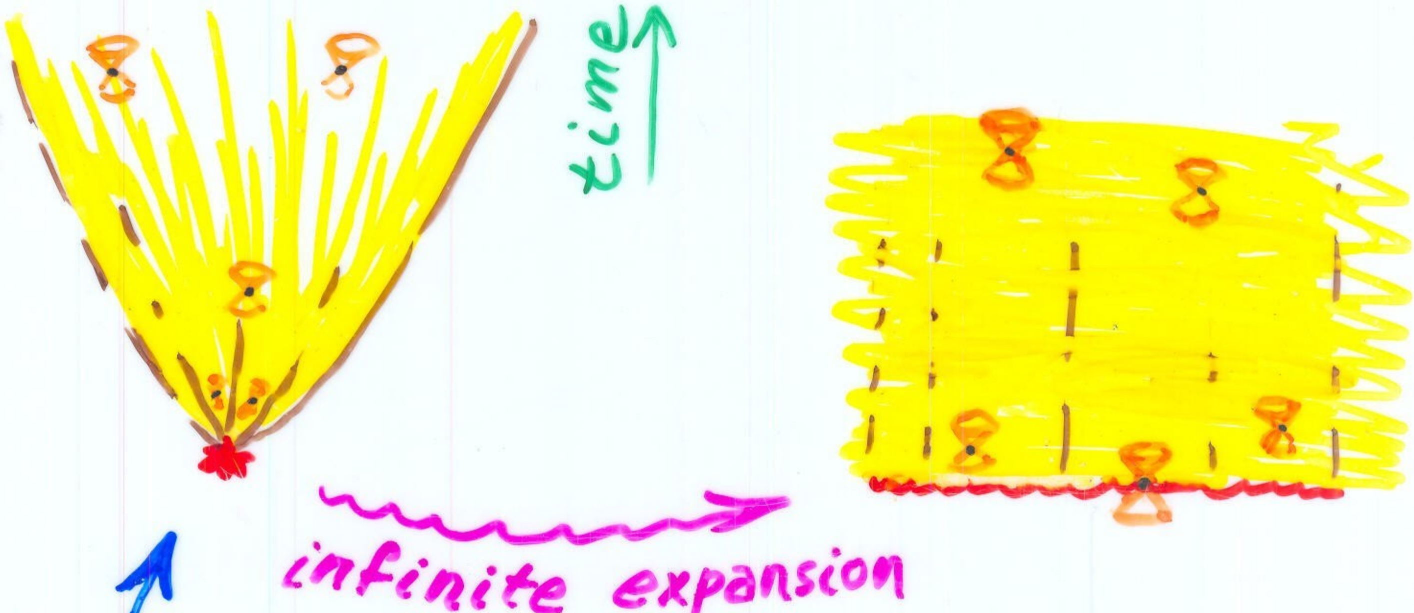
Constraint on Big Bang: 1 part in $\gg 10^{10^{123}}$ (125)

[from Bekenstein-Hawking black-hole entropy of 10^{80} protons/neutrons]

Quantum Gravity ?

Not any conventional approach (time-asymm.)

Tod's form of the Weyl curvature hypothesis



requires highly constrained Weyl curvature at the Big Bang

"mathematical trick" $\Omega \to \infty$

$\hat{g}_{..} = \Omega^2 g_{..}$

Near the big bang, energies get so great that mass becomes irrelevant and particles are effectively massless. Conformal invariance

Conformal geometry (light cones) holds

The Extremely Remote Future

Much matter collapses to black holes

Eventually, the expanding universe cools to lower than the holes' Hawking temperatures (the larger the hole, the lower the temperature — always very low!).

Then, the hole evaporates away — very slowly — until pop! it disappears!

~10^{64} yrs for $M_\odot$, ~10^{90} yrs for galactic

Provided protons, etc., eventually decay, then matter itself disappears into radiation.

Scheme appears to require:

- decay of the mass of electrons, massive neutrinos, etc., into massless ingredients — at least asymptotically, in remote future
- mass ratios (e.g. $m_e : m_p : m_N$) might evolve with time — and
- other numerical constants Nature (e.g. fine struc. const.) might evolve
- GR metric is Milne's "dynamical" metric ("atomic" metrics give finite aeon time)

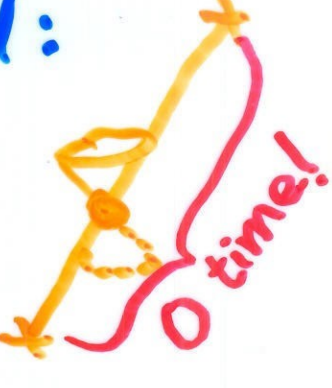
With only massless ingredients left, the universe loses track of time. All contents of the universe would be conformally invariant

No way of constructing a clock — only the light-cone structure remains

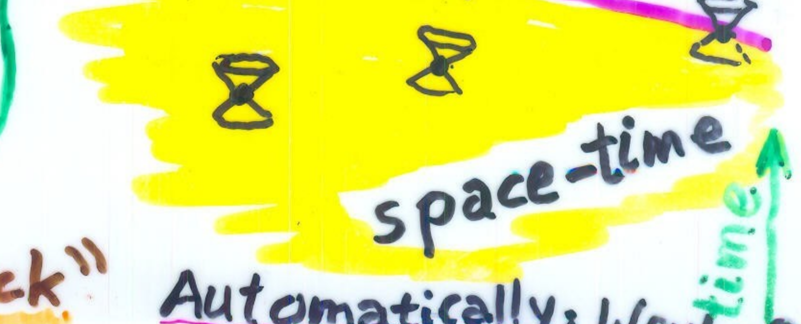
Conformal geometry

To a photon (or other massless entity) no time is experienced between beginning and end:

Eternity is no time at all, to a photon!



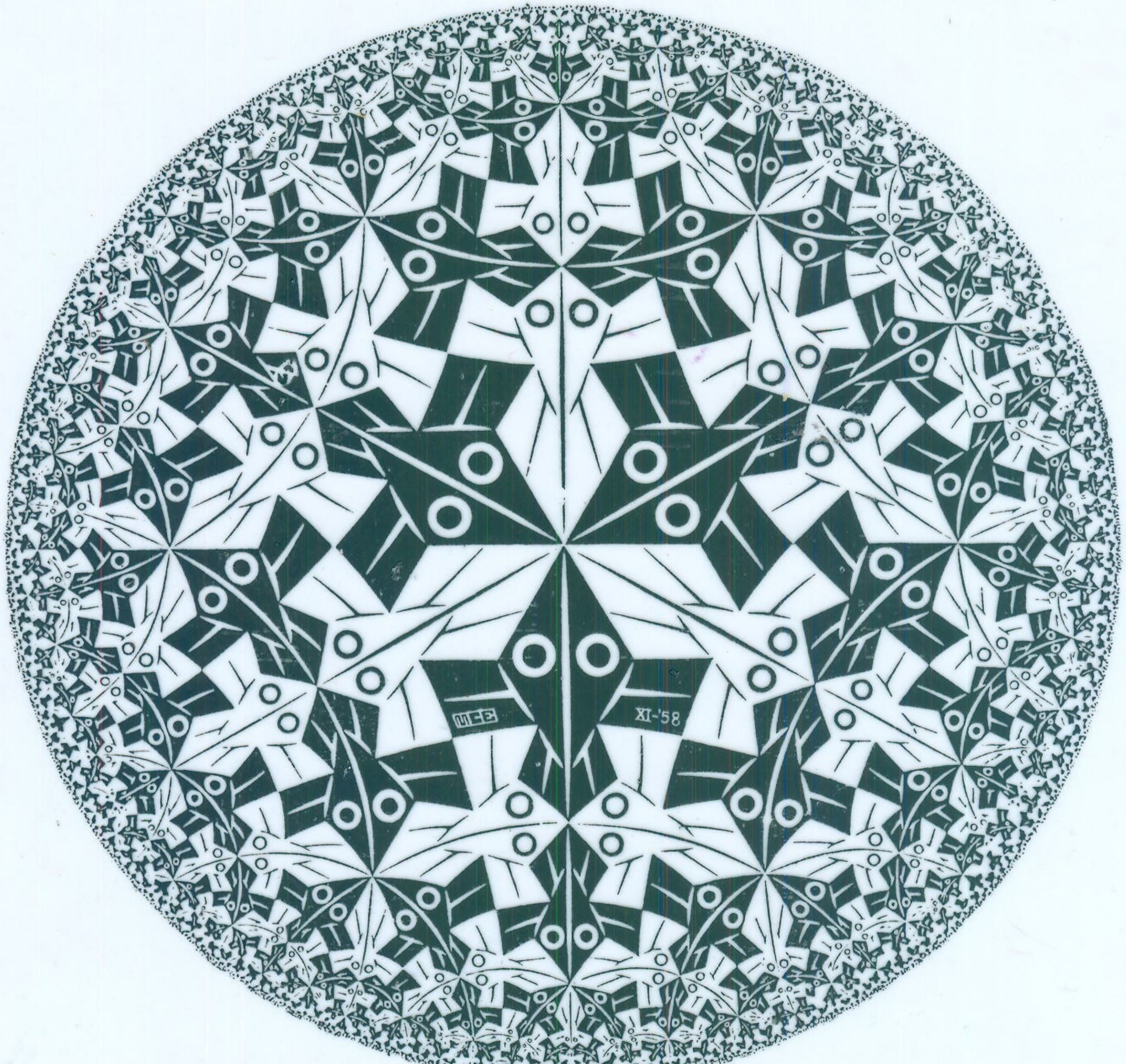
Conformal infinity $\mathscr{I}$

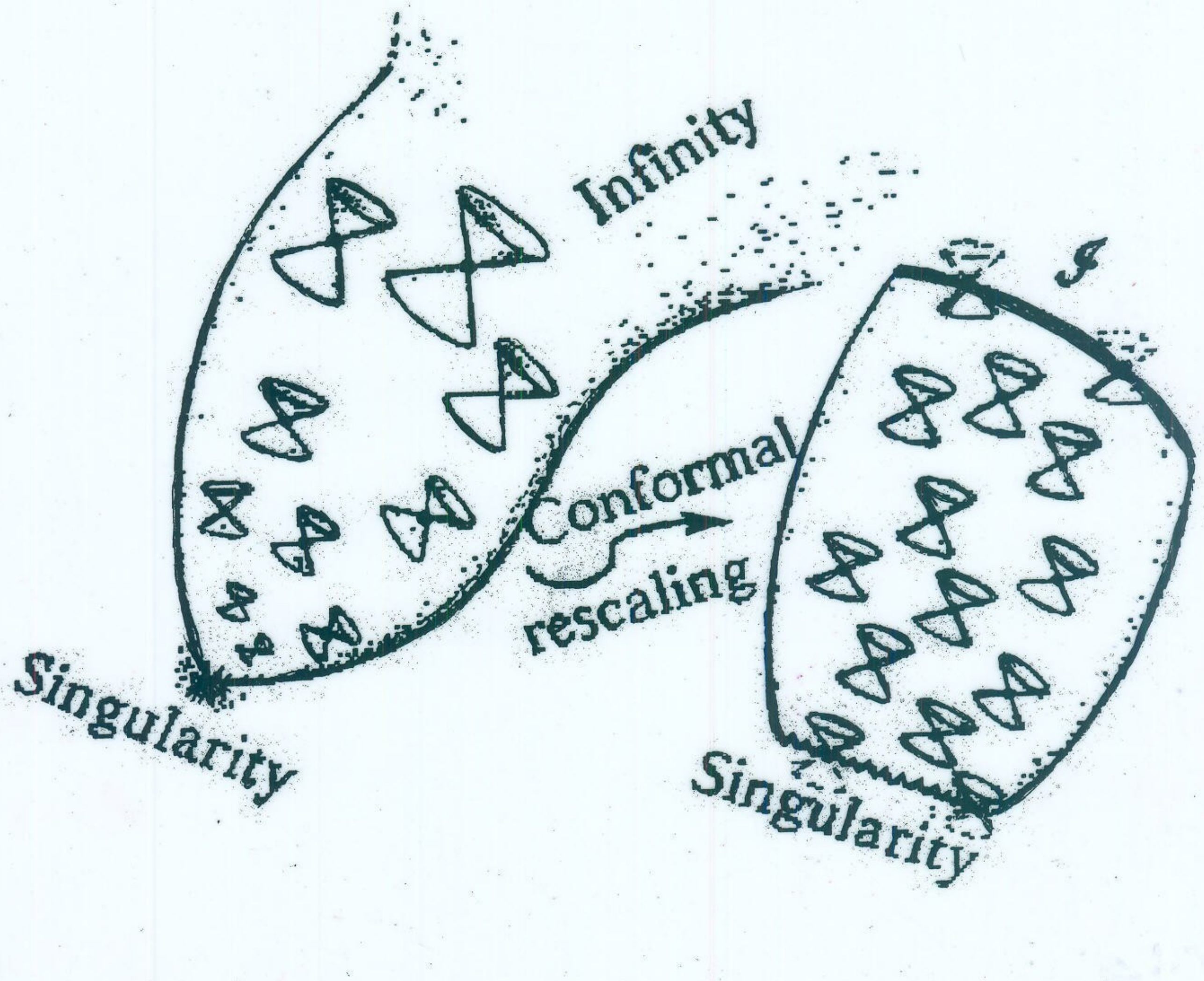


"mathematical trick"

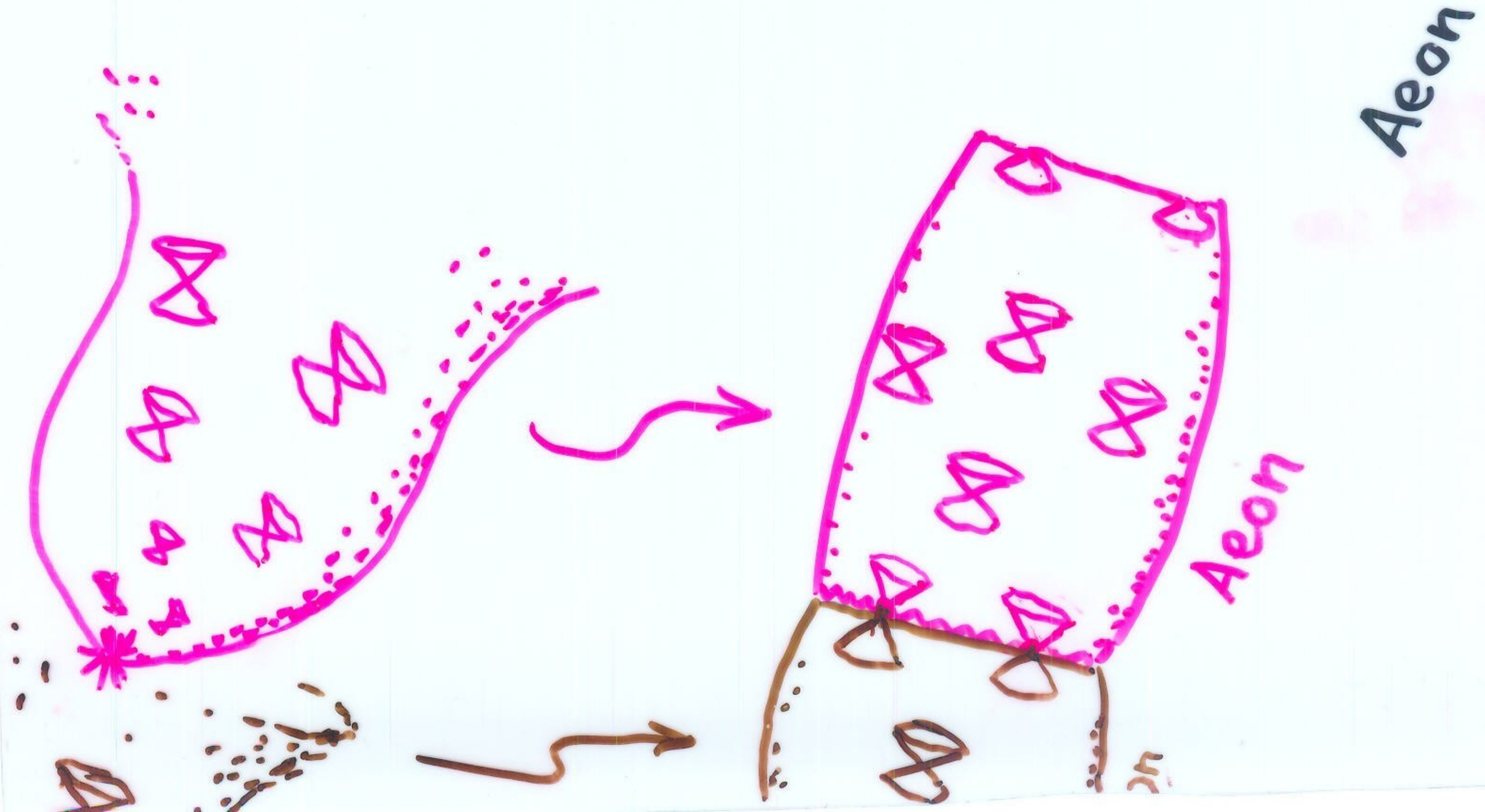
$\Omega \to 0$ $\quad \hat{g}_{..} = \Omega^2 g_{..}$

Automatically: Weyl = 0 at $\mathscr{I}^+$


XI-'58


Infinity
Conformal
rescaling
Singularity
Singularity

Conformal Cyclic Cosmology



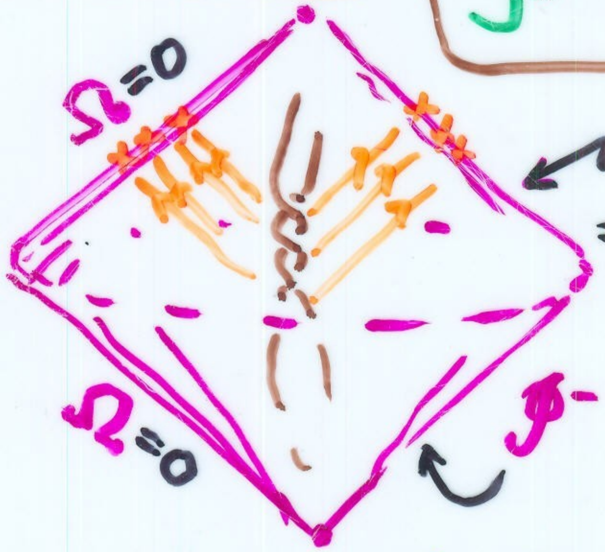
2 Conformal "Mathematical Tricks"

Gravitational radiation

physical metric

$$\hat{g}_{..} = \Omega^2 g_{..}$$

Cosmological singularities

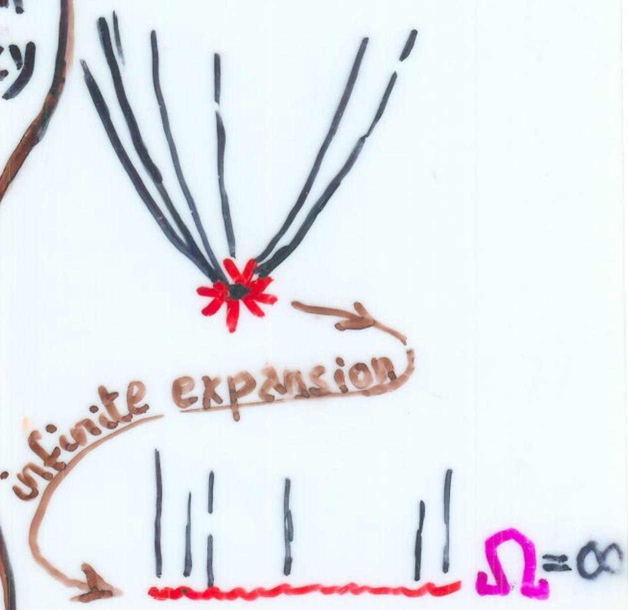


RP~1962

Asymptotically flat space-time $\Lambda=0$

Trick: shrink ∞ to a finite place by taking $\Omega=0$ there

When $\Lambda>0$, this still works (in some sense better (RP Friedrich) – easier) but then $\mathscr{I}^+$ is spacelike rather than null

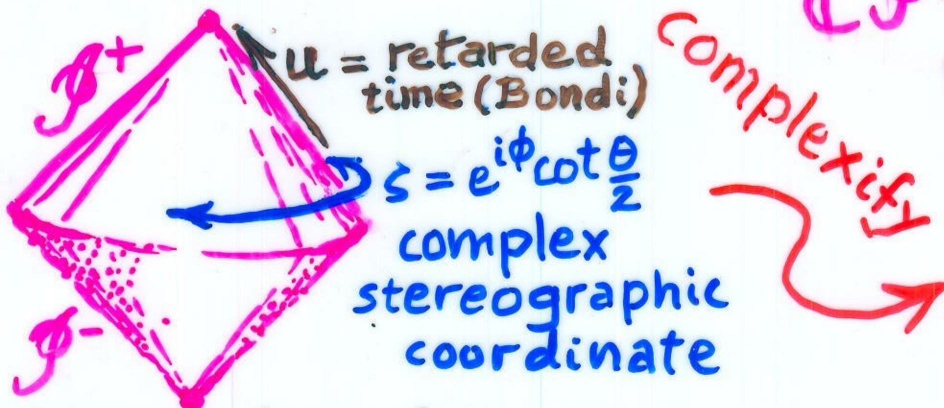


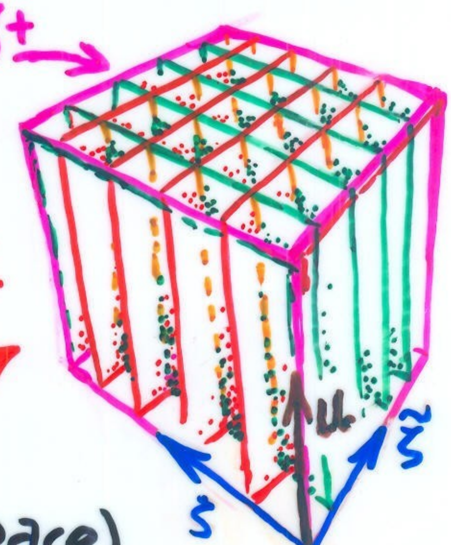
Trick: expand out singularity to obtain a conformally smooth initial hypersurface

Tod's form of Weyl curvature hypothesis:

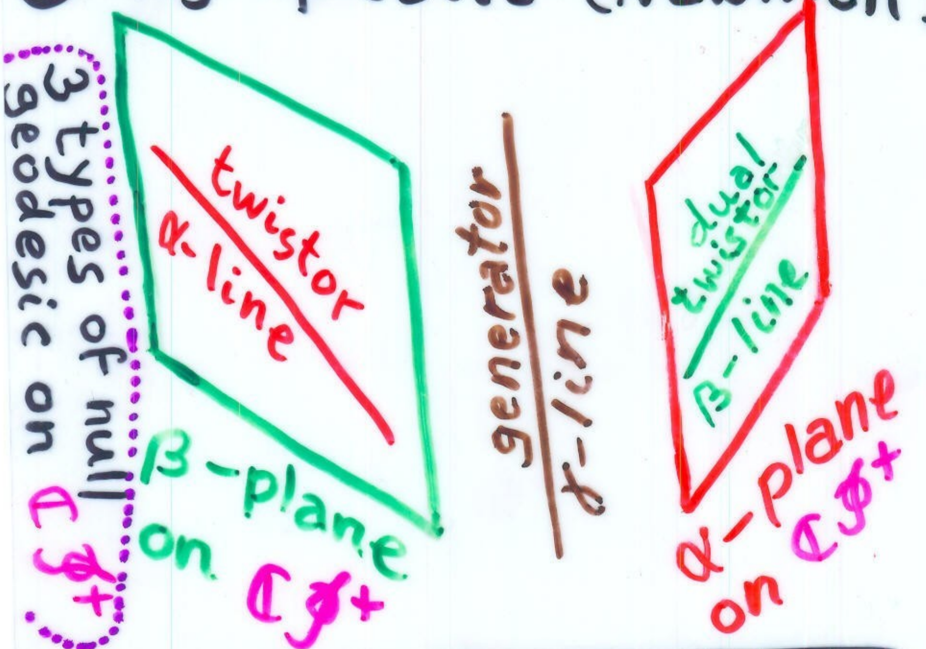
this works!

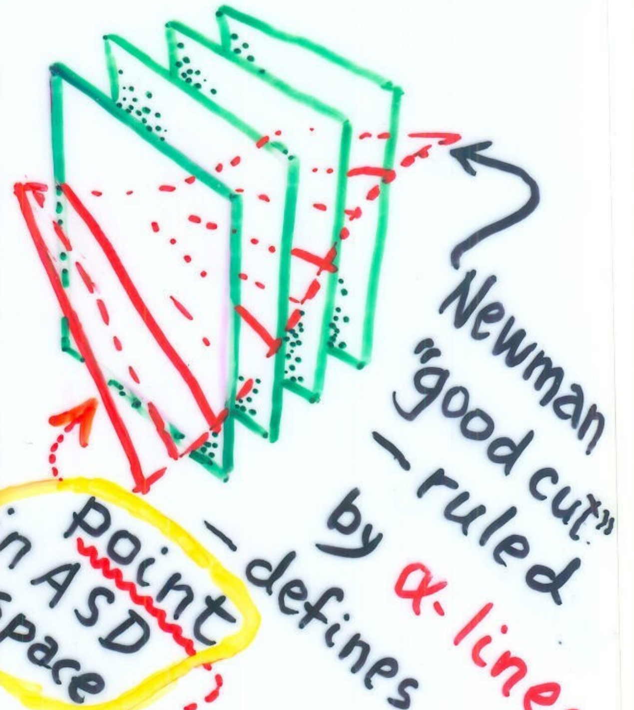
Anti-self-dual (complex) solutions of the Einstein vacuum equations: 2 original twistor approaches ($\Lambda = 0$).

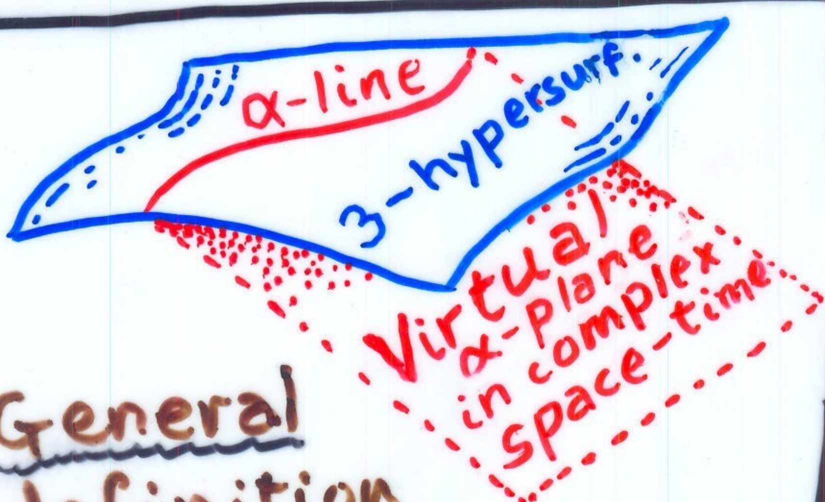




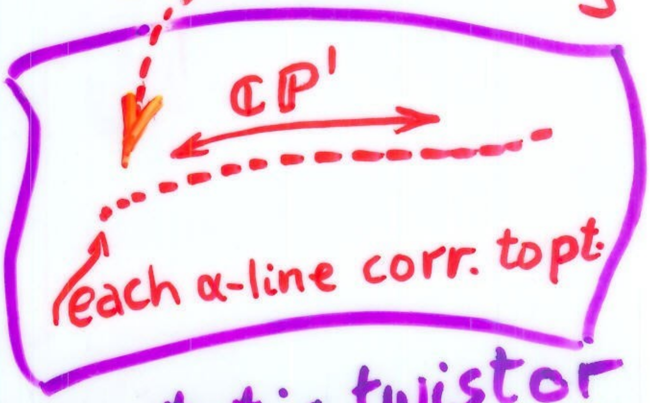
① Asymptotic (Newman $\mathcal{H}$-Space)







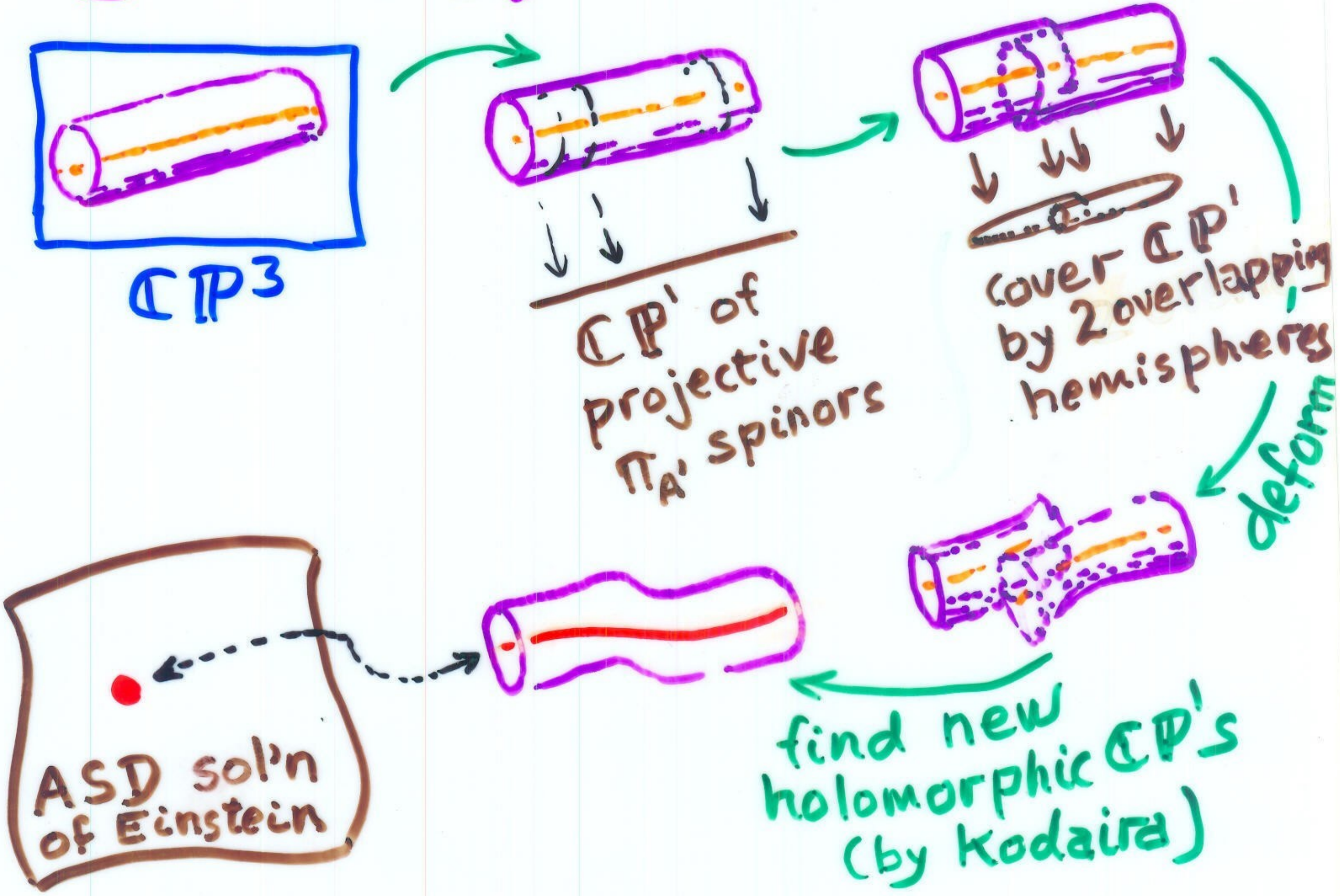
General definition of hypersurface twistor (projective) as α-line.



Asymptotic twistor space

② "Constructive" (non-linear graviton)

Produce deformed version of (a region in) $\mathbb{CP}^3$



Structure of the deformed twistor space: Let's phrase things in terms of non-projective space

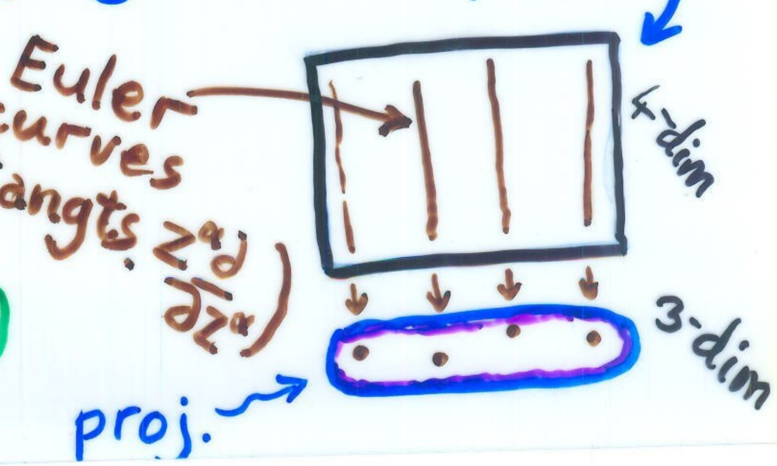
$\Upsilon \left(= Z^\alpha \frac{\partial}{\partial Z^\alpha}\right)$

$\iota \left(= \pi_{A'} d\pi^{A'}\right)$

$\tau \left(= \frac{1}{2} d\pi_{A'} \wedge d\pi^{A'}\right)$

$\theta \left(= \frac{1}{6} \epsilon_{\alpha\beta\gamma\delta} Z^\alpha dZ^\beta \wedge dZ^\gamma \wedge dZ^\delta\right)$

$\phi \left(= dZ^0 \wedge dZ^1 \wedge dZ^2 \wedge dZ^3\right)$



In the case $\Lambda=0$ (RP 1976), we have

$$d\iota = 2\tau \qquad d\theta = 4\phi$$

$$\tau \lrcorner \Upsilon = \iota \qquad \phi \lrcorner \Upsilon = \theta$$

and $\Upsilon = \theta \div \phi$ in the sense $d a_\wedge \theta = \Upsilon(a)\phi$

where τ is "simple":

$$\tau_\wedge \tau = 0 \quad \text{i.e.} \quad \iota_\wedge \tau = 0$$

We also have the homogeneity relations

$$\pounds_\Upsilon \iota = 2\iota, \; \pounds_\Upsilon \tau = 2\tau, \; \pounds_\Upsilon \theta = 4\theta, \; \pounds_\Upsilon \phi = 4\phi$$

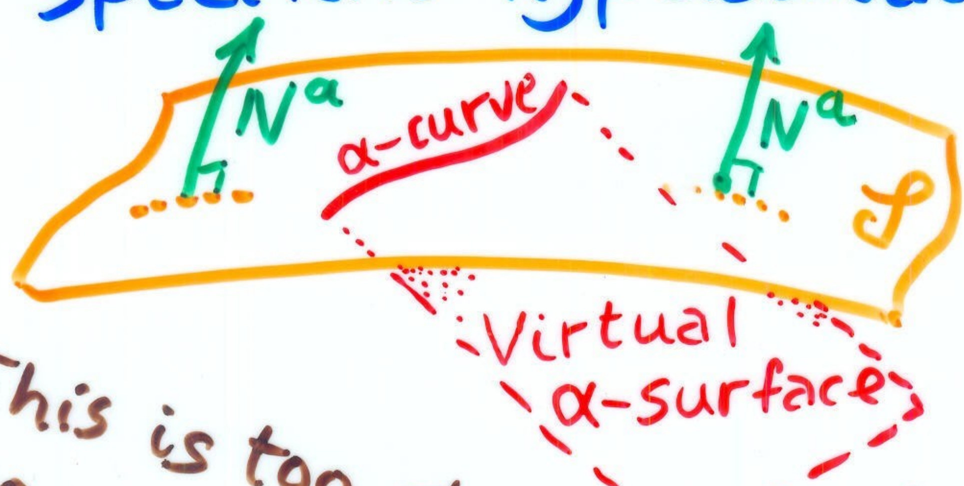
In 1980, R.S. Ward generalized this to $\Lambda \neq 0$, in effect just by relaxing the "simplicity" conditions above to:

$$\tau_\wedge \tau = \tfrac{1}{3}\Lambda\phi \quad \text{i.e.} \quad \iota_\wedge \tau = \tfrac{1}{6}\Lambda\theta$$

Question: what is the analogue of ① (Newman's "$\mathcal{H}$-space" method) in the case $\Lambda > 0$, which appears to be the cosmologically appropriate case?

When $\Lambda > 0$, we have a spacelike $\mathscr{I}^+$ (and we may also assume that the conformally "stretched out" big bang hypersurface $\mathscr{B}$ is spacelice; and according to CCC, $\mathscr{B}$'s conformal structure has the same character as that of $\mathscr{I}^+$).

Hypersurface twistors for a spacelike hypersurface $\mathscr{S}$:



An α-surface would have tangent vectors $\xi^A \pi^{A'}$ for arbitrary ξ^A where $\pi^{A'} \nabla_{AA'} \pi_{B'} = 0$

This is too strong, so we restrict to $\mathscr{S}$:

$$\underbrace{(N^{AC'}\pi_{C'})\pi^{A'}}_{\text{tangent to } \mathscr{S}} \nabla_{AA'} \pi_{B'} = 0$$

The solutions of this give us the α-curves and therefore the points of hyp. twistor space for $\mathscr{S}$

Now choose $\mathscr{S}$ to be $\mathscr{I}^+$ (or $\mathcal{B}$).
We find (rather surprisingly) that with the natural choice

$$\hat{N}_a = -\nabla_a \Omega$$

conformal factor

where

$$\hat{g}_{ab} = \Omega^2 g_{ab}$$

rescaled metric to make $\mathscr{I}^+$ finite

physical metric

$$\left(\hat{N}_a \hat{N}^a = \tfrac{1}{3}\Lambda \text{ on } \mathscr{I}^+\right)$$

that

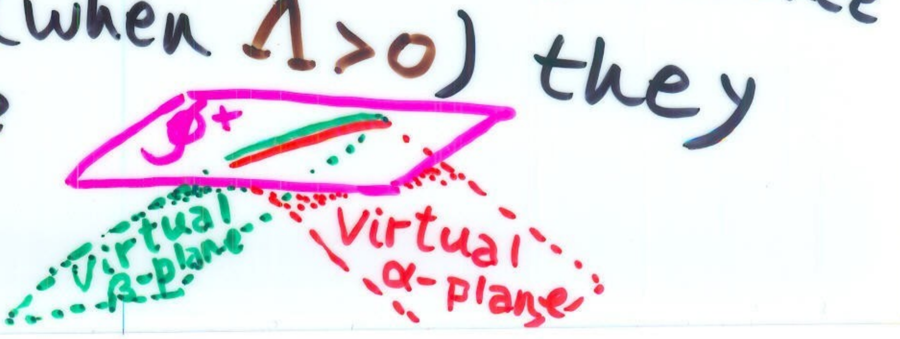
$$(N^{AC'}\pi_{C'})\pi^{A'} \nabla_{AA'}\{(N^{BD'}\pi_{D'})\pi^{B'}\} = 0$$

as a consequence of the "asymptotic Einstein condition"

$$\hat{\nabla}_{A'(A}\hat{N}_{B)B'} = 0 \text{ on } \mathscr{I}^+$$

(with only massless fields present on $\mathscr{I}^+$).

It follows that the α-lines on $\mathscr{I}^+$ are actually geodesics on $\mathscr{I}^+$ (as they are in the $\Lambda=0$ case), whence by symmetry (when $\Lambda>0$) they must also be β-lines!



This has various striking implications (apparently):

- The asymptotic twistor space is a complex-symplectic manifold [so: ι, τ exist as in Ward's constr: i.e. there's a non-degenerate $I_{\alpha\beta}$]
- We expect to find 3 distinct families of holomorphic $\mathbb{CP}^1$'s giving 3 different analogues of Newman's $\mathcal{H}$-space:
  - an SD Ward space
  - an ASD Ward space
  - a conformally flat Ward sp.

from taking various kinds of $\Lambda \to 0$ limits