Dynamics of wavepackets in crystals by multiscale analysis

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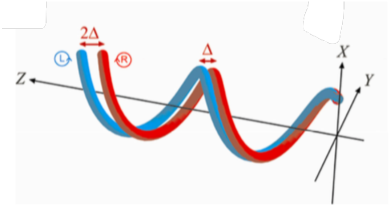
Motivation: the spin Hall effect of light

- Causes a linearly polarized beam of light (wavelength λ) in a gradient index medium (smoothly varying refractive index $n(\boldsymbol{x})$, $\lambda \ll$ scale of variation of the medium) to split into its constituent circularly polarized parts:



- Splitting $\Delta \propto$ light wavelength λ
- Experimental verification: K. Bliokh et al. 'Geometrodynamics of spinning light' Nature Photonics 2008

Ray optics doesn't explain spin Hall effect of light

- Effect appears in the regime: wavelength $\lambda \ll$ scale of variation of the medium
- Ray optics for a beam: center of mass $\boldsymbol{q}(t)$ and averaged wavevector $\boldsymbol{p}(t)$ of a beam satisfy classical equations:

$$\dot{\boldsymbol{q}}(t) = \hat{\boldsymbol{p}}(t)$$
$$\dot{\boldsymbol{p}}(t) = \nabla n(\boldsymbol{q}(t))$$

- Polarization independent, cannot explain spin Hall effect of light

Corrected ray optics equations

- Resolution: corrected ray equations, include an *anomalous velocity* proportional to λ:

$$\dot{\boldsymbol{q}}(t) = \hat{\boldsymbol{p}}(t) + \lambda \dot{\boldsymbol{p}}(t) \times \mathcal{F}_\sigma(\boldsymbol{p}(t))$$
$$\dot{\boldsymbol{p}}(t) = \nabla n(\boldsymbol{q}(t))$$

where $\sigma = \pm$ denotes the handedness of the polarization.

$$\mathcal{F}_\sigma(\boldsymbol{p}) := \nabla_{\boldsymbol{p}} \times \langle \boldsymbol{e}_\sigma(\boldsymbol{p}) | i \nabla_{\boldsymbol{p}} \boldsymbol{e}_\sigma(\boldsymbol{p}) \rangle = \sigma \frac{\boldsymbol{p}}{|\boldsymbol{p}|^3}$$

is the *Berry curvature* associated with each circular polarization state $\boldsymbol{e}_\sigma$.

Ray optics limit analogous to the semiclassical limit of quantum mechanics

- Semiclassical limit of Schrödinger's equation with potential $W(\boldsymbol{q})$:
  - seek wavepacket solutions, wavelength $\ll$ scale of variation of potential
  - center of mass and average wavevector of a wavepacket follow classical trajectories of $\mathcal{H} = \frac{1}{2}|\boldsymbol{p}|^2 + W(\boldsymbol{q})$
  - justify by 'WKB ansatz':

$$\psi(\boldsymbol{x}, t) = e^{i\phi(\boldsymbol{x},t)/\hbar} a(\boldsymbol{x}, t) + O(\hbar)$$

- Ray optics limit of Maxwell's equations in isotropic gradient-index media:
  - wavelength $\ll$ scale of variation of refractive index $n(\boldsymbol{q})$
  - center of mass and average wavevector of a beam follow classical trajectories of $\mathcal{H} = |\boldsymbol{p}| - n(\boldsymbol{q})$
  - justify by 'ray optics ansatz':

$$\boldsymbol{E}(\boldsymbol{x}, t) = e^{i\phi(\boldsymbol{x},t)/\lambda} \sum_{\sigma \in \pm} a_\sigma(\boldsymbol{x}, t) \boldsymbol{e}_\sigma(\nabla\phi) + O(\lambda)$$

Ray optics with polarization analogous to semiclassical quantum mechanics in a crystal

- Beam polarization vector $\boldsymbol{e}(\boldsymbol{p})$ must be transverse to the wavevector: $\boldsymbol{p} \cdot \boldsymbol{e}(\boldsymbol{p}) = 0$.
- Analogous to semiclassical quantum mechanics *with a periodic background* $V(\boldsymbol{x})$:

$$-\frac{1}{2}\Delta_{\boldsymbol{x}} \to -\frac{1}{2}\Delta_{\boldsymbol{x}} + V(\boldsymbol{x}),$$
$$\forall \boldsymbol{v} \in \Lambda : V(\boldsymbol{x}+\boldsymbol{v}) = V(\boldsymbol{x})$$

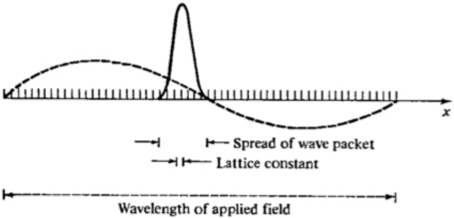
  Wavepacket solutions modulated *Bloch waves* associated with a Bloch band $E_n(\boldsymbol{p})$, with role of wavevector played by pseudo-momentum.
- 3-fold degeneracy of polarization condition at $\boldsymbol{p} = 0 \implies$ Berry curvature $\implies$ spin Hall effect of light.
- Bloch bands $E_n(\boldsymbol{p})$ may be degenerate $\implies$ Berry curvature $\implies$ anomalous velocity

Outline of talk

- ‘Semiclassical wavepacket’ asymptotic solutions of Schrödinger’s equation with a periodic background and describe range of validity of the asymptotics
- Corrections to the asymptotics which describe anomalous velocity due to Berry curvature (analogous to the spin Hall effect of light)
- Landau-Zener Bloch band crossing interactions
- Ongoing work/future directions

Model: electron dynamics in crystals

- Electrons in solids under the influence of an external potential which is *slowly-varying* relative to the lattice constant can be modelled as *wavepackets* which are localized relative to the varying potential but also spread over a few lattice periods



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[1] **Solid State Physics, Ashcroft and Mermin (1976).**

Model: electron dynamics in crystals

- Non-dimensionalized model Schrödinger equation:

$$i\partial_t \psi^\epsilon = -\frac{1}{2}\Delta_{\boldsymbol{x}}\psi^\epsilon + U(\boldsymbol{x}, \epsilon \boldsymbol{x})\psi^\epsilon$$

where two-scale potential U is periodic with respect to the lattice of ions Λ in its first argument:

$$\forall \boldsymbol{v} \in \Lambda, U(\boldsymbol{x} + \boldsymbol{v}, \boldsymbol{X}) = U(\boldsymbol{x}, \boldsymbol{X})$$

- Focus on simpler case:

$$U(\boldsymbol{x}, \epsilon \boldsymbol{x}) = V(\boldsymbol{x}) + W(\epsilon \boldsymbol{x})$$

- We consider the limit $\epsilon \ll 1$: the external potential $W(\epsilon \boldsymbol{x})$ slowly varying on the scale of the lattice period

Recap: spectral theory of periodic operators

- Recall the spectral theory of the operator with periodic potential obtained by taking $\epsilon = 0$ (no applied field):

$$h := -\frac{1}{2}\Delta_{\boldsymbol{x}} + V(\boldsymbol{x})$$
$$\forall \boldsymbol{v} \in \Lambda, V(\boldsymbol{x} + \boldsymbol{v}) = V(\boldsymbol{x})$$

- Bloch's theorem: suffices to study the eigenvalue problem on a single cell with $\boldsymbol{p}$-pseudo-periodic boundary conditions:

$$h\,\Phi(\boldsymbol{x};\boldsymbol{p}) = E(\boldsymbol{p})\Phi(\boldsymbol{x};\boldsymbol{p})$$
$$\forall \boldsymbol{v} \in \Lambda, \Phi(\boldsymbol{x} + \boldsymbol{v}) = e^{i\boldsymbol{p}\cdot\boldsymbol{v}}\Phi(\boldsymbol{x};\boldsymbol{p})$$

  symmetry of boundary condition $\implies$ we may restrict $\boldsymbol{p}$ to a primitive cell of the reciprocal lattice: first Brillouin zone $\mathcal{B}$
- Fixed $\boldsymbol{p}$, known as pseudo-momentum, self-adjoint elliptic eigenvalue problem $\implies$ discrete real spectrum:

$$E_1(\boldsymbol{p}) \leq E_2(\boldsymbol{p}) \leq ... \leq E_n(\boldsymbol{p}) \leq ...$$

Spectral theory of periodic operators

- Maps $\boldsymbol{p} \in \mathcal{B} \to E_n(\boldsymbol{p}) \in \mathbb{R}$ are the Bloch band dispersion functions
- The set of 'Bloch waves' are a basis (in $L^2(\mathbb{R}^d)$) of eigenfunctions of $h = -\frac{1}{2}\Delta_{\boldsymbol{x}} + V(\boldsymbol{x})$:

$$\left\{\Phi_n(\boldsymbol{x}; \boldsymbol{p}) := e^{i\boldsymbol{p}\cdot\boldsymbol{x}}\chi_n(\boldsymbol{x}; \boldsymbol{p}), n \in \mathbb{N}, \boldsymbol{p} \in \mathcal{B}\right\}$$

  $\chi_n(\boldsymbol{x}; \boldsymbol{p})$ satisfies another self-adjoint elliptic eigenvalue problem with periodic boundary conditions on a single cell
- The spectrum of h is then the union of real intervals swept out by the Bloch band dispersion functions $E_n(\boldsymbol{p})$

Semiclassical re-scaling

Our model is:

$$i\partial_t \psi^\epsilon = -\frac{1}{2}\Delta_{\boldsymbol{x}}\psi^\epsilon + V(\boldsymbol{x})\psi^\epsilon + W(\epsilon\boldsymbol{x})\psi^\epsilon$$

It will be convenient to work with the 'semiclassical re-scaling':

$$\begin{aligned} &\boldsymbol{x}' := \epsilon\boldsymbol{x}, t' := \epsilon t, \\ &\psi^{\epsilon\prime}(\boldsymbol{x}', t') := \psi^\epsilon(\boldsymbol{x}, t). \end{aligned}$$

After dropping the primes we obtain:

$$i\epsilon\partial_t \psi^\epsilon = -\epsilon^2\frac{1}{2}\Delta_{\boldsymbol{x}}\psi^\epsilon + V\left(\frac{\boldsymbol{x}}{\epsilon}\right)\psi^\epsilon + W(\boldsymbol{x})\psi^\epsilon$$

Theorem (Carles-Sparber 2008, Hagedorn 1980)

Let $(\boldsymbol{p}(t), \boldsymbol{q}(t))$ denote classical trajectories generated by the Bloch band Hamiltonian $\mathcal{H} = E_n(\boldsymbol{p}) + W(\boldsymbol{q})$, and assume the band E_n is isolated at each $\boldsymbol{p}(t)$:

$$\forall t \geq 0, E_{n-1}(\boldsymbol{p}(t)) < E_n(\boldsymbol{p}(t)) < E_{n+1}(\boldsymbol{p}(t)).$$

Then there exists a solution $\psi^\epsilon(\boldsymbol{x}, t)$ of the PDE:

$$i\epsilon\partial_t\psi^\epsilon = -\epsilon^2\frac{1}{2}\Delta_{\boldsymbol{x}}\psi^\epsilon + V\left(\frac{\boldsymbol{x}}{\epsilon}\right)\psi^\epsilon + W(\boldsymbol{x})\psi^\epsilon$$

which is asymptotic as $\epsilon \downarrow 0$ to a 'semiclassical wavepacket' up to 'Ehrenfest time' $t \sim \ln 1/\epsilon$:

$$\psi^\epsilon(\boldsymbol{x}, t) =$$

$$\epsilon^{-d/4}e^{iS(t)/\epsilon}e^{-i\boldsymbol{p}(t)\cdot\boldsymbol{q}(t)/\epsilon}a\left(\frac{\boldsymbol{x}-\boldsymbol{q}(t)}{\epsilon^{1/2}}, t\right)e^{i\boldsymbol{p}(t)\cdot\boldsymbol{x}/\epsilon}\chi_n\left(\frac{\boldsymbol{x}}{\epsilon}; \boldsymbol{p}(t)\right)$$

$$+ O_{L^2_x(\mathbb{R}^d)}(\epsilon^{1/2}e^{Ct}).$$

Precise interpretation of functions $(\boldsymbol{q}(t), \boldsymbol{p}(t))$

Writing the solution in terms of the multiscale variables:

$$\psi^\epsilon(\boldsymbol{x}, t) = \tilde{\psi}^\epsilon(\boldsymbol{y}, \boldsymbol{z}, t)\Big|_{\boldsymbol{y}=\frac{\boldsymbol{x}-\boldsymbol{q}(t)}{\epsilon^{1/2}}, \boldsymbol{z}=\frac{x}{\epsilon}} + O_{L^2_x(\mathbb{R}^d)}(\epsilon^{1/2})$$

$\boldsymbol{q}(t), \boldsymbol{p}(t)$ nothing but the center of mass and expected pseudo-momentum of the wavepacket, to leading order in $\epsilon^{1/2}$:

$$\mathcal{Q}^\epsilon(t) := \int_{\mathbb{R}^d} \boldsymbol{x} |\tilde{\psi}^\epsilon(\boldsymbol{y}, \boldsymbol{z}, t)|^2_{\boldsymbol{y}=\frac{\boldsymbol{x}-\boldsymbol{q}(t)}{\epsilon^{1/2}}, \boldsymbol{z}=\frac{\boldsymbol{x}}{\epsilon}} \, \mathrm{d}\boldsymbol{x}$$

$$= \boldsymbol{q}(t) + O(\epsilon^{1/2})$$

$$\mathcal{P}^\epsilon(t) := \int_{\mathbb{R}^d} \overline{\tilde{\psi}^\epsilon(\boldsymbol{y}, \boldsymbol{z}, t)} (-i\epsilon^{1/2}\nabla_{\boldsymbol{y}}) \tilde{\psi}^\epsilon(\boldsymbol{y}, \boldsymbol{z}, t)\Big|_{\boldsymbol{y}=\frac{\boldsymbol{x}-\boldsymbol{q}(t)}{\epsilon^{1/2}}, \boldsymbol{z}=\frac{\boldsymbol{x}}{\epsilon}} \, \mathrm{d}\boldsymbol{x}$$

$$= \boldsymbol{p}(t) + O(\epsilon^{1/2})$$

$O(\epsilon)$ corrections to dynamics of observables

- Expect correction to equations of motion of center of mass and expected pseudo-momentum due to Berry curvature of Bloch band $\propto$ wavelength ϵ, analogous to the spin Hall effect of light[2]

[2] **M. C. Chang and Q. Niu**, **'Berry phase**, **hyperorbits**, **and the Hofstader's Spectrum' Phys. Rev. Lett. 199 Phys. Rev. Lett. 1995.**

Theorem (Watson-Weinstein-Lu 2016)

1) The observables $\mathcal{Q}^\epsilon(t)$ and $\mathcal{P}^\epsilon(t)$, the center of mass and average pseudo-momentum, satisfy the equations of motion:

$$\dot{\mathcal{Q}}^\epsilon(t) = \nabla_{\mathcal{P}^\epsilon} E_n(\mathcal{P}^\epsilon(t)) + \epsilon \boldsymbol{C}_1[a^\epsilon](t) - \epsilon \dot{\mathcal{P}}^\epsilon(t) \times \boldsymbol{\mathcal{F}}_n(\mathcal{P}^\epsilon(t)) + O(\epsilon^{3/2})$$

$$\dot{\mathcal{P}}^\epsilon(t) = -\nabla_{\mathcal{Q}^\epsilon} W(\mathcal{Q}^\epsilon(t)) + \epsilon \boldsymbol{C}_2[a^\epsilon](t) + O(\epsilon^{3/2})$$

where $\mathcal{F}_n(\mathcal{P}^\epsilon)$ is the Berry curvature of the Bloch band. $\boldsymbol{C}_1[a^\epsilon](t), \boldsymbol{C}_2[a^\epsilon](t)$ describe coupling to the wavepacket envelope $a^\epsilon(\boldsymbol{y}, t)$, which satisfies:

$$i\partial_t a^\epsilon(\boldsymbol{y}, t) = -\frac{1}{2}\nabla_{\boldsymbol{y}} \cdot \nabla_{\mathcal{P}^\epsilon} \nabla_{\mathcal{P}^\epsilon} E_n(\mathcal{P}^\epsilon(t)) \cdot \nabla_{\boldsymbol{y}} a^\epsilon(\boldsymbol{y}, t) + \frac{1}{2}\boldsymbol{y} \cdot \nabla_{\mathcal{Q}^\epsilon} \nabla_{\mathcal{Q}^\epsilon} W(\mathcal{Q}^\epsilon(t)) \cdot \boldsymbol{y} a^\epsilon(\boldsymbol{y}, t) + O_{L^2_{\boldsymbol{y}}(\mathbb{R}^d)}(\epsilon^{1/2})$$

Theorem (Watson-Weinstein-Lu 2016 continued)

2) After an appropriate change of variables, the coupled dynamics of $\mathcal{Q}^\epsilon(t), \mathcal{P}^\epsilon(t), a^\epsilon(\mathbf{y}, t)$ can be derived from the ϵ-dependent Hamiltonian:

$$\begin{aligned}
\mathcal{H}^\epsilon &:= E_n(\mathcal{P}^\epsilon) + W(\mathcal{Q}^\epsilon) + \epsilon \nabla_{\mathcal{Q}^\epsilon} W(\mathcal{Q}^\epsilon) \mathcal{A}_n(\mathcal{P}^\epsilon) \\
&+ \epsilon \frac{1}{2} \int_{\mathbb{R}^d} \nabla_{\mathbf{y}} \overline{a^\epsilon(\mathbf{y}, t)} \cdot \nabla_{\mathcal{P}^\epsilon} \nabla_{\mathcal{P}^\epsilon} E_n(\mathcal{P}^\epsilon) \cdot \nabla_{\mathbf{y}} a^\epsilon(\mathbf{y}, t) \, d\mathbf{y} \\
&+ \epsilon \frac{1}{2} \int_{\mathbb{R}^d} \mathbf{y} \overline{a^\epsilon(\mathbf{y}, t)} \cdot \nabla_{\mathcal{Q}^\epsilon} \nabla_{\mathcal{Q}^\epsilon} W(\mathcal{Q}^\epsilon) \cdot \mathbf{y} a^\epsilon(\mathbf{y}, t) \, d\mathbf{y} + O(\epsilon^{3/2})
\end{aligned}$$

where $\mathcal{A}_n(\mathcal{P}^\epsilon)$ is the n-th band Berry connection. The equations of motion can then be written:

$$\dot{\mathcal{Q}}^\epsilon = \nabla_{\mathcal{P}^\epsilon} \mathcal{H}^\epsilon \qquad i\partial_t a^\epsilon = \frac{\delta \mathcal{H}}{\delta \overline{a^\epsilon}}$$
$$\dot{\mathcal{P}}^\epsilon = -\nabla_{\mathcal{Q}^\epsilon} \mathcal{H}^\epsilon$$

Gaussian reduction of envelope equation

- The equation satisfied by the wavepacket envelope:

$$i\partial_t a^\epsilon(\boldsymbol{y}, t) = -\frac{1}{2}\nabla_{\boldsymbol{y}} \cdot \nabla_{\boldsymbol{\mathcal{P}}^\epsilon} \nabla_{\boldsymbol{\mathcal{P}}^\epsilon} E_n(\boldsymbol{\mathcal{P}}^\epsilon(t)) \cdot \nabla_{\boldsymbol{y}} a^\epsilon(\boldsymbol{y}, t)$$
$$+ \frac{1}{2}\boldsymbol{y} \cdot \nabla_{\boldsymbol{\mathcal{Q}}^\epsilon} \nabla_{\boldsymbol{\mathcal{Q}}^\epsilon} W(\boldsymbol{\mathcal{Q}}^\epsilon(t)) \cdot \boldsymbol{y} a^\epsilon(\boldsymbol{y}, t)$$

has basis of exact solutions: e.g. time-dependent Gaussians[3]:

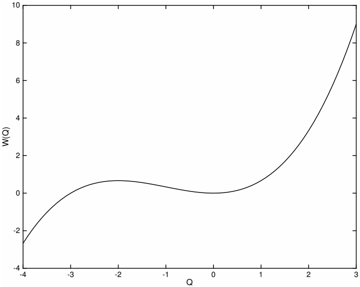
$$a^\epsilon(\boldsymbol{y}, t) = \frac{1}{[\det A^\epsilon(t)]^{1/2}} \exp\left(-\frac{1}{2}\boldsymbol{y} \cdot B^\epsilon(t) A^{\epsilon -1}(t) \boldsymbol{y}\right)$$
$$\dot{A}^\epsilon(t) = i\nabla_{\boldsymbol{\mathcal{P}}^\epsilon} \nabla_{\boldsymbol{\mathcal{P}}^\epsilon} E_n(\boldsymbol{\mathcal{P}}^\epsilon) B^\epsilon(t)$$
$$\dot{B}^\epsilon(t) = i\nabla_{\boldsymbol{\mathcal{Q}}^\epsilon} \nabla_{\boldsymbol{\mathcal{Q}}^\epsilon} W(\boldsymbol{\mathcal{Q}}^\epsilon) A^\epsilon(t)$$

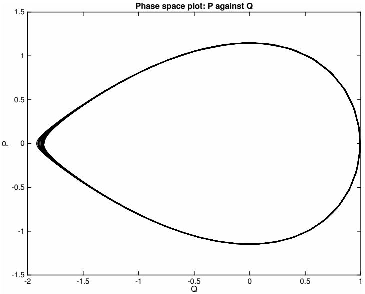
[3] **Raising and lowering operators for semiclassical wave packets**, **G. A. Hagedorn**, **Annals of Physics 1998**.

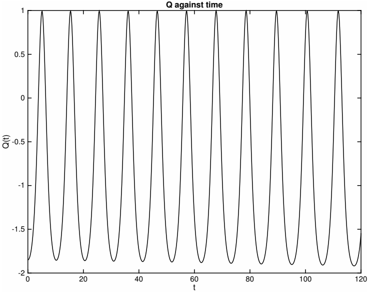
Numerical simulation: $\epsilon = 0$, decoupled system

Study coupling of observables to wave-field:

- One-dimensional: $d = 1$
- Uniform background: $V\left(\frac{x}{\epsilon}\right) = 0$
- Gaussian envelope
- Applied potential $W(\mathcal{Q}) = \frac{1}{6}\mathcal{Q}^3 + \frac{1}{2}\mathcal{Q}^2$



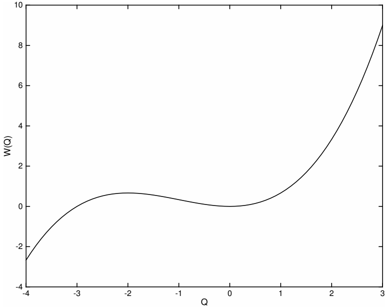


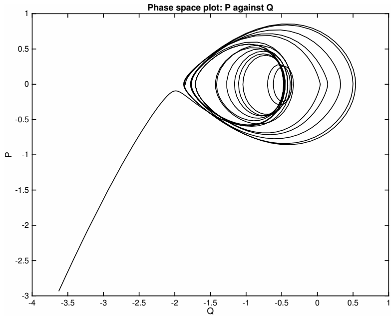


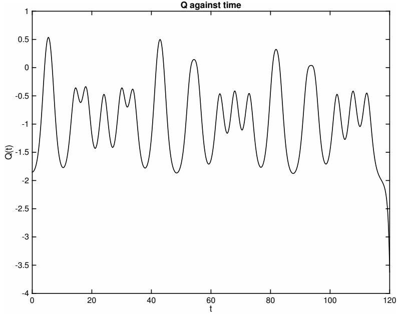
Numerical simulation: $\epsilon \neq 0$, coupled system

Simulation of full coupled system:

- Wave-field coupling has destabilizing effect on periodic orbits
- Wavepacket may escape potential well to $\mathcal{Q}^{\epsilon} = -\infty$



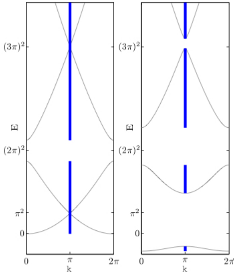




Band crossing dynamics in $d = 1$

- Would like to relax the 'isolated band' assumption:

$$\forall t \geq 0, E_{n-1}(\boldsymbol{p}(t)) < E_n(\boldsymbol{p}(t)) < E_{n+1}(\boldsymbol{p}(t))$$



[4]

- At crossings the Bloch band functions: $\boldsymbol{p} \to E_n(\boldsymbol{p})$ are not smooth in general. In $d = 1$ there exists a 'smooth choice' of bands in a neighborhood of the crossing: $E_+(p), E_-(p)$.

[4] **Topologically protected states in one-dimensional continuous systems and Di**
C. L. Fefferman, J. P. Lee-Thorp, M. I. Weinstein, PNAS 2014.

Theorem (Watson-Weinstein 2016)

Let $E_+(p), E_-(p)$ denote smooth band functions in a neighborhood of a crossing point p^*. Let $(q_+(t), p_+(t))$ denote a classical trajectory of the $+$-band Hamiltonian $E_+(p) + W(q)$ which passes through the crossing point at $t = 0$: $p_+(0) = p^*, \partial_q W(q_+(0)) \neq 0$. Then, if $\psi^\epsilon(x,t)$ solves the PDE on the interval $t \in [-T, T]$ and is associated with the $+$-band at $t = -T$:

$$\psi^\epsilon(x, -T) =$$

$$\epsilon^{-1/4} e^{ip_+(-T)(x - q_+(-T))/\epsilon} a_+ \left(\frac{x - q_+(-T)}{\epsilon^{1/2}}, -T \right) \chi_+ \left(\frac{x}{\epsilon}; p_+(-T) \right)$$

then at $t = T$, the wavepacket remains to leading order associated with the $+$-band:

$$\psi^\epsilon(x, T) =$$

$$\epsilon^{-1/4} e^{iS(T)/\epsilon} e^{ip_+(T)(x - q_+(T))/\epsilon} a_+ \left(\frac{x - q_+(T)}{\epsilon^{1/2}}, T \right) \chi_+ \left(\frac{x}{\epsilon}; p_+(T) \right)$$

$$+ O_{L^2_x(\mathbb{R})}(\epsilon^{1/2})$$

Theorem (Watson-Weinstein 2016 continued)

At the crossing time $t = 0$, a wavepacket associated with E_- is excited whose observables $(q_-(t), p_-(t))$ follow a classical trajectory of the band Hamiltonian $E_-(p) + W(q)$ with initial data:

$$q_-(0) = q_+(0)$$
$$p_-(0) = p_+(0) = p^*.$$

This wavepacket has magnitude (in $L^2_x(\mathbb{R})$) proportional to $\epsilon^{1/2}$.

Remarks on band crossing result

- Proof is by matched asymptotic expansion: error in single-band approximation blows up as $t \to 0$, resolution by making more general ansatz for asymptotic solution which includes contributions from the band $E_- \implies$ excited wave
- Since $\partial_p E_+(p^*) = -\partial_p E_-(p^*)$, the wavepacket 'excited' at the crossing has opposite group velocity. Call this a 'reflected wave'
- Our result can be seen as an analog of those obtained by Hagedorn[5] in the context of Born-Oppenheimer approximation of molecular dynamics

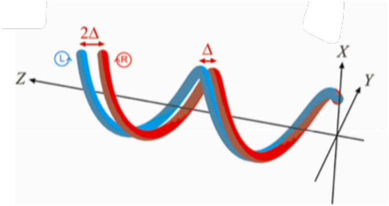
[5] **Molecular propagation through electron energy level crossings**, **Hagedorn G.**, **Memoirs of the American Mathematical Society (1994)**.

Recap of talk

- 'Semiclassical wavepacket' asymptotic solutions of Schrödinger's equation with a periodic background
- Corrections to the asymptotics which describe anomalous velocity due to Berry curvature (analogous to the spin Hall effect of light) and particle-field coupling between physical observables and the shape of the wavepacket envelope
- Landau-Zener Bloch band crossing interactions
- Ongoing work/future directions

Ongoing work

- Rigorous derivation of spin Hall effect for circularly polarized 'ray optics wavepackets'



Ongoing work

- Important *non-separable* example in $d = 2$:

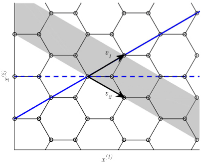
$$U(\boldsymbol{x}, \epsilon\boldsymbol{x}) = V_{h,e}(\boldsymbol{x}) + \kappa(\boldsymbol{k} \cdot \epsilon\boldsymbol{x}) V_{h,o}(\boldsymbol{x})$$

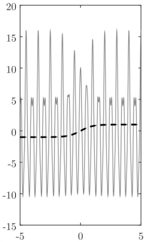
where $V_{h,e}(\boldsymbol{x})$, $V_{h,o}(\boldsymbol{x})$ have symmetry of a honeycomb lattice Λ_h, and satisfy:

$$V_{h,e}(-\boldsymbol{x}) = V_{h,e}(\boldsymbol{x}), V_{h,o}(-\boldsymbol{x}) = -V_{h,o}(\boldsymbol{x})$$

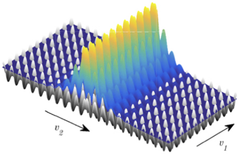
and $\kappa(\zeta)$ models a domain wall along the line $\boldsymbol{k} \cdot \boldsymbol{x} = 0$:

$$\lim_{\zeta \to -\infty} \kappa(\zeta) = -\kappa_\infty < 0, \kappa(0) = 0, \lim_{\zeta \to \infty} \kappa(\zeta) = \kappa_\infty > 0$$





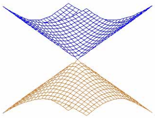
- System shown to support robust edge states by Fefferman, Lee-Thorp and Weinstein 'Edge states in honeycomb structures', Arxiv 1506:06111



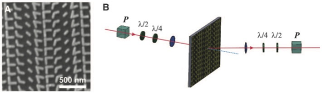
- Study *semiclassical wavepackets* localized near to the edge: anomalous velocity due to Berry curvature along the edge

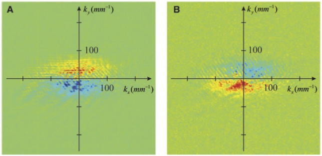
Future directions

- Spin Hall effect in anisotropic media, biaxial crystals: dispersion surfaces conically degenerate along optic axis



- Crossing result in higher dimensions: 'smooth bands' $E_+(\boldsymbol{p}), E_-(\boldsymbol{p})$ in a neighborhood of the crossing may not exist
- Metamaterials: how can we generalize results when features vary over scale of the wavelength?[6]





[6] **X. Yin, Z. Ye, Y. Wang, X. Zhang 'Photonic spin Hall effect at metasurfaces' Science 2013.**

Thanks for listening!