QUANTUM FIELDS IN CURVED SPACETIME: THE STRONG COUPLING STORY

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Non-perturbative Techniques in Field Theory

LMS-EPSRC Durham Symposium

July 22, 2010

OUTLINE

- Motivation
- QFTs in curved spacetime
- The gauge/gravity toolkit
- QFTs on asymptotically flat black holes
- Cosmological phase transitions, holographically
- QFTs on AdS black holes
- Discussion

Motivation

Field theory motivation:

- Strongly coupled fields in curved spacetime.
- Stepping stone for full-blown quantum gravity.

AdS/CFT motivation:

- General constraints on bulk spacetimes.
- New insights into the detailed workings of the correspondence.

Phenomenological motivation:

- Limiting case of induced gravity (brane-world) models.

An invitation to QFTs on curved spacetime

- Quantum fields in curved spacetime are rife with many interesting physical phenomena:
  - nature of the vacuum
  - particle production
- The details are important to :
  - understand inflationary cosmology
  - Hawking radiation

- Learnt lots in the last 4 decades, but at the level of free fields.

An invitation to QFTs on curved spacetime

- Can one tackle strongly coupled QFTs on curved spacetime backgrounds?
- Are there qualitative differences from the behaviour of weakly coupled quantum fields?
- Can there be non-trivial phase structure/transitions?
- Derive quantitative results for vacuum polarization effects?

- e.g. expectation value of expectation values of appropriate operators (stress tensor).

A sampling of possibilities

- Explore a class of strongly coupled field theories on various curved manifolds.
- Main tool: the holographic gauge/gravity duality.

- QFTs on asymptotically flat backgrounds $(\Lambda = 0)$
- QFTs in cosmological spacetimes $(\Lambda > 0)$
- QFTs in negatively curved backgrounds $(\Lambda < 0)$

Hubeny, Marolf, MR

Marolf, MR, van Raamsdonk

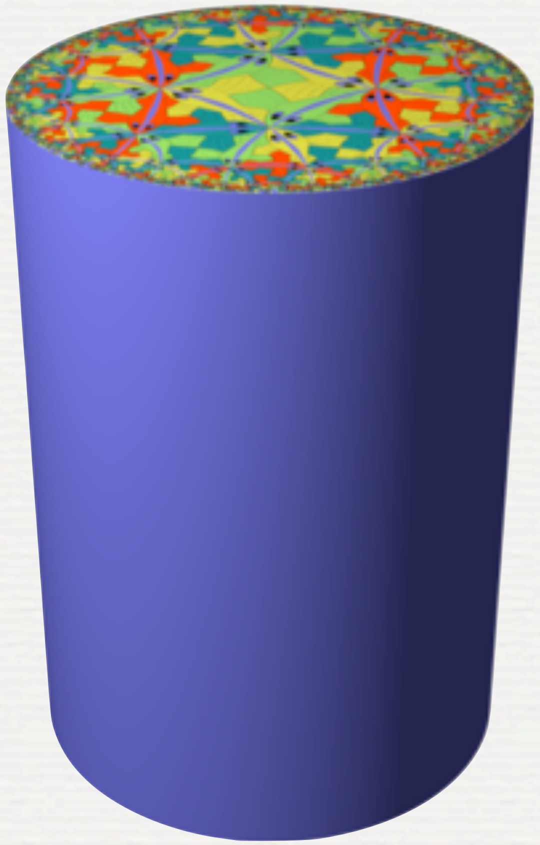
Aharony, Marolf, MR

The AdS/CFT correspondence: Basic dictionary

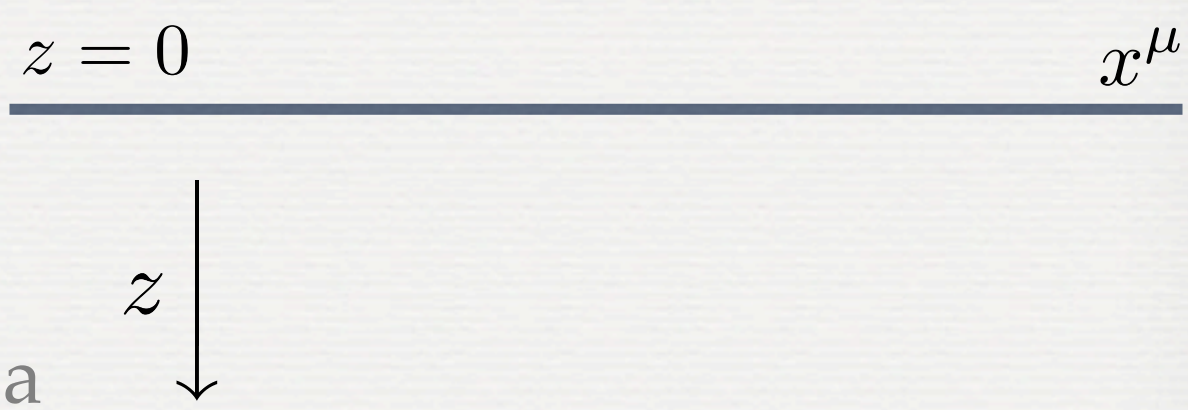
- AdS/CFT relates dynamics of a class of strongly coupled field theories to string theory in an asymptotically AdS spacetime.
- However, in a suitable limit *c or N >> 1*, λ >>1, restrict attention to the massless closed string modes → gravity limit.
- Canonical example $\mathcal{N} = 4$ SYM in four dimensions which is dual to gravity on AdS_5 X $\mathbf{S}^5$.

- Strong-weak duality allows one to probe dynamics in strongly coupled gauge theories.
- Phase structure of the field theories maps to the classical phase structure of gravitational solutions.

The AdS/CFT correspondence: Basic dictionary



Global AdS is like a cylinder with a time-like boundary which is a copy of the Einstein Static Universe (Lorentzian cylinder).



We will also have occasion to use the Poincare patch where the boundary is a copy of Minkowski spacetime.

AdS/CFT's role in strongly coupled field theories

- Consider a boundary field theory on a non-dynamical curved spacetime background with a prescribed metric $\gamma_{\mu\nu}$.
- QFT dynamics is governed at strong coupling by ``asymptotically locally AdS'' geometries.

- Focus on situations where we turn on a non-trivial (non-dynamical) gravity background for our field theory.
- Non-normalizable mode (source) for gravity

- $\Rightarrow$ restrict attention to the universal sub-sector (consistent truncation) involving only bulk metric dynamics.

AdS/CFT's role in strongly coupled field theories

- Want solutions, $\mathscr{M}_{d+1}$, to Einstein's equations with negative cc with the bulk metric asymptoting to $\mathscr{B}_d$ with chosen metric $\gamma_{\mu\nu}$.

$$\mathcal{S}_{\rm bulk} = \frac{1}{16\pi\, G_N^{(d+1)}} \int d^{d+1}x\, \sqrt{-g}\, (R - 2\,\Lambda)$$

$$ds^2 = \frac{dz^2 + \left(\gamma_{\mu\nu} + \cdots + z^d\, T_{\mu\nu}\, dw^\mu\, dw^\nu + \cdots\right)}{z^2}$$

- Can't get all the data from a Fefferman-Graham expansion
- Don't know & would like to compute the response of the field theory to the background curvature: $\langle T_{\mu\nu} \rangle$.

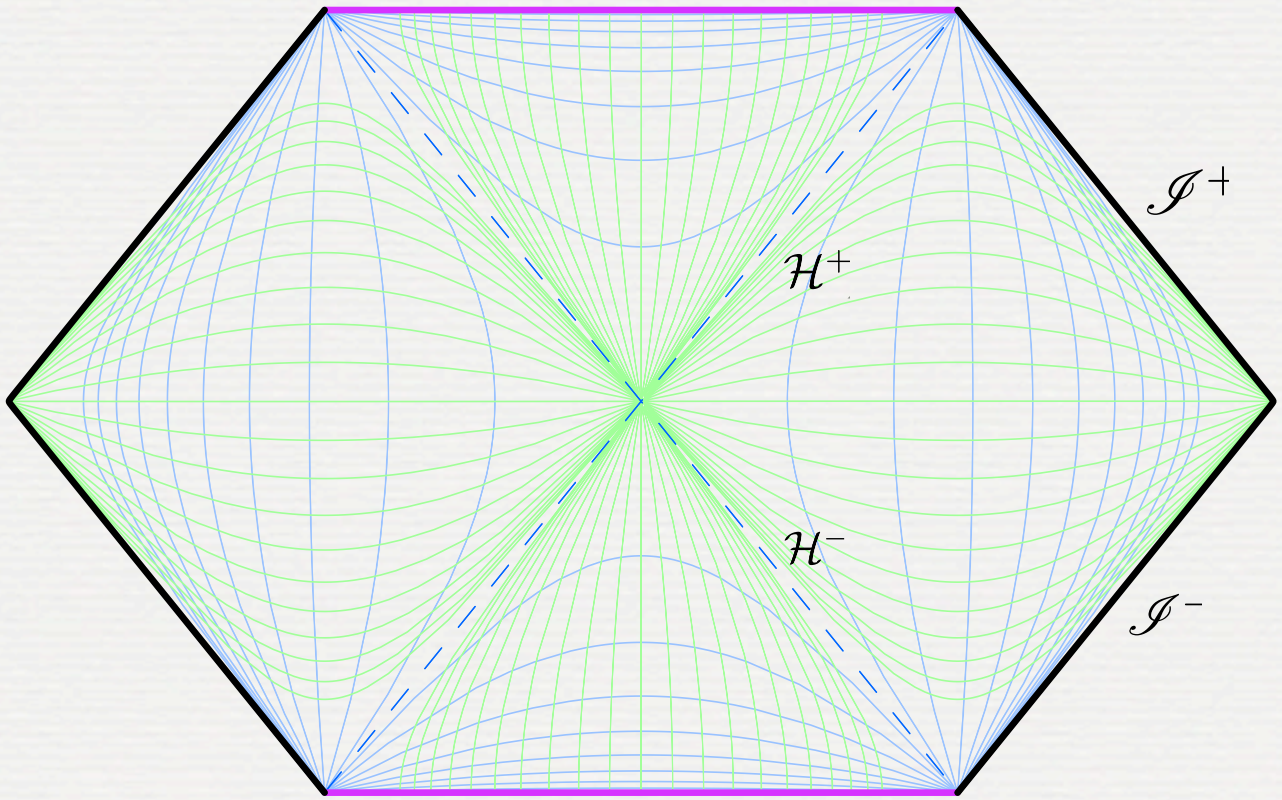
cf. Janik's talk

A sample of previous work

- Studies of $\mathcal{N} = 4$ SYM on various backgrounds:

- squashed spheres
  - Copsey, Horowitz
- near horizon geometry of extreme black holes, $AdS_2 \times \mathbf{S}^2$.
  - Kaus, Reall
  - Suzuki, Shiromizu, Tanahaski
- perturbed boundary metric (isotropization)
  - Chesler, Yaffe
  - Bhattacharyya, Minwalla
  - Beuf, Heller, Janik, Peschanski

QFTs in black hole backgrounds



Study quantum fields on black hole backgrounds, say the asymptotically flat Schwarzschild black hole with a horizon at r_+.

$$ds^2_\partial = \gamma_{\mu\nu}\, dx^\mu\, dx^\nu = -f(r)\, dt^2 + \frac{dr^2}{f(r)} + r^2\, d\Omega^2_{d-2}$$

$$f(r) = 1 - \frac{r_+^{d-3}}{r^{d-3}}$$

QFTs on asymptotically flat bh spacetimes

- Hawking: black holes radiate thermally.
- Equilibrium state: Hartle-Hawking vacuum is thermal.
- For Schwarzschild black hole

$$T_H = \frac{d-3}{4\pi}\,\frac{1}{r_+}$$

- ∃ other states of interest, e.g., stationary Unruh state (relevant for stationary Kerr bhs).

- More generally, consider spacetimes, with multiple length scales:
  - horizon size R
  - temperature T

QFTs on asymptotically flat bh spacetimes

- Extract <$T_{\mu\nu}$> using heat kernel techniques for free fields.
- Conformally coupled scalar field in 4 dimensions:

$$T^{\mu}_{\nu} = \frac{\pi^2 T^4}{90} \left[A\left(\frac{r_+}{r}\right) \left(\delta^{\mu}_{\nu} - 4\,\delta^{\mu}_{0}\,\delta^{0}_{\nu}\right) + B\left(\frac{r_+}{r}\right) \left(3\,\delta^{\mu}_{0}\,\delta^{0}_{\nu} + \delta^{\mu}_{1}\,\delta^{1}_{\nu}\right) \right]$$

$$A(x) = \frac{1-(4-3\,x)^2\,x^6}{(1-x)^2}\,, \qquad B(x) = 24\,x^6$$

- Thermal far from the black hole and regular on the horizon.
- Local energy density is negative near the horizon (due to vacuum polarization).

Strongly coupled CFTs on asymptotically flat bh spacetimes

- Consider strongly coupled QFT (*SU(N)* $\mathcal{N}=4$ SYM) on Schwarzschild background.
- For the Hartle-Hawking state of the field theory:
- $\langle T_{\mu\nu}\rangle \sim N^2\,(T_H)^4 \;\Rightarrow$ theory in deconfined phase.
- $\mathcal{N}=4$ SYM is a CFT and the only scale is set by the temperature.

- Finite temperature $\mathcal{N}=4$ SYM on Minkowski spacetime has a holographic dual which is a black hole in AdS_5.

$$ {}^{d}\mathfrak{f}(z) = 1 - \frac{z^d}{z_0^d} $$

$$ ds^2_{\text{planar BH}} = \frac{1}{z^2}\left(-{}^{d}\mathfrak{f}(z)\,dt^2 + d\mathbf{x}^2_{d-1} + \frac{dz^2}{{}^{d}\mathfrak{f}(z)}\right) $$

$$ T = \frac{d}{4\pi\, z_0} $$

Strongly coupled CFTs on asymptotically flat bh spacetimes

- First guess for $\mathcal{N}=4$ on Schwarzschild: bulk horizon is given by local temperature

$$T_{\text{local}} = \frac{1}{\sqrt{f(r)}} T_H$$

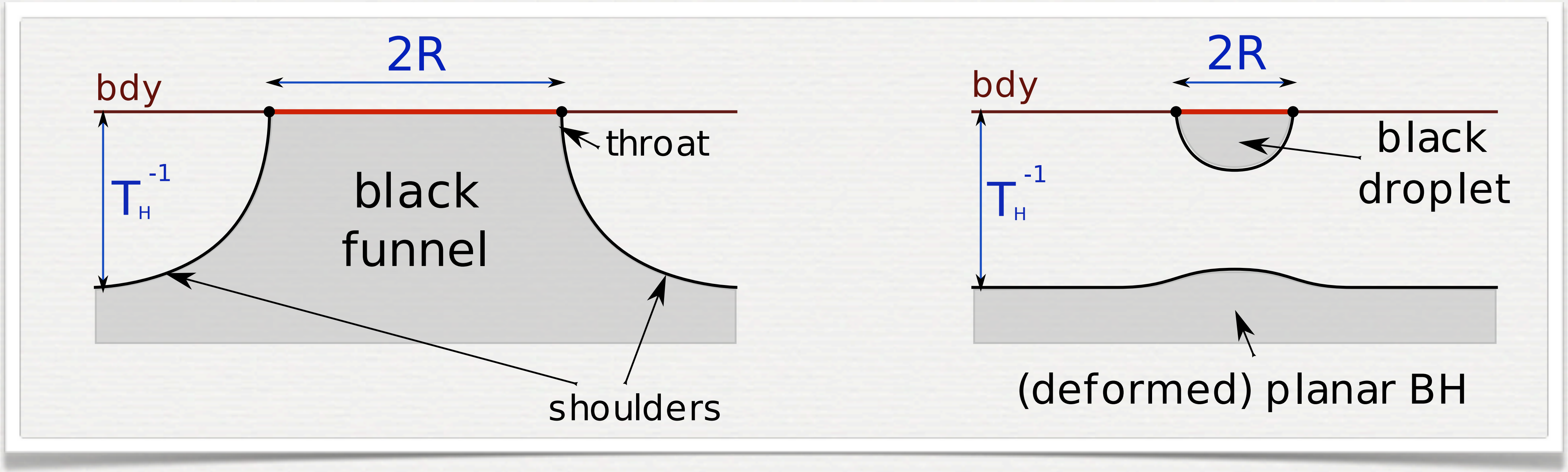
$z = 0$

$r \to \infty$

$z \downarrow$

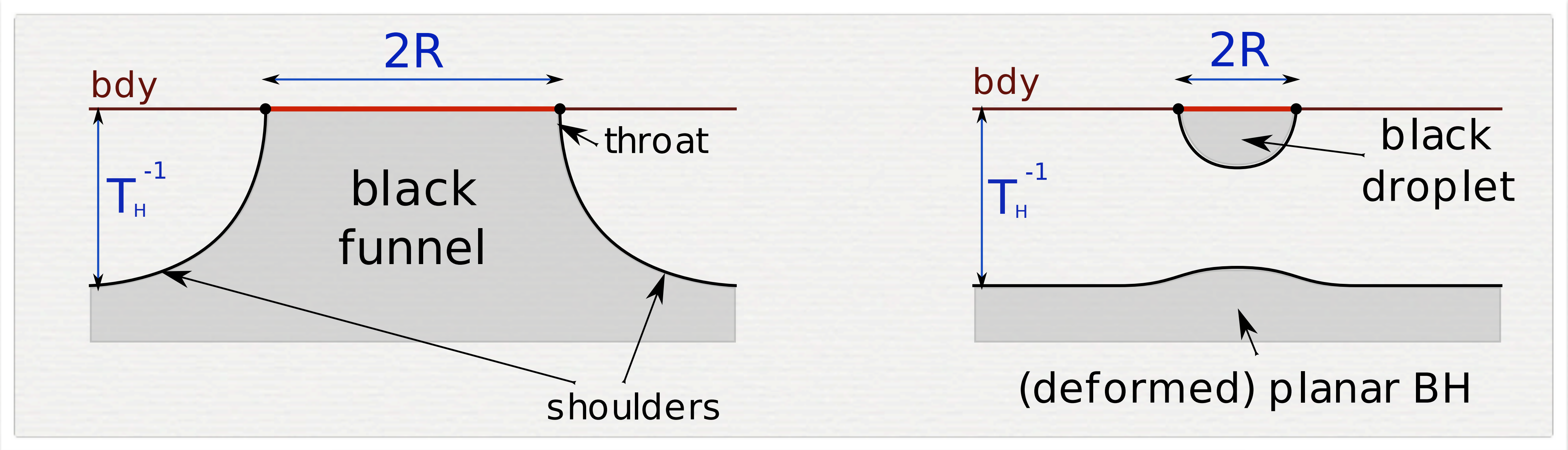
- Necks get thinner; instability?
- Schwarzschild is tricky: decouple T_H and R.

New black hole spacetimes in AdS



- Dominant solution for any given non-dynamical boundary metric depends on the dimensionless combination of
  - characteristic horizon size
  - boundary Hawking temperature
- Phase transitions as we move in the space of boundary metrics?
- Schwarzschild exactly on the boundary $T_H R = 1$.

Qualitative new behaviour of QFTs



- Expect field theory for large T_H to be *a deconfined plasma:*
  - Funnel phase: plasma couples strongly to the black hole.
  - Droplet phase: plasma couples weakly to the black hole.
- Interaction between the (deformed) planar bh and the droplet is suppressed by powers of *c or N* $\rightarrow$ achieved by *bulk Hawking radiation.*

Explicit examples

- 1+1 dim CFT on a 2 dim black hole background:

$$ds^2 = -\tanh^2 r\, dt^2 + dr^2$$

- In Fefferman-Graham gauge with the bulk metric ansatz:

$$ds^2 = \frac{1}{z^2}\left(-f(r,z)\, dt^2 + g(r,z)\, dr^2 + dz^2\right)$$

- Einstein's equations can be solved analytically

$$f(r,z) = \frac{1}{16}\left(4\tanh r + z^2\, \frac{1 - 2\,r_+ \cosh^4 r}{\sinh r\, \cosh^3 r}\right)^2$$

$$g(r,z) = \left(1 + z^2\, \frac{2\,r_+ \cosh^4 r - 1 - 4\sinh^2 r}{4\sinh^2 r\, \cosh^2 r}\right)^2$$

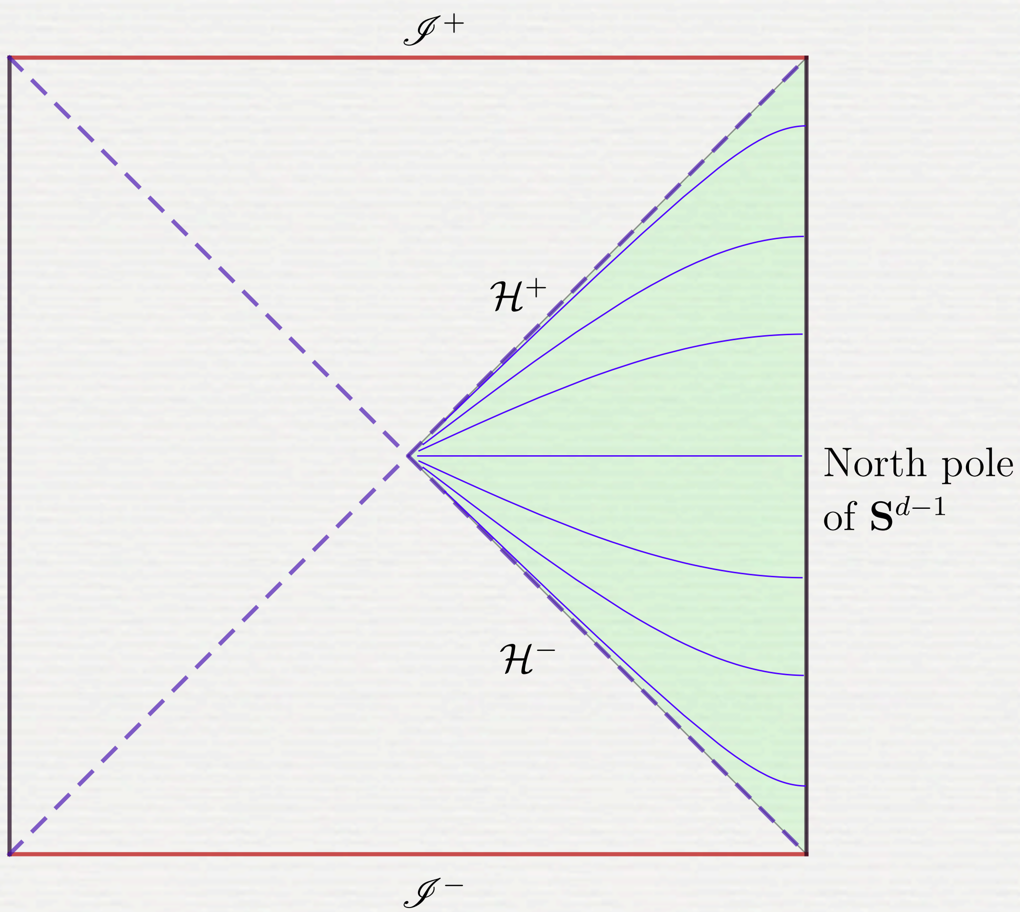
Explicit examples

- Bulk event horizon at:

$$z_H(r) = \frac{2\,\cosh r}{\sqrt{\cosh^2 r + 1}}\,, \quad \& \quad r = 0$$

- ∃ one-parameter family of solutions where the bulk horizon has a constant temperature, *different* from that of the boundary bh.

QFTs in de Sitter



Study quantum fields on de Sitter with Hubble scale H.
Concentrate on the static patch.

$$ds^2_\partial = \gamma_{\mu\nu}\, dx^\mu\, dx^\nu = -f(r)\, dt^2 + \frac{dr^2}{f(r)} + r^2\, d\Omega^2_{d-2}$$

$$f(r) = 1 - H^2\, r^2$$

Field theories in de Sitter

- Vacuum state of global de Sitter obtained by analytically continuing the Euclidean Bunch-Davies vacuum.
- Static observer sees the Gibbons-Hawking vacuum which is thermal at the de Sitter temperature.

$$T_{\mathrm{d}S} = \frac{1}{2\pi} H$$

- Consider a QFT which is known to undergo a confinement-deconfinement transition in Minkowski spacetime.
- What happens when we put this theory on de Sitter with time varying cosmological constant?
- Is there a phase transition as Hubble radius increases/decreases past a critical value?

Warm up: CFTs in the static patch

- A CFT in de Sitter sees only the thermal scale associated with cosmic acceleration T_{dS}.
- What happens if we restrict to the static patch and heat up a CFT to a temperature different from T_{dS}?

- For free field theories this clearly makes sense: imagine a heat source located on/just behind the cosmological horizon.
- In fact, makes sense even for strongly coupled field theories.

- However, phase structure of the theory is trivial despite existence of a dimensionless scale T/H.

Warm up: CFTs in the static patch

- Dual solutions are hyperbolic AdS black holes interpreted in a conformal frame, where boundary is de Sitter

$$ds^2 = \frac{\rho^2}{1-H^2\,r^2}\left(-\frac{f(\rho)}{\rho^2}\,(1-H^2\,r^2)\,dt^2 + \frac{dr^2}{1-H^2\,r^2} + r^2\,d\Omega^2_{d-2}\right) + \frac{d\rho^2}{f(\rho)}$$

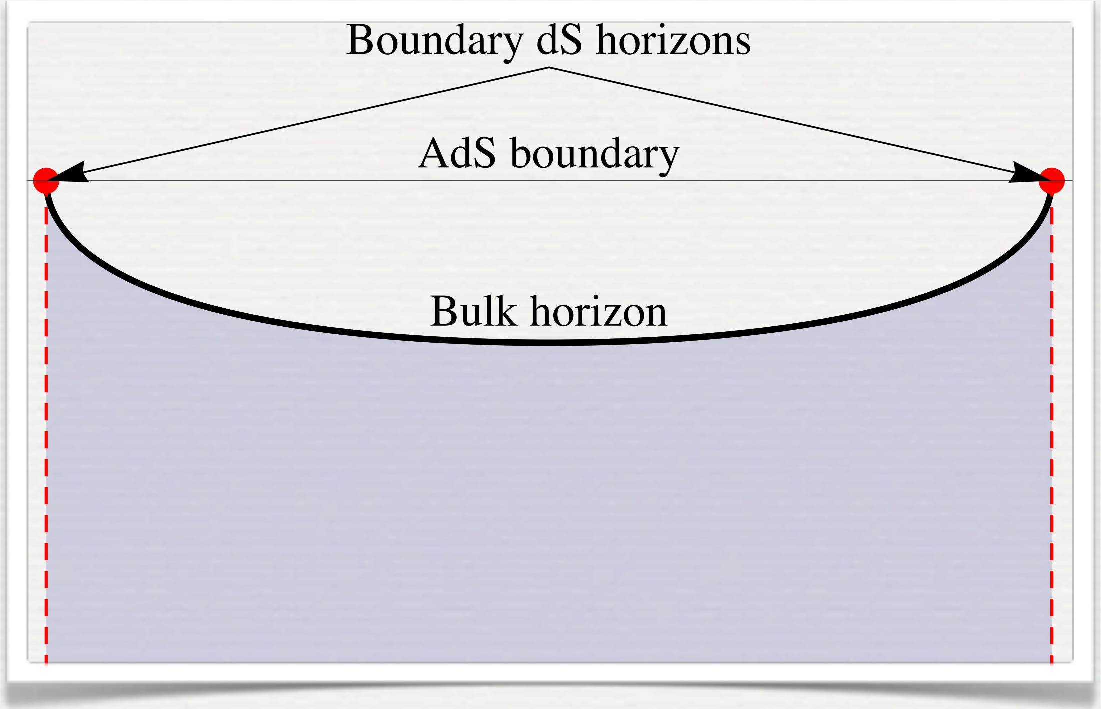
$$f(\rho) = \frac{\rho^2}{\ell^2_{d+1}} - 1 - \frac{\rho_+^{d-2}}{\rho^{d-2}}\left(\frac{\rho_+^2}{\ell^2_{d+1}} - 1\right) \qquad \beta \equiv T^{-1} = \frac{4\pi\,\ell^2_{d+1}\,\rho_+}{d\,\rho_+^2 - (d-2)\,\ell^2_{d+1}}$$

- Bulk solutions are completely regular, but the stress tensor induced on the boundary diverges on the cosmological horizon.

$$T_\nu^{\ \mu} = c\,\frac{H^d}{(1-H^2\,r^2)^{\frac{d}{2}}}\,\frac{\rho_+^{d-2}}{\ell^{d-2}_{d+1}}\left(\frac{\rho_+^2}{\ell^2_{d+1}} - 1\right)\,\mathrm{diag}\Big\{1-d, 1, 1, \cdots, 1\Big\}$$

Visulaizing the solutions

- The bulk solutions schematically look like:



- Very curious feature: bulk horizon knows about T not T_{dS}.
- NB: cosmological horizon is continuously connected to the bulk black hole horizon.

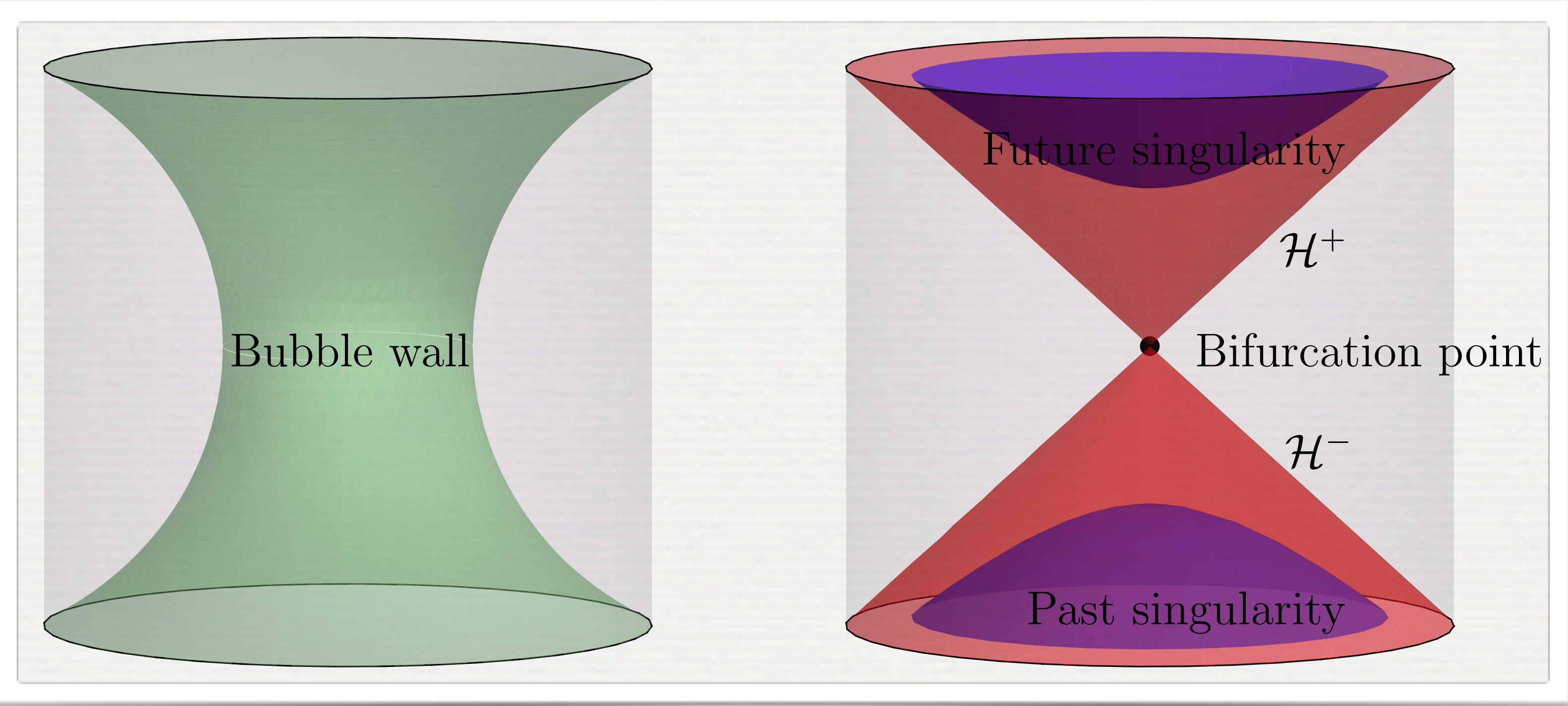
Confining theories on de Sitter

- A simple model of confining theories on de Sitter can be obtained by Scherk-Schwarz compactification (on a circle of radius *R*) of CFTs.
  - e.g. $\mathcal{N} = 4$ SYM on $dS_3 \times \mathbf{S}^1$.
- Holographic duals simple the → boundary geometry is simply a double Wick rotation of the Einstein Static Universe.
- Two geometries with the chosen boundary which exchange dominance at a critical value of *H* (for fixed *R*).

Aharony, Fabinger, Horowitz, Silverstein | Balasubramanian, Ross | Ross, Titchener | Balasubramanian, Larjo, Simon | Hutasoit, Kumar, Rafferty

Confining theories on de Sitter

- Lorentzian geometries in question are:



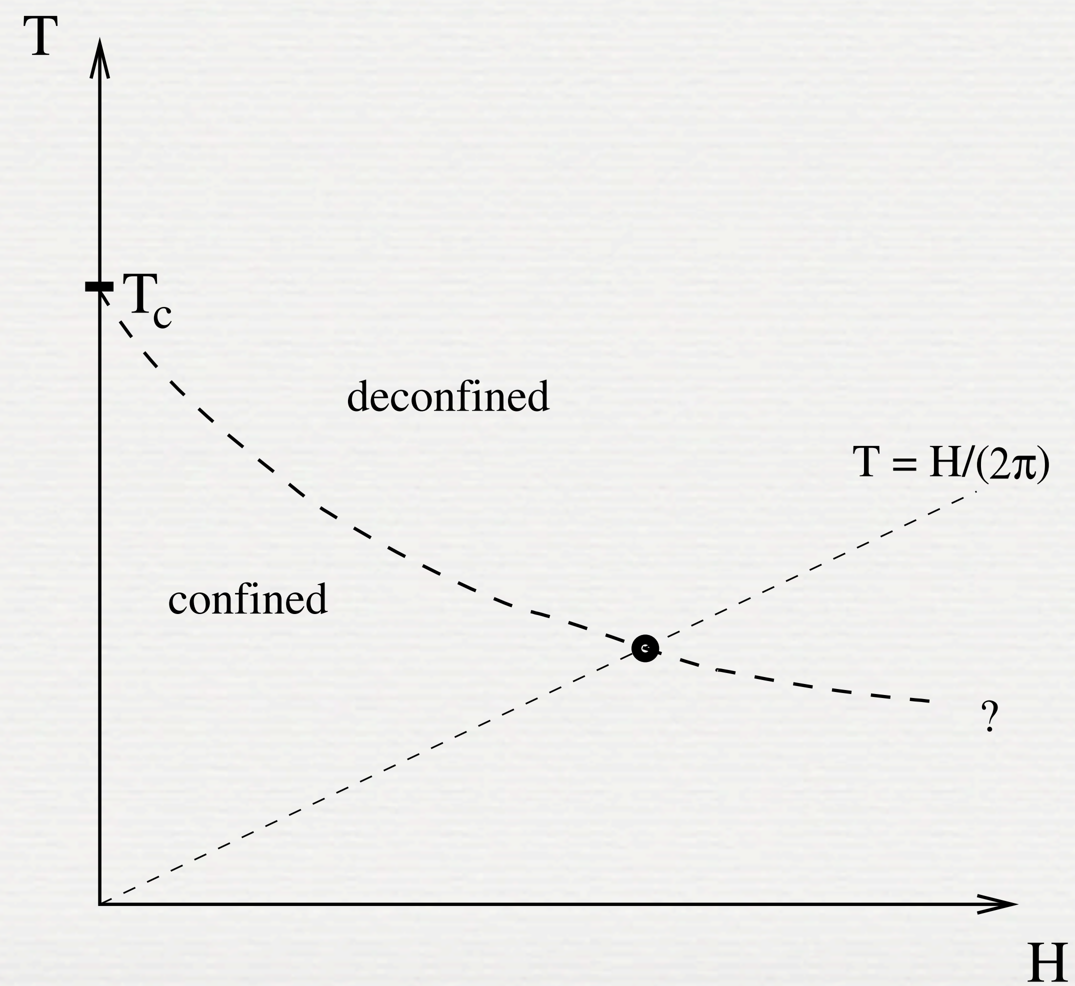
bubble of nothing　　　　topological bh

- The phase transition happens at a lower value of cosmic temperature (benchmark against the Minkowski transition point).

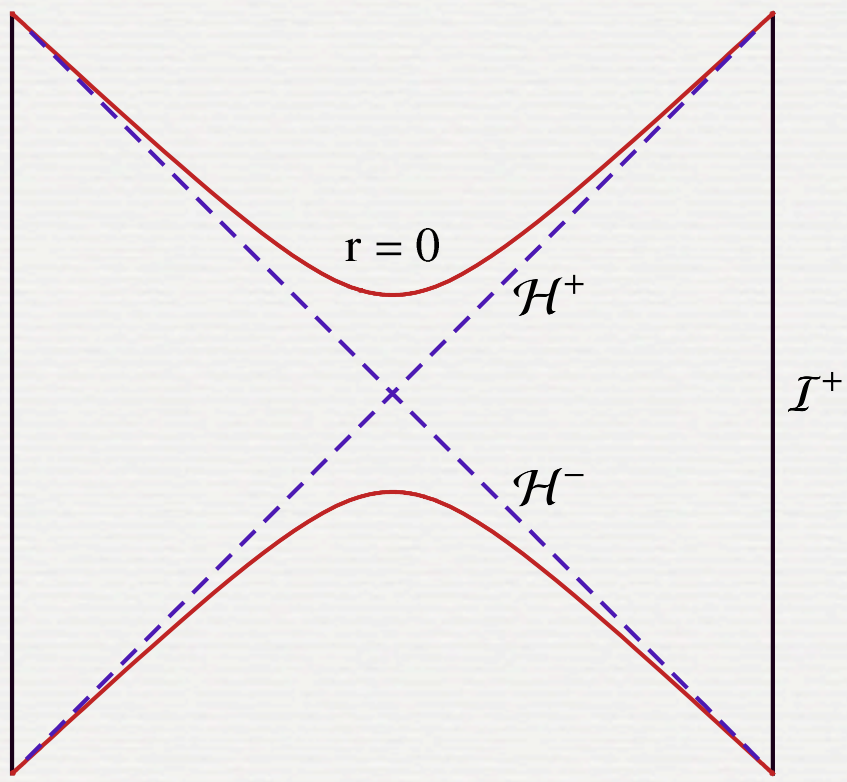
$$H = \frac{1}{d\,R} \qquad\qquad T_{dS} = \frac{T_c}{d}$$

Interpolating from Minkowski to de Sitter

- Continously connect Minkowski confinement/deconfinement transition to the cosmological phase transition?
- Indications from perturbation theory in H
- Again de Sitter QFTs with T different from T_{dS}.



QFTs on AdS black holes



Interacting field theories on AdS black hole backgrounds e.g. Schwarzshild AdS (SAdS).

$$ds^2_\partial = \gamma_{\mu\nu}\, dx^\mu\, dx^\nu = -f(r)\, dt^2 + \frac{dr^2}{f(r)} + r^2\, d\Omega^2_{d-2}$$

$$f(r) = \frac{r^2}{\ell^2} + 1 - \left(\frac{r_+}{r}\right)^{d-3}\left(\frac{r_+^2}{\ell^2} + 1\right) \qquad T_H = \frac{(d-1)\, r_+^2 + (d-3)\, \ell^2}{4\pi\, r_+\, \ell^2}$$

QFTs on asymptotically AdS bh spacetimes

- SU(N) N = 4 SYM on AdS black hole background.
  - Nature of the Hartle-Hawking state?
  - Is it thermal?
  - Boundary conditions? (AdS is not globally hyperbolic)

- Role of AdS asymptopia:
  - thermodynamics of AdS bhs is quite different
  - global properties differ, e.g. rotating AdS bhs admit Hartle-Hawking state (small rotation).

- Will use *transparent boundary conditions*.

Local Quantum Fields in AdS bh background

- Local quantum field in SAdS does not see the Hawking temperature due to the divergent red-shift.
- Equilibrium state around a SAdS black hole won't necessarily be thermal.
- $\mathcal{N} = 4$ SYM on a large radius SAdS background $<T_{\mu\nu}> \sim \mathcal{O}(1)$ in the Hartle-Hawking state.
- Naively might have expected $<T_{\mu\nu}> \sim \mathcal{O}(N^2)$.

Fitzpatrick, Randall, Wiseman

Gregory, Ross & Zegers

- *Aside*: AdS/CFT works around this by instructing us to conformally rescale the boundary data which compensates for the red-shift.

3d CFTs on BTZ: dual geometries

- Consider the static, spherically symmetric global AdS_4 spacetime:

$$ds^2 = -f(\rho)\, dT^2 + \frac{d\rho^2}{f(\rho)} + \rho^2 \left(d\theta^2 + \sin^2\theta\, d\Phi^2\right)$$

- Can be used to construct the duals of 2+1 CFTs on BTZ with transparent boundary conditions.
- Achieved by a simple change of boundary conformal frame:

$$ds^2 = \frac{\rho^2\, r_+^2}{r^2\, \ell^2} \left[-\frac{r^2 - r_+^2}{\ell^2}\, dt^2 + \frac{\ell^2}{r^2 - r_+^2}\, dr^2 + r^2\, \frac{L^2\, f(\rho)}{\rho^2}\, d\phi^2\right] + \frac{d\rho^2}{f(\rho)}\ .$$

- Pure AdS_4 $\rightarrow$ BTZ black string.
- $SAdS_4$ $\rightarrow$ AdS bubble of nothing.

3d CFTs on BTZ: dual geometries

- CFT on large BTZ is dual to the BTZ black string.
- $\Rightarrow$ for $T_{BTZ} >> 1$ one has $<T_{\mu\nu}> \sim \mathcal{O}(1)$.

- CFT on small BTZ is dual to the AdS bubble of nothing
- $\Rightarrow$ for $T_{BTZ} << 1$ has $<T_{\mu\nu}> \sim \mathcal{O}(c)$.

$$\mathcal{T}_\mu^{\ \nu} = c\,\frac{\mu}{L\,\ell^3}\,\frac{r_+^3}{3\,r^3}\,\{1,1,-2\}$$

$$\mu(r_+) = \frac{4\,L\,\ell^3}{27\,r_+^3}\left[1+\left(1+\frac{3\,r_+^2}{2\,\ell^2}\right)\sqrt{1-\frac{3\,r_+^2}{\ell^2}}\right]$$

- Free field result (conformally coupled scalar):

Steif

$$\langle\,\mathcal{T}_\nu^\mu\,\rangle_{HH} = \frac{A(r_+)}{r^3}\,\mathrm{diag}\{1,1,-2\} \quad \text{for} \quad A(r_+) = \frac{2}{32\,\pi}\sum_{n=1}^{\infty}\frac{\cosh 2\pi\,n\,r_+ + 3}{(\cosh 2\pi\,n\,r_+ - 1)^{3/2}}$$

Summary

- Holographic gauge/gravity duality provides a useful tool to obtain quantitative results for strongly coupled fields in curved spacetime.

- Interesting new classes of gravitational solutions in asymptotically locally AdS spacetimes.
- Allows quantitative predictions for physical quantities, e.g. $<T_{\mu\nu}>$.
- Interesting to compute correlation functions of local operators

- Stepping stone towards understanding induced gravity (brane-world) models.