# Scattering Amplitudes, MHV Diagrams, and Wilson Loops

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| | |
|---|---|
| Brandhuber, Spence, GT | hep-th/0612007 |
| Brandhuber, Spence, Zoubos, GT | 0704.0245 [hep-th] |
| Nasti, GT | 0706.0976 [hep-th] |
| Brandhuber, Heslop, GT | 0707.1153 [hep-th] |
| Brandhuber, Heslop, Spence, GT | in preparation |

Twistors, Strings, and Scattering Amplitudes, LMS Durham Symposium
Durham, August 2007

# Outline

- MHV diagrams (Cachazo, Svrcek, Witten)
  - Loop MHV diagrams (Brandhuber, Spence, GT)
- Amplitudes in pure Yang-Mills from MHV diagrams (Brandhuber, Spence, Zoubos, GT)
  - All-plus amplitude
- MHV amplitudes in N=4 SYM from a Wilson loop calculation at weak coupling
  - One-loop calculation at $n$ points (Brandhuber, Heslop, GT)
  - Higher loops (Brandhuber, Heslop, Spence, GT, in preparation)

# Motivations

- Unifying theme is simplicity of amplitudes
  - Geometry in Twistor Space
- unexplained by Feynman diagrams
  - Parke-Taylor formula for Maximally Helicity Violating amplitude of gluons (helicities are a permutation of $--++ \ldots +$)
- New methods account for this simplicity, and allow for very efficient calculations

☞ Number of Feynman diagrams for scattering $gg \longrightarrow n\, g$ :

| $n$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| # of diagrams | 4 | 25 | 220 | 2485 | 34300 | 559405 | 10525900 |

(tree level)

☞ Result is: $\mathcal{A}(1^{\pm}, 2^{+}, \ldots n^{+}) = 0$

at tree level

$$\mathcal{A}_{\mathrm{MHV}}(1^{+} \ldots i^{-} \ldots j^{-} \ldots n^{+}) = \frac{\langle i\, j\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

Large numbers of Feynman diagrams combine to produce unexpectedly and mysteriously simple expressions


LHC is coming !

# Amplitudes

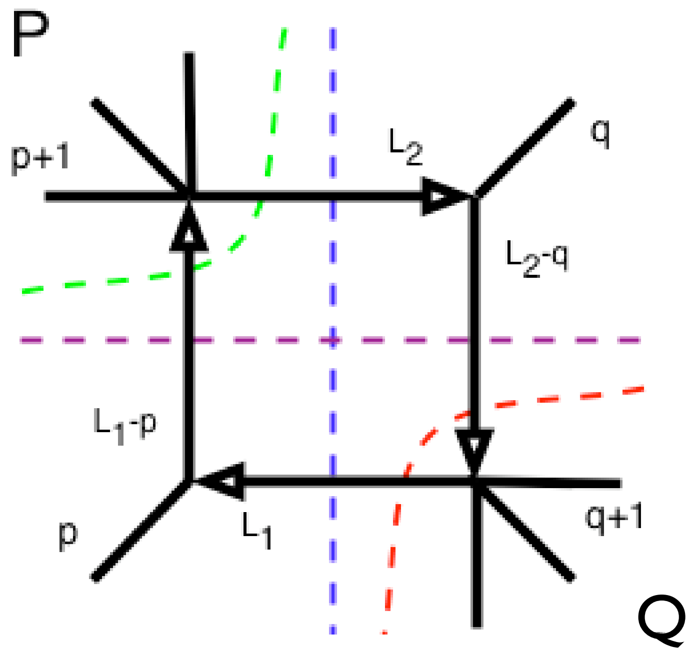
$$\mathcal{A} = \mathcal{A}(\{\lambda_i, \tilde{\lambda}_i, ; h_i\})$$

- Colour-ordered partial amplitudes
  - momenta and polarisation vectors expressed in terms of spinors and helicities
  - colour indices stripped off
- Planar theory

# Simplicity of amplitudes persists at loop level:

- $n$-point MHV amplitude in N=4 SYM at one loop:

$$\mathcal{A}_{\rm MHV}^{1-\rm loop} = \mathcal{A}_{\rm MHV}^{\rm tree} \sum$$




- Sum of two-mass easy box functions, all with coefficient 1

- Computed in 1994 by Bern, Dixon, Dunbar, Kosower using unitarity
- Rederived in 2004 with loop MHV diagrams...

  (Brandhuber, Spence, GT)
- ...and, more recently, with a weakly-coupled Wilson loop calculation, with the Alday-Maldacena polygonal contour (Brandhuber, Heslop, GT)

# All-loop conjecture of Bern, Dixon, Smirnov

Zvi Bern and Anastasia Volovich's talks

- $n$-point MHV amplitudes in N=4 SYM

$$\mathcal{M}_n = \exp\left[\sum_{L=1}^{\infty} a^L \left( f^{(L)}(\varepsilon)\mathcal{M}_n^{(1)}(L\varepsilon) + C^{(L)} + E_n^{(L)}(\varepsilon)\right)\right]$$

- $\mathcal{M}_n := \mathcal{A}_{n,\mathrm{MHV}} / \mathcal{A}_{n,\mathrm{MHV}}^{\mathrm{tree}}$

- $\mathcal{M}_n^{(1)}(\varepsilon)$ is the all-orders in $\varepsilon$ one-loop amplitude

- $f^{(L)}(\varepsilon) = f_0^{(L)} + \varepsilon f_1^{(L)} + \varepsilon^2 f_2^{(L)}$

anomalous dimension of twist-two operators at large spin

- $C^{(L)},\ E_n^{(L)}(\varepsilon)$

More on this later...

# Another intriguing, simple amplitude:

- All-plus amplitude in pure Yang-Mills, 1 loop

$$\mathcal{A}_n^{1-\text{loop}}(1^+,\ldots,n^+) = \frac{-i}{48\pi^2} \sum_{1\le l_1<l_2<l_3<l_4\le n} \frac{\mathrm{Tr}(\frac{1-\gamma^5}{2}\hat{l}_1\,\hat{l}_2\,\hat{l}_3\,\hat{l}_4)}{\langle 1\,2\rangle\,\langle 2\,3\rangle\cdots\langle n\,1\rangle}$$

finite, rational

  - like MHV amplitude, no multiparticle poles
  - all-plus equivalently computed in Self-Dual Yang-Mills
  - vanishes in supersymmetric theories
  - dimension shifting relations (Bern, Dixon, Dunbar, Kosower)

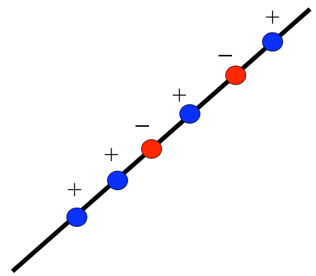
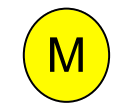
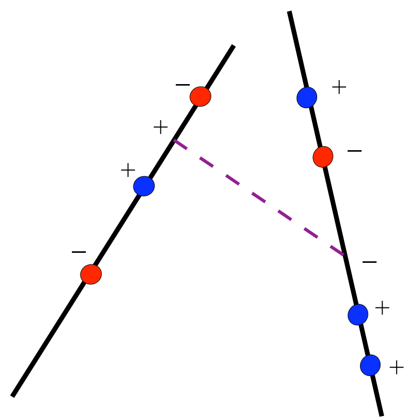
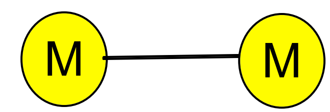
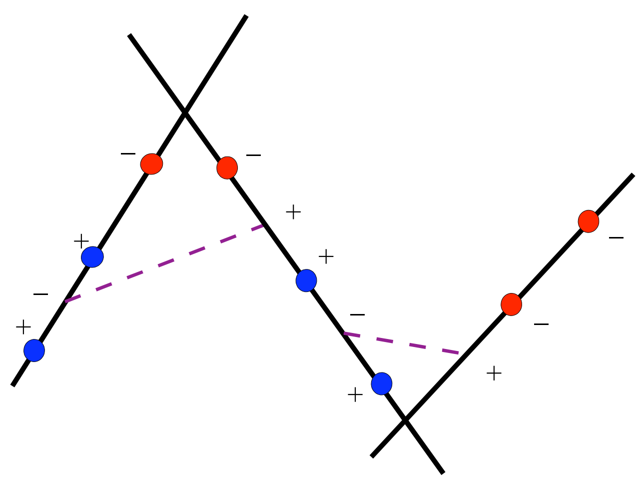
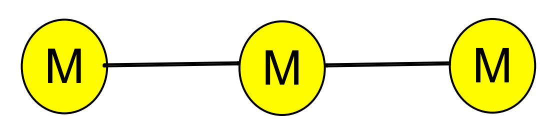
- Escapes naive application of MHV rules !

One-loop vertex ?

# Amplitudes in Twistor Space

(Witten, 2003)

- Scattering amplitudes are supported on algebraic curves in Penrose's twistor space
- $d = q - 1 + l$ $\quad$ $q$ = # negative helicity gluons, $l$ = # loops
- $g \leq l$
  - Tree MHV: $q=2,\ l=0$ ➡ $d=1,\ g=0$ (complex line)

| Amplitude | Twistor space structure | MHV diagrams |
| --- | --- | --- |
| MHV |  |  |
| nMHV |  |  |
| nnMHV |  |  |

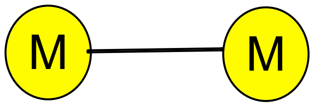
# Why MHV diagrams

- MHV amplitudes ➡ complex lines in twistor space (Witten)
- Line in twistor space ➡ point in Minkowski space (Penrose)
- MHV amplitude ➡ local interaction in spacetime ! (Cachazo, Svrcek, Witten)
    - Locality in lightcone formulation (Mansfield; Gorsky & Rosly)

# MHV Rules

(Cachazo, Svrcek, Witten)

- MHV amplitude ➜ MHV vertex
- Off-shell continuation for internal (possibly loop) momenta needed
  - Same as in lightcone Yang-Mills




- Internal momentum is off-shell
- Need to define spinor $\lambda$ for an off-shell vector!

- Scalar propagators connect MHV vertices

- Off-shell prescription:

$$L_{a\dot{a}} = l_{a\dot{a}} + z\eta_{a\dot{a}}$$

  - $l_{a\dot{a}} := l_a \tilde{l}_{\dot{a}}$ is the off-shell continuation
  - $\eta$ is a reference vector

- Draw all diagrams obtained by sewing $d = q - 1 + l$ MHV vertices

  $q$ = # negative helicity gluons,

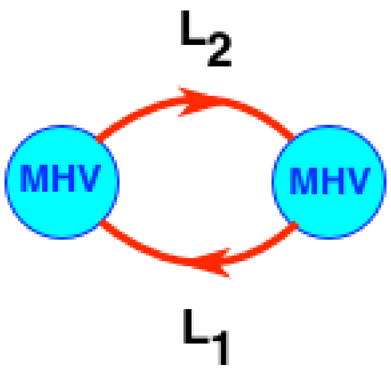
  $l$ = # loops

- Examples:
  - MHV: $q=2, \quad l=1 \qquad d=2$
  - All minus: $q=n, \quad l=1 \qquad d=n$
  - All plus: $q=0, \quad l=1 \qquad d=0$ ??

# One-loop MHV amplitudes in N=4

(Brandhuber, Spence, GT)

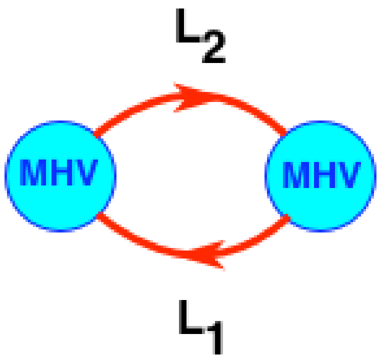
$$\sum \int d\mathcal{M}$$




- Sum over
  - all possible MHV diagrams
  - internal particle species (g, f, s) and helicities
- $d\mathcal{M}$ = phase space measure X dispersive measure
- Different from unitarity-based approach of BDDK

# The integration measure

- $P_L$ is the momentum on the left




$$d\mathcal{M} := \frac{d^4L_1}{L_1^2+i\varepsilon}\frac{d^4L_2}{L_2^2+i\varepsilon}\,\delta^{(4)}(L_2-L_1+P_L)$$

- Use $L = l + z\eta$ , and $L \to (l, z)$

$$\Rightarrow \quad \frac{d^4L}{L^2+i\varepsilon} = \frac{dz}{z+i\,\mathrm{sgn}\,(l_0\eta_0)\varepsilon}\;\frac{d^3l}{2l_0}$$

dispersive measure X phase-space measure (Nair measure)

# Applications (with supersymmetry)

- One-loop MHV amplitudes in N=4 SYM

  (Brandhuber, Spence, GT)

- One-loop MHV amplitudes in N=1,2 SYM

  (Bedford, Brandhuber, Spence, GT; Quigley, Rozali)

  - No twistor string theory for N=1 SYM, nevertheless MHV diagram method works

# Proving MHV diagrams at one loop

Supersymmetric theories

(Brandhuber, Spence, GT)

- Covariance ($\eta$-independence)

  Feynman Tree Theorem

- Correct singularity structure
  - Discontinuities across (generalised) cuts
  - Soft, collinear
  - Multiparticle

- Use tree-level BCFW proof at one loop:
  - If all singularities match, and the amplitude is covariant, then $\mathcal{A}_{\mathrm{MHV}} - \mathcal{A}_{Feynman}$ is a polynomial in the external momenta whose dimension is 4 − # particles ➡

$$\mathcal{A}_{\mathrm{MHV}} = \mathcal{A}_{Feynman}$$

- Proof from field redefinition on lightcone Yang-Mills action (Mansfield)
- Proof from twistor actions (Boels, Mason, Skinner)

Tim Morris and Rutger Boels talks tomorrow

- Relation with BCFW recursion relation (tree level) (Risager)

# Without supersymmetry

- Cut-constructible part of one-loop MHV amplitudes in pure Yang-Mills
(Bedford, Brandhuber, Spence, GT)

- Rational terms in non-supersymmetric amplitudes missed by MHV diagrams

    - Non-supersymmetric amplitudes are not cut-constructible in four dimensions

    - use recursive techniques to derive rational terms
(Bern, Dixon, Kosower; Bern, Berger, Dixon, Forde, Kosower)

# The all-minus amplitude

(Brandhuber, Spence, GT)

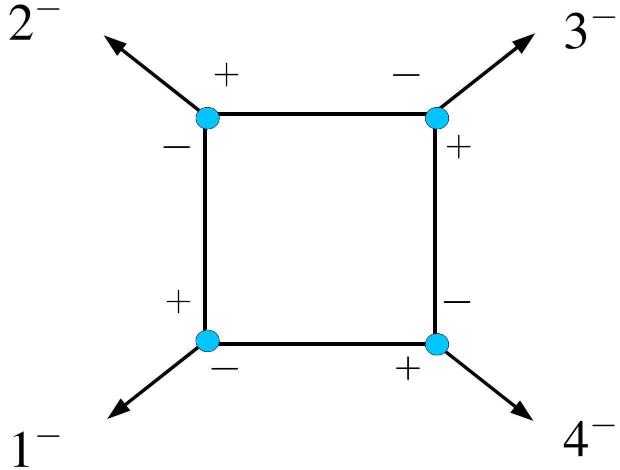
- $n$ three-point MHV vertices (for $A(1^- \cdots n^-)$ )
- Key observation: three-point MHV vertices are the same as lightcone vertices ➡ result is a priori correct

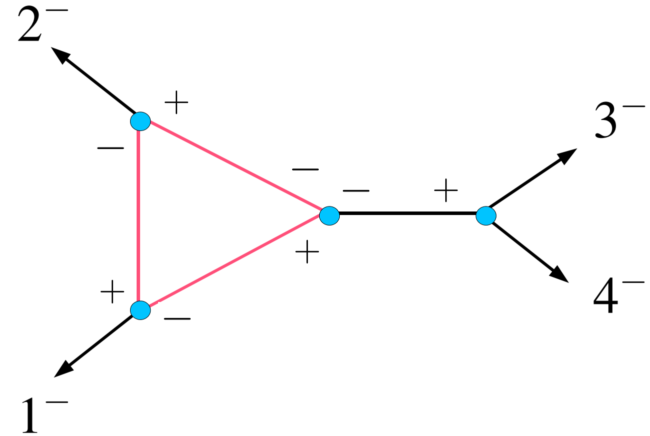
# Explicit calculation

- Use supersymmetric decomposition:

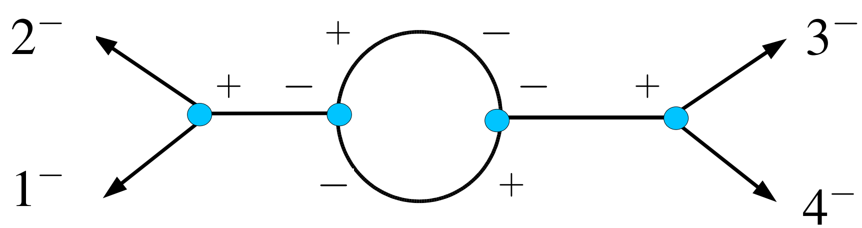
$$\mathcal{A}_g = (\mathcal{A}_g + 4\mathcal{A}_f + 3\mathcal{A}_s) - 4(\mathcal{A}_f + \mathcal{A}_s) + \mathcal{A}_s$$

- N=4 and N=1 contributions vanish
- Gluon ➡ scalar running in the loop
    - simpler to calculate




+ pms




+ pm

Result ~ $\dfrac{\langle 1\,2\rangle\,\langle 3\,4\rangle}{[1\,2]\,[3\,4]}\,K_4$

$$K_4 = -\varepsilon(1-\varepsilon)\,I_4^{D=8-2\varepsilon} \xrightarrow[\varepsilon\to 0]{} -\frac{1}{6}$$

(Originally derived by Bern & Kosower, and Bern & Morgan)

# Finiteness of the all-minus amplitude

- Define $L_D = L_4 + L_{-2\varepsilon}$ with $L_D^2 = L_4^2 + L_{-2\varepsilon}^2 := L_4^2 - \mu^2$
- A finite, non-zero result arises from incomplete cancellations of propagators

$$\frac{L_4^2}{L_D^2} = \frac{L_4^2 - \mu^2 + \mu^2}{L_D^2} = 1 + \frac{\mu^2}{L_D^2}$$

  - MHV vertices are 4-dimensional
  - D-dimensional propagators

- Naive calculation directly in 4d gives zero
- Finite, non-zero result related to an anomaly ?
  - Finiteness arises as $\varepsilon/\varepsilon$ effect

    Reminiscent of an anomaly...
  - Anomaly in worldsheet conformal symmetry in N=2 open strings (Chalmers, Siegel)

- All-minus amplitude understood within MHV diagram method
- All-plus amplitude
    - Parity conjugate of all-minus, but MHV method treats the two helicities differently
- Longstanding speculations on a one-loop all-plus vertex
    - All-plus amplitude has no multiparticle poles (as MHV)
    - Twistor space geometry seems to confirm this

# Where is the all-plus amplitude ?

## Go back to the path integral !

- Mansfield's procedure: (in a nutshell)
  - Start from lightcone quantisation of YM, $A^- = 0$
  - integrate out $A^+$ (no derivatives wrt lightcone time $x^-$)
  - $A_z, A_{\bar{z}}$ correspond to physical polarisations
- Action is $S = S^{-+} + S^{--+} + S^{++-} + S^{--++}$

(Scherk, Schwarz)

anti MHV

- Change variables in path integral: $A_z, A_{\bar{z}} \to B_+, B_-$

$$(S^{-+} + S^{-++})[A_z, A_{\bar{z}}] = S^{-+}[B_+, B_-]$$

· LHS is SDYM action

· Bäcklund transformation

- Further require:
  - Transformation is canonical, with $A_z = A_z[B_+]$
  - Canonicality ➡ Jacobian equal to 1 (classically)
    - Subtleties related to $\det \partial_+$

- Plug $$A_z \sim B_+ + B_+^2 + B_+^3 + \cdots$$

$$A_{\bar{z}} \sim B_-(1 + B_+ + B_+^2 + B_+^3 + \cdots)$$

in $$(S^{--+} + S^{--++})[A_z, A_{\bar{z}}]$$

- Result is

$$S[B_+, B_-] = S^{-+} + S^{--+} + S^{--++} + S^{--+++} + \cdots$$

- Vertices have MHV helicity configuration

# Comments

- Jacobian for $A_z, A_{\bar{z}} \rightarrow B_+, B_-$ is 1 (classically)
- Equivalence Theorem:
  - Green's functions of the B fields are different from those of the A fields, however
  - S-matrix elements are the same modulo a wave-function renormalisation...
  - ...equal to 1 at one loop (Ettle & Morris)
- We can equivalently calculate amplitudes with B fields insertions

# One missing thing !

- We have just mapped Self-Dual Yang-Mills to a free theory...
- ...with the consequence of eliminating the all-plus amplitude
- Potential sources of problems:
  - Jacobian
  - Equivalence Theorem
  - Regularisation

# Our solution

(Brandhuber, Spence, Zoubos, GT)

- Use Thorn worldsheet friendly regularisation
  - inherently four-dimensional
- Perform Mansfield-Bäcklund transformation on the regularised, 4d action
  - SDYM classically integrable only in 4d
- New one-loop effective interactions from regularisation, plus
- Usual MHV vertices

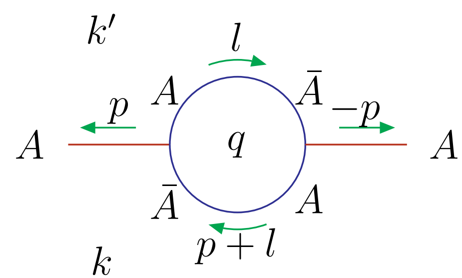
simple...

- Worldsheet friendly regulator: $\exp(-\delta \sum_{i=1}^{n} \mathbf{q}_i^2)$

  $\mathbf{q}^2 = 2 q_z q_{\bar{z}}$

  - $\delta$ is sent to zero at the end of calculation

- $q_i$ are loop region (T-dual) momenta

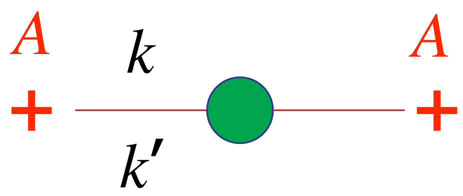



$p = k' - k$

- Regularisation generate Lorentz-violating processes

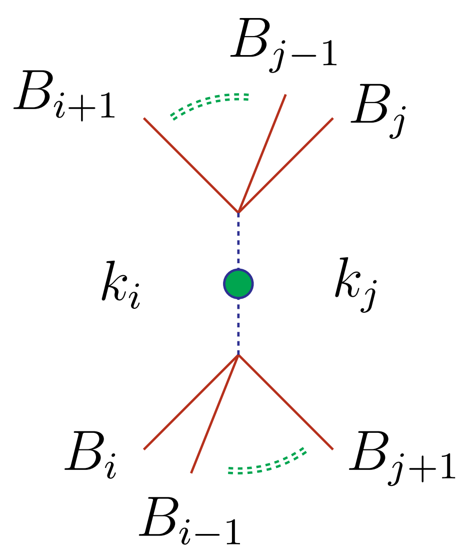
  - cancel with appropriate ++ counterterm

    (Chakrabarti, Qiu, Thorn)

- 

counterterm ~ $\frac{g^2 N}{12\pi^2}\left((k_{\bar{z}})^2 + (k'_{\bar{z}})^2 + k_{\bar{z}}k'_{\bar{z}}\right)$

- Applying Mansfield transformation on counterterm generates all-plus amplitudes:




Reminders: $A = A(B)$ holomorphic

$A$ is positive-helicity gluon

Equivalence Theorem: $A \to B$

Counterterm acts as a generating functional of all-plus amplitudes

- Explicit check at four points
- Soft, collinear limits

# A complementary solution

(Ettle, Fu, Fudger, Mansfield, Morris)

- Use dimensional regularisation
  - new interactions due to the regularisation
  - vanish as $\varepsilon \to 0$
- Perform Mansfield-Bäcklund transformation on the full D-dimensional action
- Violations of the equivalence theorem produce the missing amplitudes

Tim Morris's talk tomorrow

# Next tasks

- Calculate more general amplitudes, including rational terms
- First example: $-++.....+$

# Gravity

Michael Green's talk
Zvi Bern's talk on Friday

- Simplicity of gravity amplitudes
  - Twistor space structure (Bern, Bjerrum-Bohr, Dunbar)
  - Tree-level MHV rules from recursion relations (Bjerrum-Bohr, Dunbar, Ita, Perkins, Risager)
  - Applications to one-loop MHV diagrams (Nasti, GT)
  - Field redefinitions on lightcone gravity action (Ananth, Theisen)
  - Recursion relations (Bedford, Brandhuber, Spence, GT; Cachazo, Svrcek; Benincasa, Boucher-Veronneau, Cachazo)
- Finiteness of N=8 ? (Bern, Dixon, Roiban; Bern, Carrasco, Dixon, Johansson, Kosower, Roiban; Green, Russo, Vanhove)
- Surprises even without supersymmetry ! (Bern, Carrasco, Forde, Ita, Johansson)

# Back to N=4 Super Yang-Mills

# Amplitudes and Wilson Loops

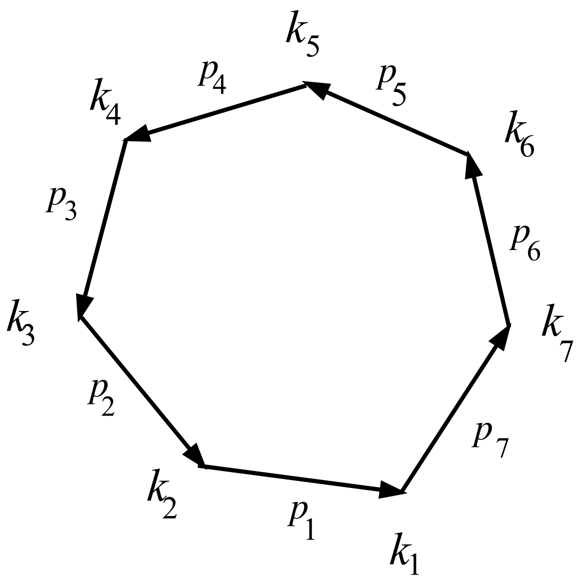
(Brandhuber, Heslop, GT; Brandhuber, Heslop, Spence, GT)

- We wish to calculate $< W[C] >$ at weak coupling

$$W[C] := \mathrm{Tr}\,\mathrm{P}\exp\left[ig\oint_C d\tau\Big(A_\mu(x(\tau))\dot{x}^\mu(\tau)+\phi_i(x(\tau))\dot{y}^i(\tau)\Big)\right]$$

  - Contour $C$ as in Alday-Maldacena calculation (next slide)
  - When $\dot{x}^2 = \dot{y}^2$ Wilson loop is locally supersymmetric
  - Choose $\dot{x}^2 = 0$ (lightlike momenta) and $\dot{y} = 0$
  - In general, supersymmetry is broken globally

- Contour $C$ in the strong-coupling calculation of A&M
  - Dictated by the momenta of the scattered gluons




$$p_i = k_i - k_{i+1}$$

$k$'s are T-dual (region) momenta

$$\sum_{i=1}^{n} p_i = 0$$

Contour is closed

# Motivation

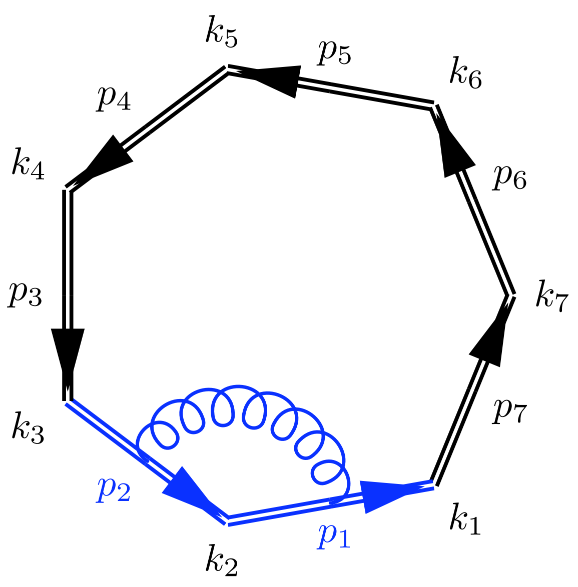
- Computation of amplitudes at strong coupling (Alday and Maldacena) (Fernando Alday's talk)
  - dual to that of the area of a string ending on a lightlike polygonal loop embedded in the boundary of AdS
  - scattering in AdS is at fixed angle, large energy ➡ similar to Gross-Mende calculation
  - leads to an exponential of classical string action
  - calculation in the T-dual variables is equivalent to that of a lightlike Wilson loop at strong coupling (Maldacena; Rey and Yee)

- Calculate $\langle W[C] \rangle$ at weak coupling for $n$ points
  - One loop (two-loop calculation in preparation)
  - Four-point case addressed by Drummond, Korchemsky, Sokatchev
- Result: $\langle W[C] \rangle$ gives the $n$-point MHV amplitude in N=4 SYM ! (modulo tree-level prefactor)
- Conjecture that equality $\langle W[C] \rangle = \mathcal{M}$ persists at higher loops

# $< W[C] >$ at one loop

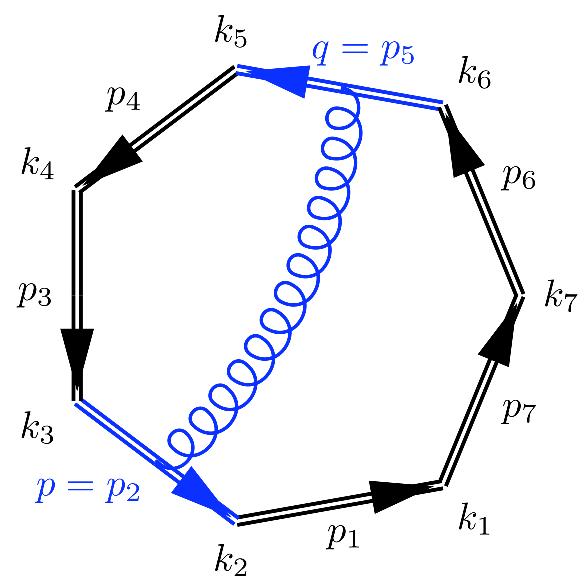
(Brandhuber, Heslop, GT)

- Calculation done (almost) instantly.
  Two classes of diagrams:




Gluon stretched between two segments meeting at a cusp

A. Infrared divergent




Gluon stretched between two non-adjacent segments

B. Infrared finite

- Clean separation between infrared-divergent and infrared-finite terms
  - Important advantage, as $\varepsilon$ can be set to zero in the finite parts from the start
- From diagrams in class A :

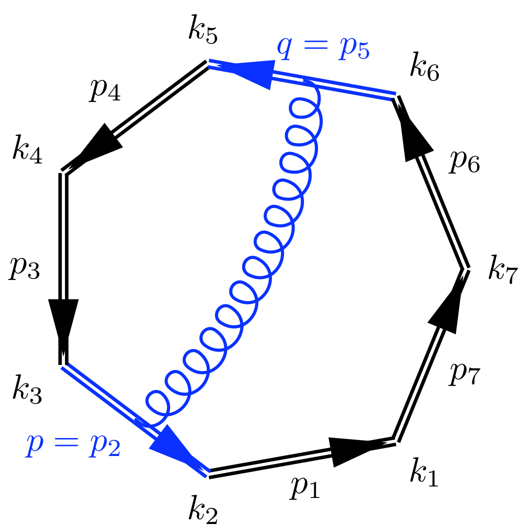
$$\mathcal{M}_n^{(1)}\big|_{IR} = -\frac{1}{\varepsilon^2}\sum_{i=1}^{n}\left(\frac{-s_{i,i+1}}{\mu^2}\right)^{-\varepsilon}$$

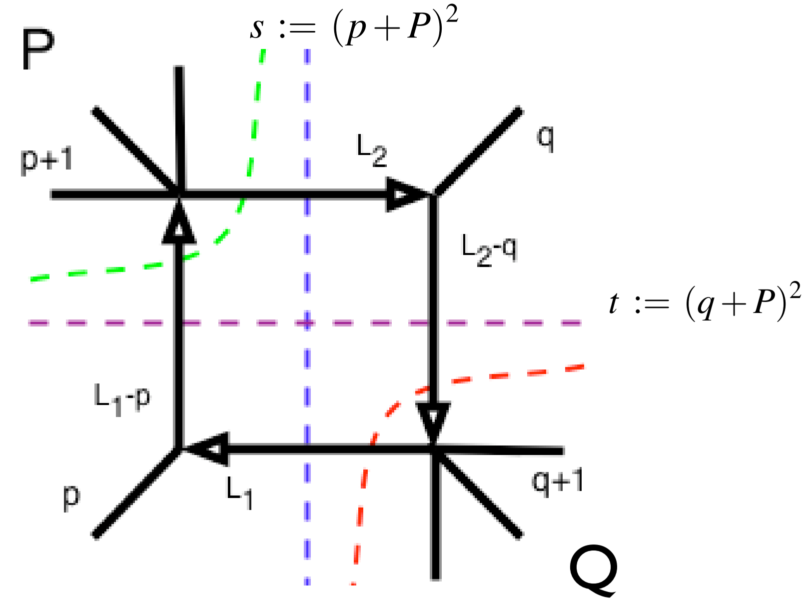
  - $s_{i,i+1} = (p_i + p_{i+1})^2$ is the invariant formed with the momenta meeting at the cusp

- Diagram in class B, with gluon stretched between $p$ and $q$ gives a result proportional to

$$\mathcal{F}_\varepsilon(s,t,P,Q) = \int_0^1 d\tau_p d\tau_q \frac{P^2+Q^2-s-t}{\left[-\left(P^2+(s-P^2)\tau_p+(t-P^2)\tau_q+(-s-t+P^2+Q^2)\tau_p\tau_q\right)\right]^{1+\varepsilon}}$$

- Explicit evaluation shows that this is equal to the finite part of a 2-mass easy box function:




- In the example: $p = p_2 \quad q = p_5$

  $P = p_3 + p_4\,, \quad Q = p_6 + p_7 + p_1$

- One-to-one correspondence between Wilson loop diagrams and finite parts of 2-mass easy box functions

- Explains why each box function appears with coefficient equal to 1 in the expression of the one-loop N=4 MHV amplitude

- Explicit calculation gives:

$$a := \frac{2(pq)}{P^2Q^2 - st}$$

$$\mathcal{F}_\varepsilon = -\frac{1}{\varepsilon^2}$$

$$\Bigg[\left(\frac{a}{1-aP^2}\right)^\varepsilon {}_2F_1\left(\varepsilon,\varepsilon,1+\varepsilon,\frac{1}{1-aP^2}\right) + \left(\frac{a}{1-aQ^2}\right)^\varepsilon {}_2F_1\left(\varepsilon,\varepsilon,1+\varepsilon,\frac{1}{1-aQ^2}\right)$$

$$- \left(\frac{a}{1-as}\right)^\varepsilon {}_2F_1\left(\varepsilon,\varepsilon,1+\varepsilon,\frac{1}{1-as}\right) - \left(\frac{a}{1-at}\right)^\varepsilon {}_2F_1\left(\varepsilon,\varepsilon,1+\varepsilon,\frac{1}{1-at}\right)\Bigg]$$

- At $\varepsilon \longrightarrow 0$ : $\mathcal{F}_{\varepsilon=0} = -\mathrm{Li}_2(1-as) - \mathrm{Li}_2(1-at) + \mathrm{Li}_2(1-aP^2) + \mathrm{Li}_2(1-aQ^2)$

  - Box function in the same compact form derived from dispersion integrals using one-loop MHV diagrams

    (Brandhuber, Spence, GT)

- At 4 points, all-orders in $\varepsilon$ result:

$$\mathcal{M}_4^{(1)}(\varepsilon) = -\frac{2}{\varepsilon^2}\left[\left(\frac{-s}{\mu^2}\right)^{-\varepsilon} {}_2F_1\left(1,-\varepsilon,1-\varepsilon,1+\frac{s}{t}\right) + \left(\frac{-t}{\mu^2}\right)^{-\varepsilon} {}_2F_1\left(1,-\varepsilon,1-\varepsilon,1+\frac{t}{s}\right)\right]$$

  - Agrees with result of Green, Schwarz and Brink

- For $n > 4$, missing topologies (vanish as $\varepsilon \longrightarrow 0$)

  - E.g. $n > 5$, get only parity-even part

# $< W[C] >$ at higher loops

(Brandhuber, Heslop, Spence, GT, in preparation)

- Key result: non-abelian exponentiation theorem (Gatheral; Frenkel and Taylor)

$$\langle W[C]\rangle := 1 + \sum_{L=1}^{\infty} a^L W^{(L)} = \exp \sum_{L=1}^{\infty} a^L w^{(L)}$$

- $w$'s are calculated by keeping only terms containing maximal non-abelian colour factor
  - subset of all possible diagrams

- BDS's Exponential Ansatz naturally emerges

$$\mathcal{M}_n := 1 + \sum_{L=1}^{\infty} a^L \mathcal{M}_n^{(L)} = \exp\left[\sum_{L=1}^{\infty} a^L \left( f^{(L)}(\varepsilon)\mathcal{M}_n^{(1)}(L\varepsilon) + C^{(L)} + E_n^{(L)}(\varepsilon)\right)\right]$$

$$\langle W_n[C]\rangle := 1 + \sum_{L=1}^{\infty} a^L W_n^{(L)} = \exp \sum_{L=1}^{\infty} a^L w_n^{(L)}$$

- If $< W[C] > = \mathcal{M}$ , then

$$w_n^{(L)} = f^{(L)}(\varepsilon)\mathcal{M}_n^{(1)}(L\varepsilon) + C^{(L)} + O(\varepsilon)$$

- Calculation of $w$ at two loops almost completed Stay tuned !

- Four-point MHV amplitude fixed using dual conformal invariance and factorisation of infrared divergences (Drummond, Korchemsky, Sokatchev)

  - appears to be not restrictive enough for $n > 4$
  - issues with anomalous dimension of twist 2 operators

# Summary

- Simplicity of scattering amplitudes ➡ geometry in Twistor Space

- New, efficient methods to derive amplitudes
  - MHV diagrams
  - recursion relations, generalised unitarity...

- MHV diagrams: provide a new diagrammatic method to calculate scattering amplitudes at tree and one-loop level in super Yang-Mills

- Progress in non-supersymmetric Yang-Mills
    - All-minus amplitude, all-plus amplitude
    - 4d Mansfield-Bäcklund transformation
    - worldsheet friendly regularisation

- MHV amplitude in N=4 SYM from a Wilson loop calculation at weak coupling
  - One loop
  - Higher loops

# Some of the pressing questions...

- Rational terms in pure YM amplitudes
- Higher loops
- Relation to integrability
- Wilson loop calculations to higher loops
- What about correlators of gauge-invariant operators ?

...and many more...