LONDON MATHEMATICAL SOCIETY
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LMS-EPSRC Durham Symposium: Mathematical and Computational Aspects of Maxwell's Equations

Controlling electromagnetic waves in a class of invisible materials

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- Patrick Bradley, Wu Biyi, Clive Parini, Luigi La Spada, Raj Mittra, Rob Foster etc.
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ANTENNAS & ELECTROMAGNETICS RESEARCH GROUP

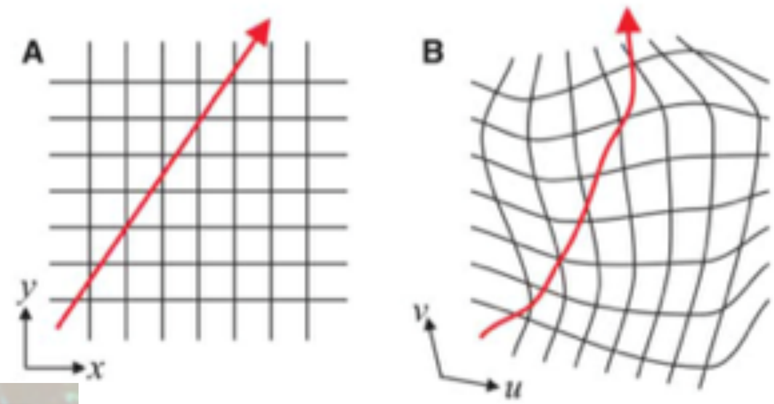
EPSRC

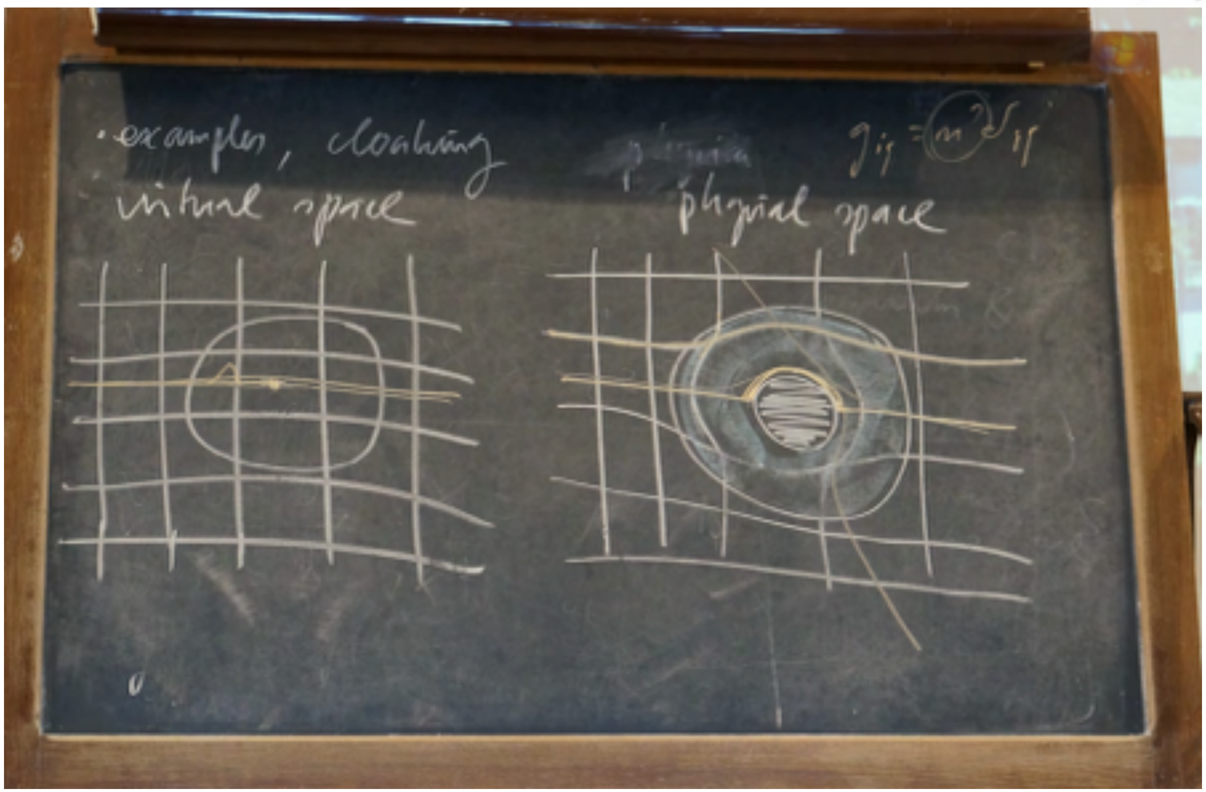
Engineering and Physical Sciences Research Council

1. How to control EM waves ? (Transformation Optics)

U. Leonhardt, Science 2006, **312,** 1777-80; J. B. Pendry. Science 2006, 312 (5781): 1780-2

- To control wave flow arbitrarily
  - Transformation optics,
  - anisotropicity of material parameters





U. Leonhardt, “The Science of Light”,
Slides of the Lecture, 2 Jul 2014,
School of Phys. Enrico Fermi Varenna.

1.1 How to control EM waves ? (hint from GO)

$$\left[\nabla^2 + \frac{\omega^2}{c^2} n^2(\boldsymbol{r}, \omega)\right] \psi(\boldsymbol{r}, \omega) = 0. \qquad (1)$$

$$\psi(\boldsymbol{r}, \omega) = R(\boldsymbol{r}) \mathrm{e}^{\mathrm{i}S(\boldsymbol{r})}, \qquad (2)$$

$$(\nabla S)^2 - \frac{\nabla^2 R}{R} - \frac{\omega^2}{c^2} n^2(\boldsymbol{r}, \omega) = 0, \qquad (3)$$

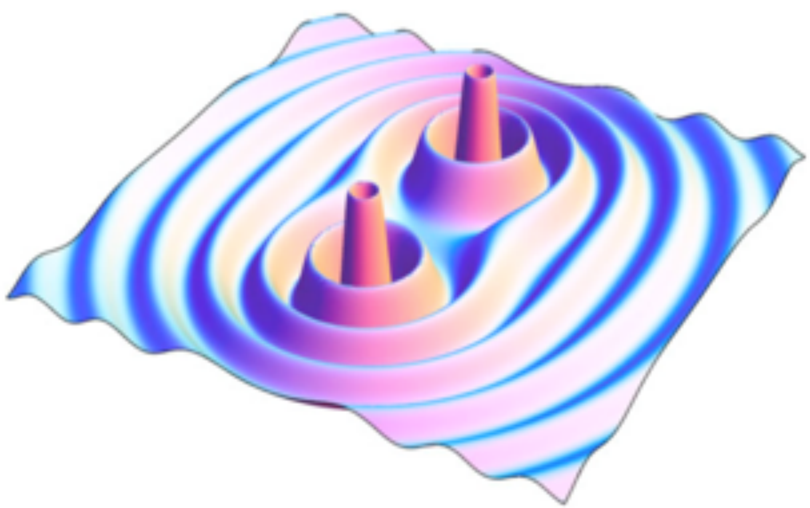
$$\nabla \cdot (R^2 \nabla S) = 0. \qquad (4)$$

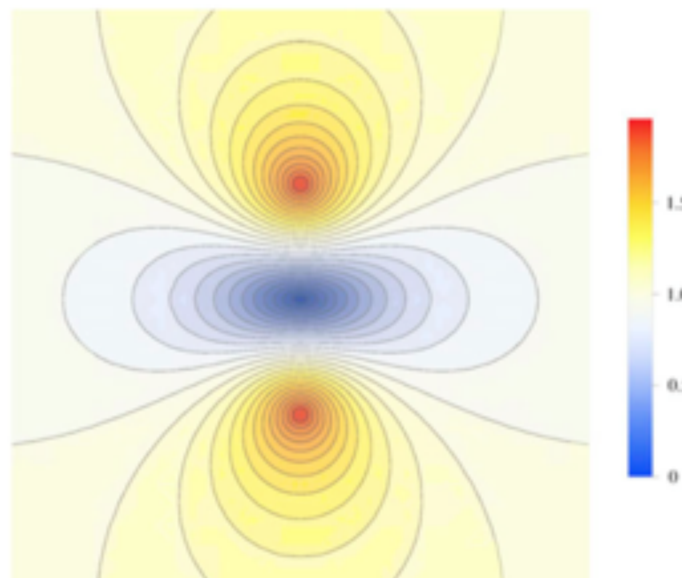
Note that if

$$S(\boldsymbol{r}) \propto \frac{1}{R(\boldsymbol{r})}$$

then the exact wave Equation (4) becomes

$$\nabla^2 R = 0,$$





Philbin, T. G. *Making geometrical optics exact*, Journal of Modern Optics (2014): 1-6, arxiv1402.2811

1.2 How to control EM waves ?

- To control wave flow arbitrarily
  - Transformation optics, mother design->daughter design
  - sometimes, anisotropicity of material parameters

- More direct — start from wave

$$(\nabla S)^2 - \frac{\nabla^2 R}{R} - \frac{\omega^2}{c^2} n^2(\boldsymbol{r}, \omega) = 0,$$
$$\nabla \cdot (R^2 \nabla S) = 0.$$

2D scenario

$$\nabla \cdot \left(\frac{1}{\xi} \nabla F\right) + k_0^2 \chi F = 0, \text{ in polar form as } F = A e^{\mathrm{i}\phi},$$

where $F = E_z$, $\chi = \varepsilon$, $\xi = \mu$ for the TE case

$F = H_z$, $\chi = \mu$, $\xi = \varepsilon$ for the TM case.

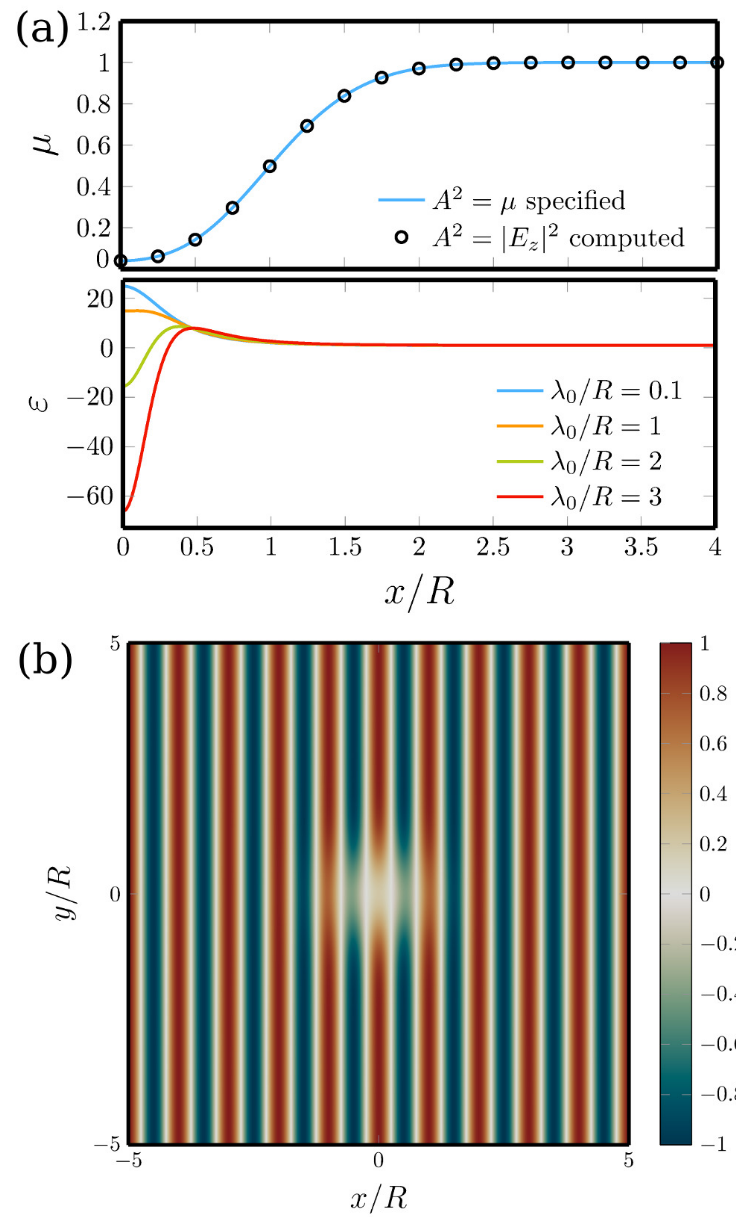
2. Amplitude modulator—planar wave keeper

TE case

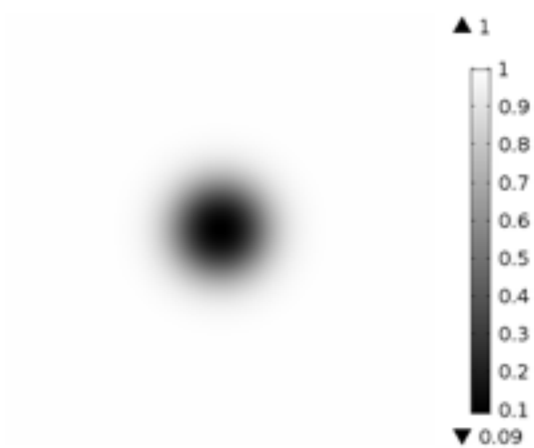
$$\begin{cases} \frac{\nabla^2 A}{A} - |\nabla\phi|^2 + k_0^2\epsilon\mu - \frac{\nabla\mu}{\mu}\cdot\frac{\nabla A}{A} = 0, \\ \nabla\cdot\left(\frac{A^2}{\mu}\nabla\phi\right) = 0. \end{cases}$$

$\mu = A^2$ ⇨ planar phase

$$A = 1 - f\exp(-r^2/R^2),$$



μ



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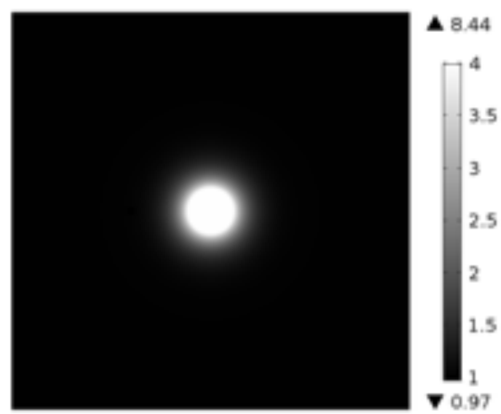
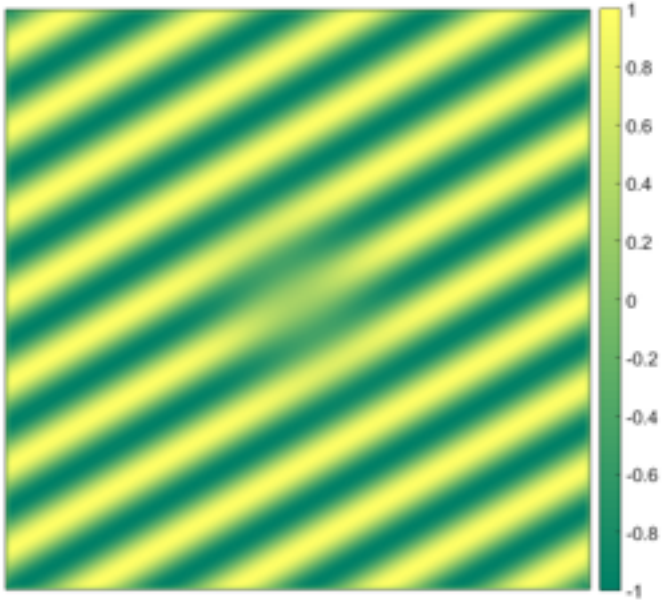


—B. Vial, Y. Liu, etc, paper1 submitting

2.1 Amplitude modulator— planar wave keeper

$$F = A\mathrm{e}^{i\phi}$$

F - incident planar wave



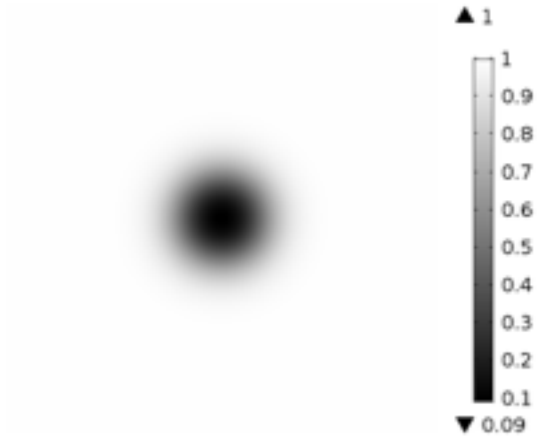
$$\mu = A^2$$

$$\epsilon = \frac{1}{k_0^2 A^2}\left(k_0^2 + 2\frac{(\nabla A)^2}{A^2} - \frac{\nabla^2 A}{A}\right)$$

—Y. Liu and B. Vial, etc, paper2 submitting

2.2 cylindrical wave keeper: surprise-without singularity

μ



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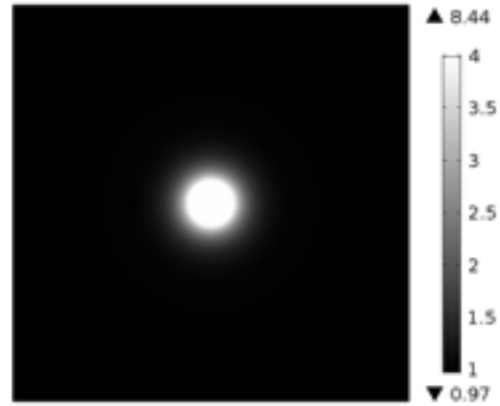
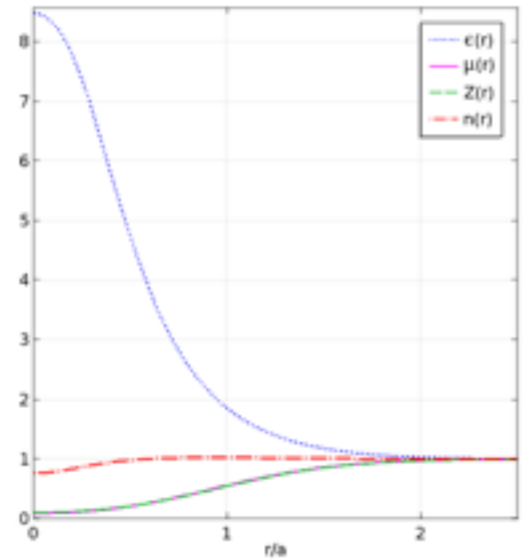
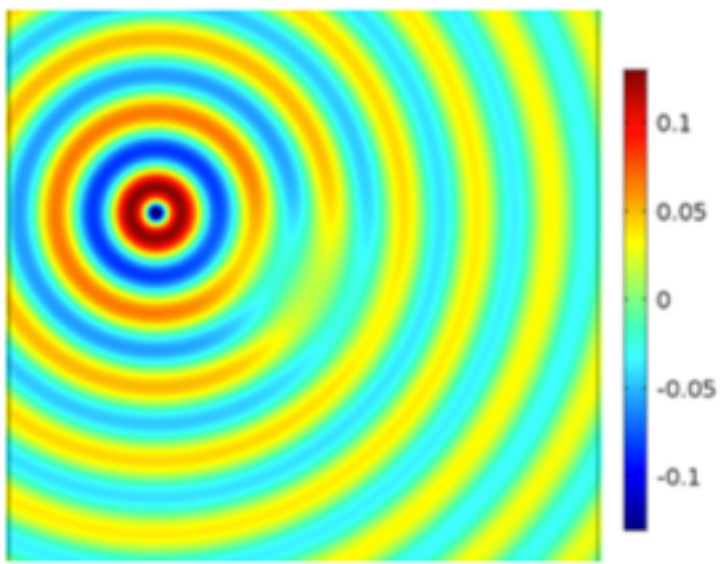
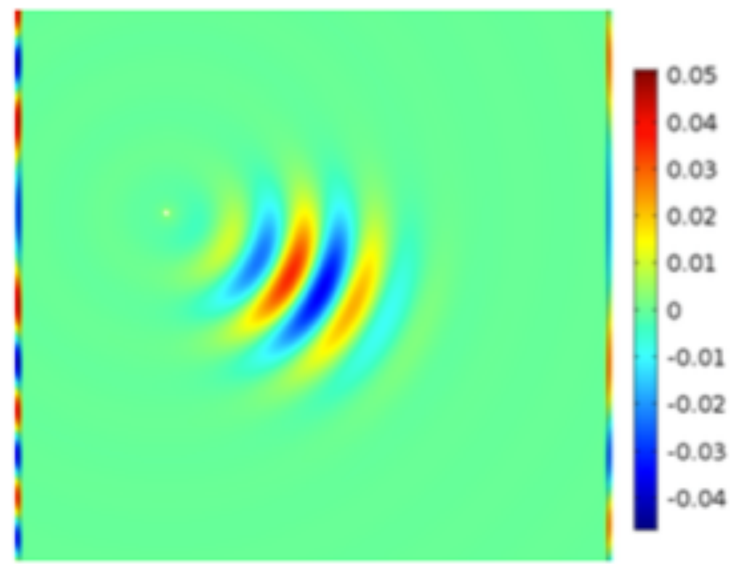




almost arbitrary position of source



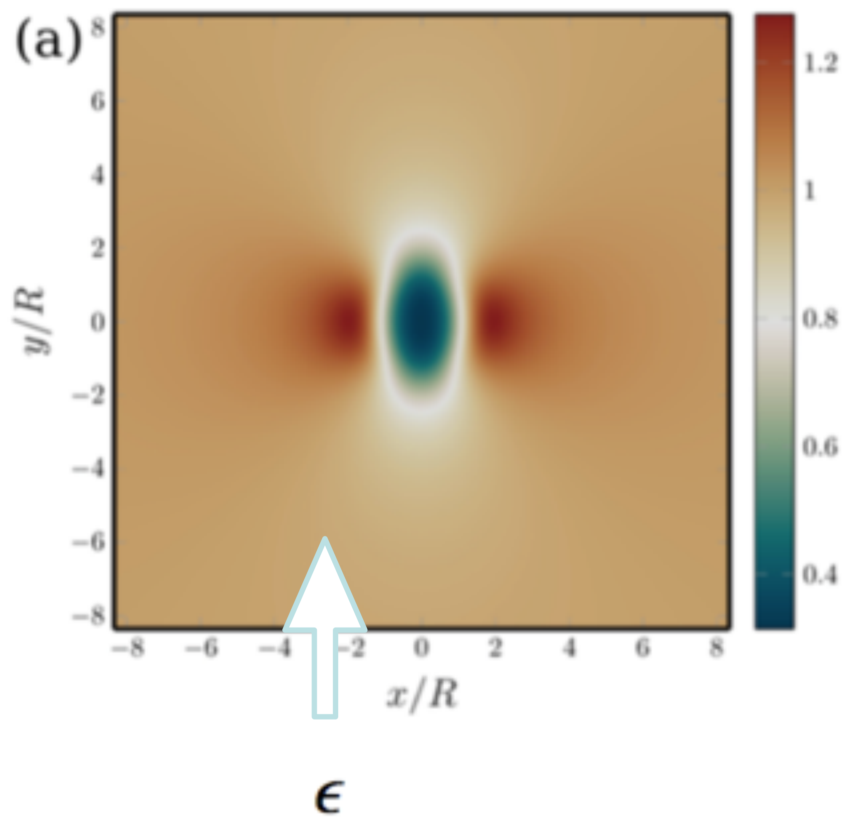


$F = Ae^{i\phi}$

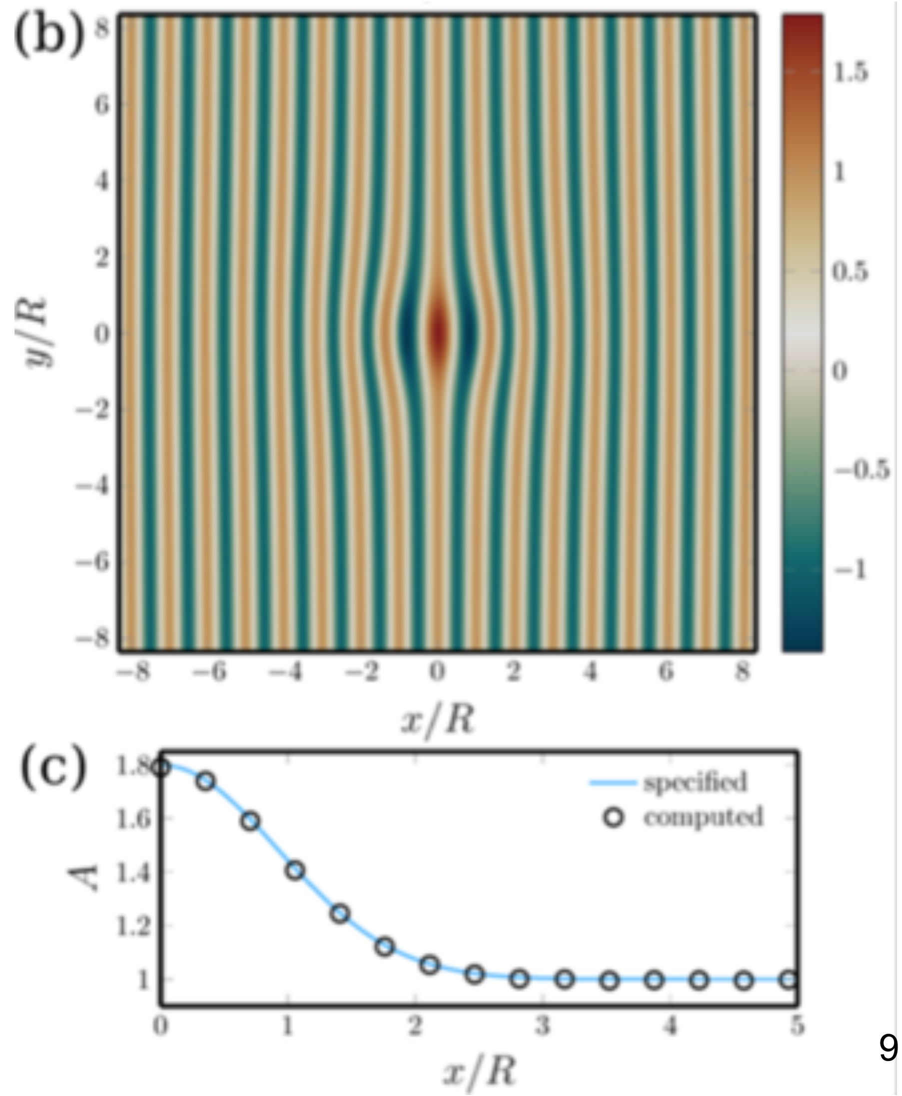
F - incident planar wave

2.3 Amplitude modulator—non-magnetic planar wave keeper

$$\mu = 1 \quad \begin{cases} \nabla \cdot \left(A^2 \nabla \phi\right) = 0, \\ \epsilon = \frac{1}{k_0^2}\left(|\nabla \phi|^2 - \frac{\nabla^2 A}{A}\right). \end{cases}$$



$$F = Ae^{i\phi}$$



—B. Vial, Y. Liu, etc, paper1 preparing

3. phase shaper: cylindrical into planar — smooth profile

$$\begin{cases} \nabla \cdot \left(A^2 \nabla \phi\right) = 0, \\ \epsilon = \frac{1}{k_0^2}\left(|\nabla \phi|^2 - \frac{\nabla^2 A}{A}\right). \end{cases} \Longrightarrow A = \frac{1}{\phi}$$

$$S_{\mathrm{i}} = \arctan[Y_0(k_0\rho), -J_0(k_0\rho)],$$

$$\phi(x,y) = \left(S_{\mathrm{i}}(x,y)[1 - sx(x)] + sx(x)\rho[(x+b)\cos\theta + y\sin\theta]\right), sx(x) := \frac{1+\tanh(\beta x)}{2}, \quad (29)$$

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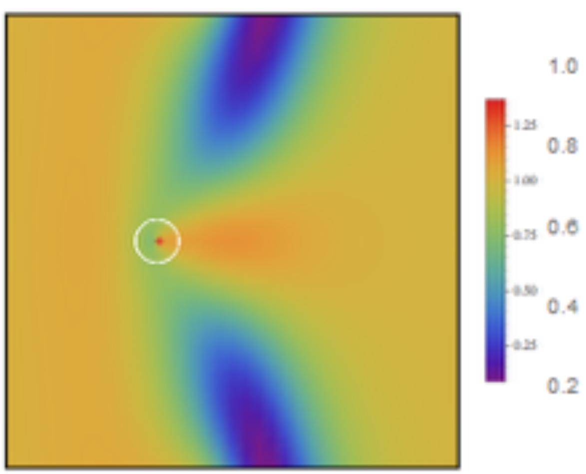
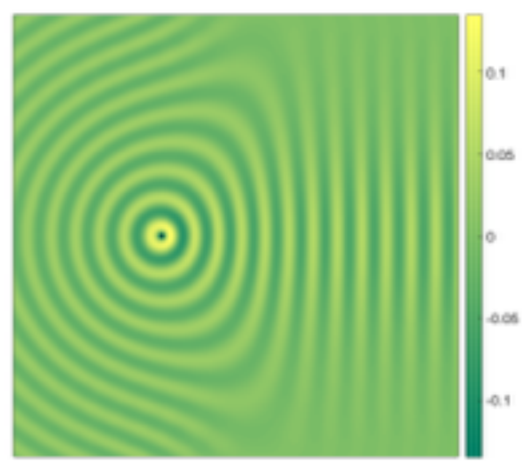
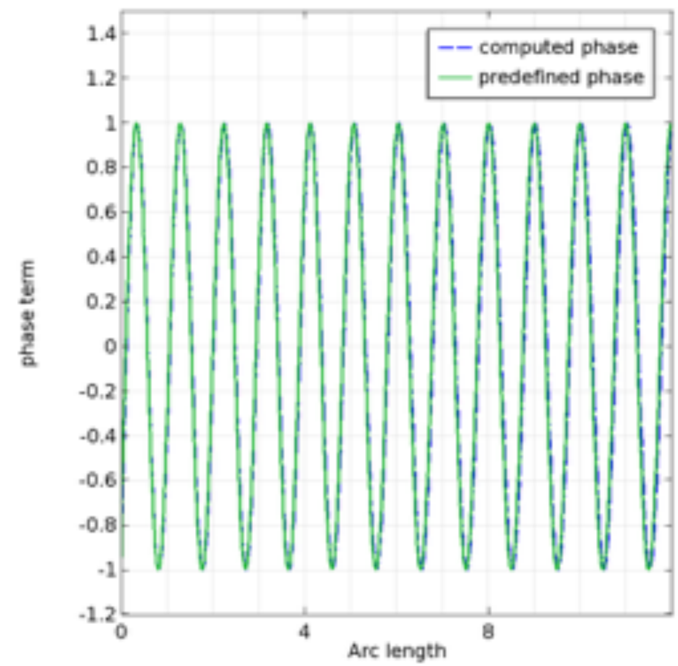


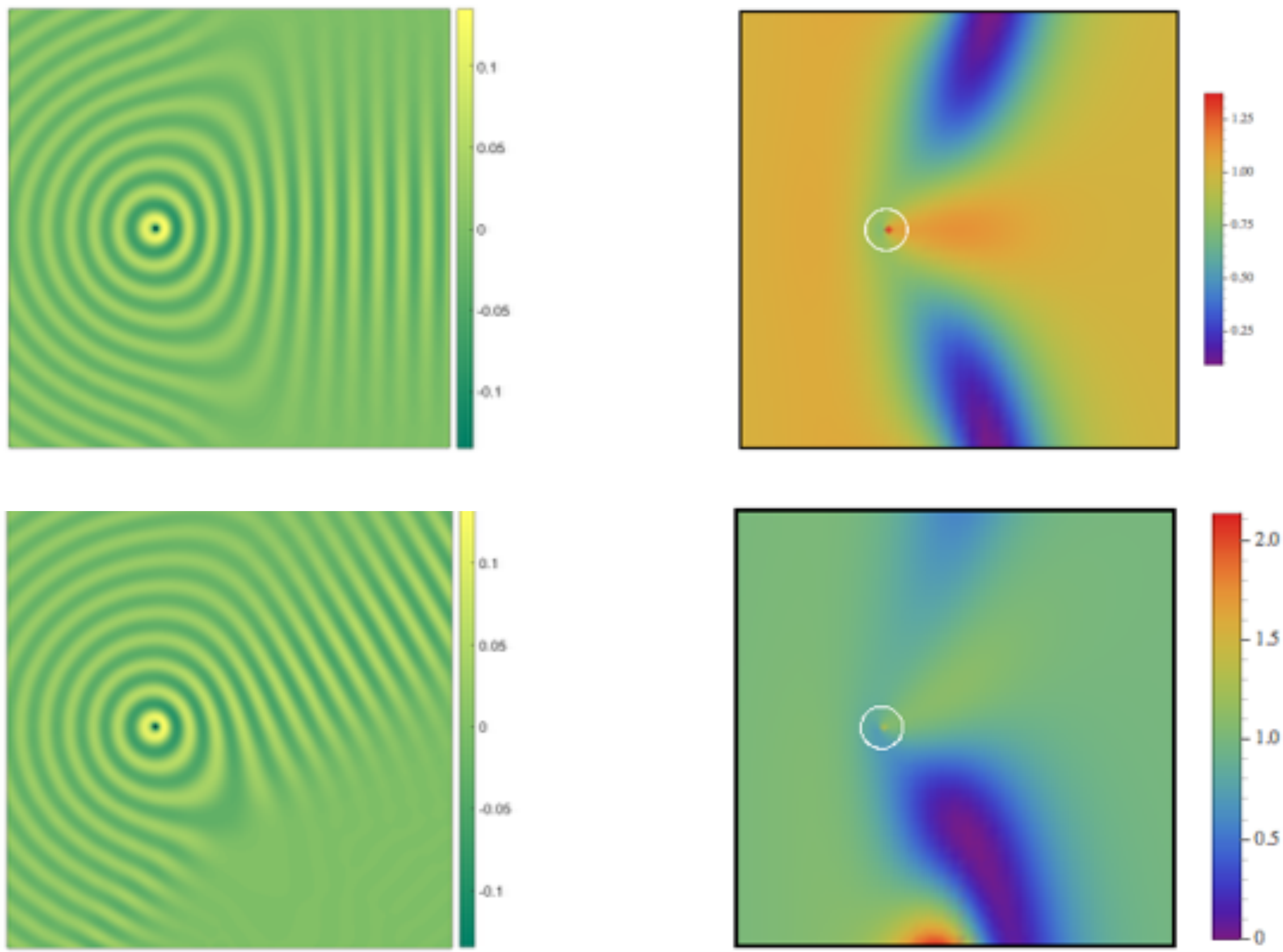
$F = Ae^{i\phi}$





a point source

3.1 cylindrical wave to planar wave —oblique output beam

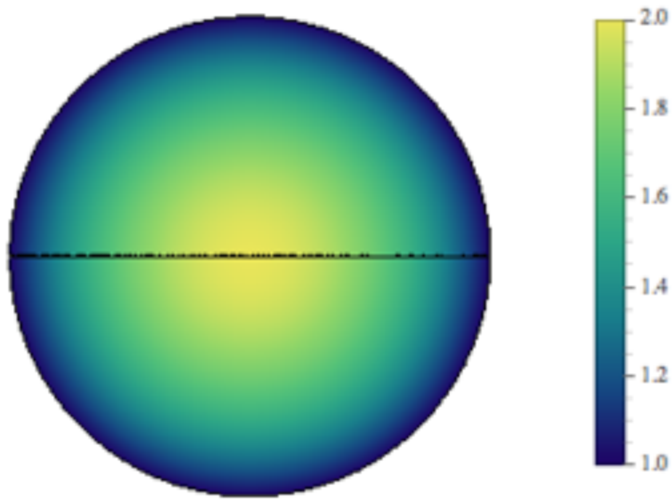


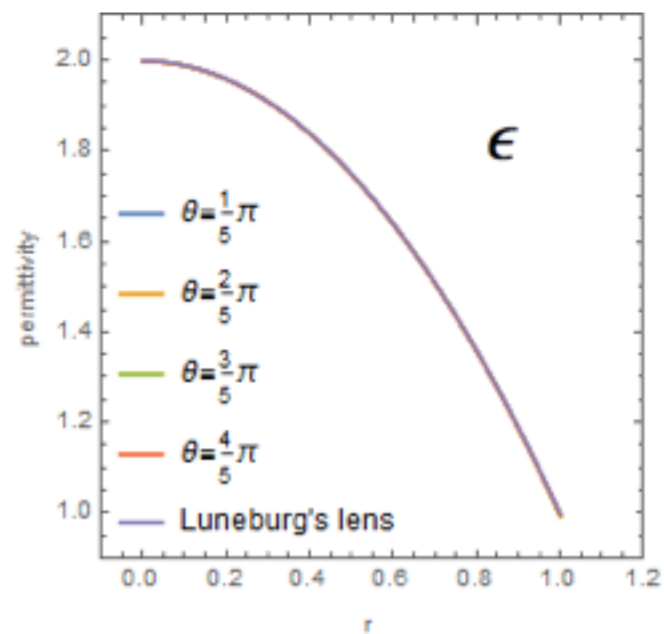
3.2 verification—recovered Lüneburg lens

$$n = \sqrt{\epsilon_r} = \sqrt{2 - \left(\frac{r}{R}\right)^2}$$

phase

ϵ





4. TM case, nonmagnetic material: coupled PDEs
Predefine amplitude of TM wave, and solve permittivity

The governing equations for TM polarization are:

$$\begin{cases} \nabla \cdot \left(\frac{A^2}{\varepsilon} \nabla \phi \right) = 0 & (1) \\ (\nabla \phi)^2 - k_0^2 \varepsilon \mu - \frac{\nabla^2 A}{A} + \frac{\nabla \varepsilon}{\varepsilon} \cdot \frac{\nabla A}{A} = 0 & (2) \end{cases}$$

and we write

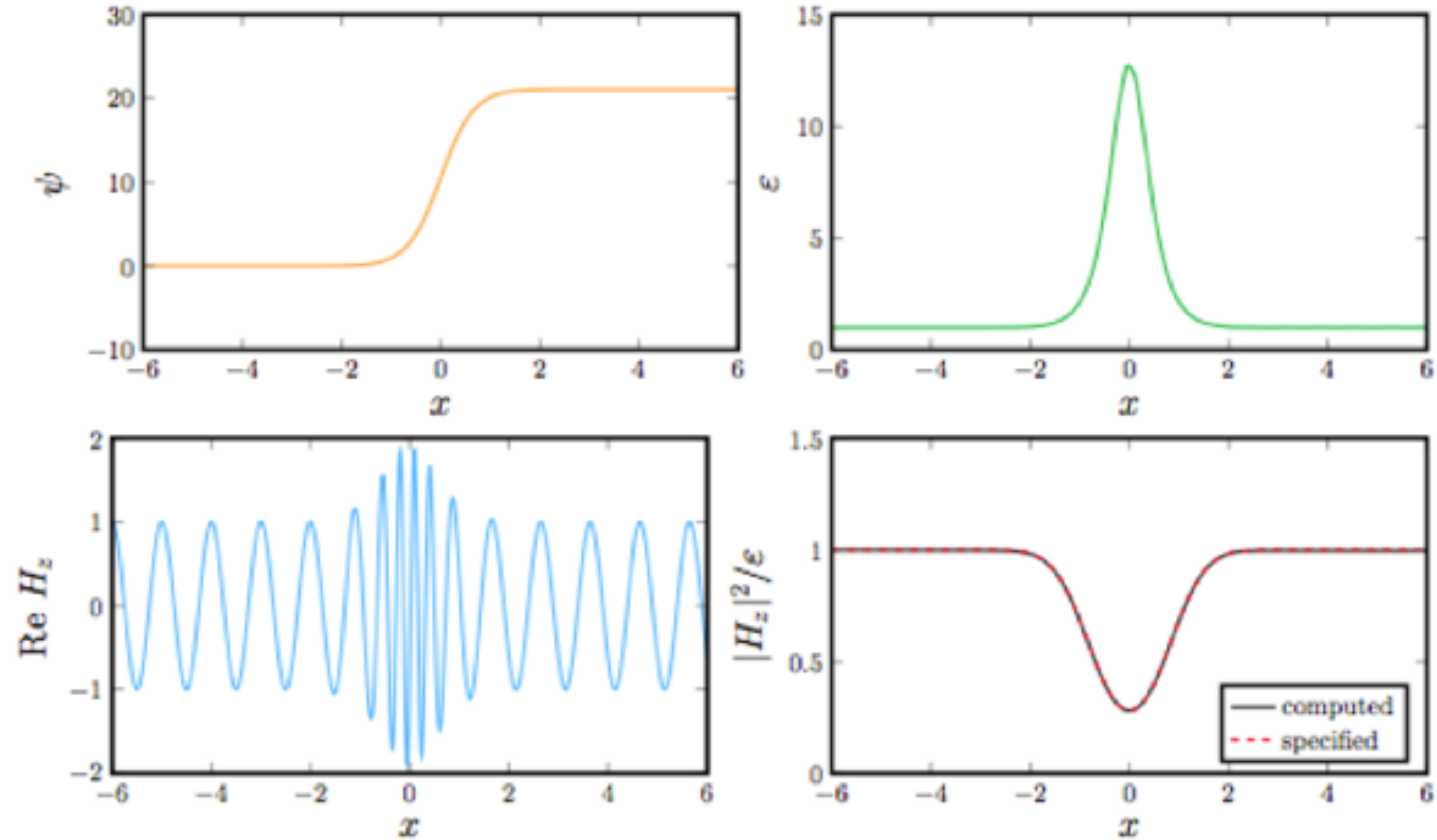
$$\nabla \phi = \boldsymbol{n} k_0 + \nabla \psi. \qquad (3)$$

We actually specify $a^2 = A^2/\varepsilon$ and the solve for ϕ in Eq.(1).
Then plug ϕ and $A = a\sqrt{\varepsilon}$ into Eq.(2) and solve for ε.
Example with $a = 1 - f e^{-x^2/R^2}$, $R = \lambda_0$, $f = 0.47$.

4. TM case, nonmagnetic material: 1D

CONTROLING THE AMPLITUDE OF EM WAVE, WITHOUT SCATTERING

Numerical solution



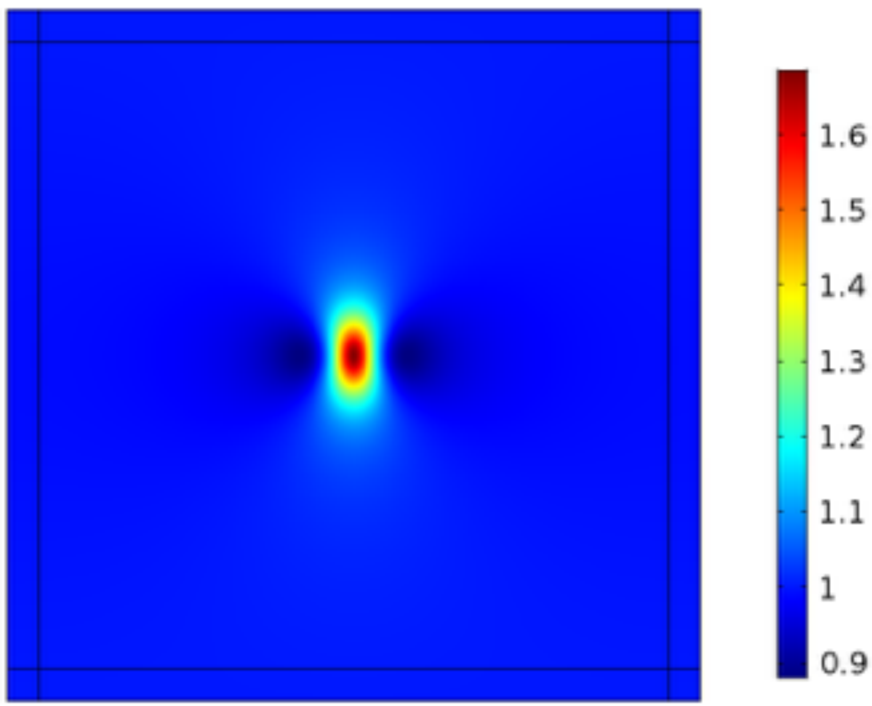
Credit: Ben Vial

4. TM case, nonmagnetic material: 2D coupled PDEs
Predefine amplitude of TM wave, and solve permittivity

$$\nabla \cdot \left(\frac{1}{\xi} \nabla F \right) + k_0^2 \chi F = 0, \qquad \text{in polar form as } F = A e^{i\phi},$$

where $F = E_z$, $\chi = \varepsilon$, $\xi = \mu$ for the TE case

$F = H_z$, $\chi = \mu$, $\xi = \varepsilon$ for the TM case.



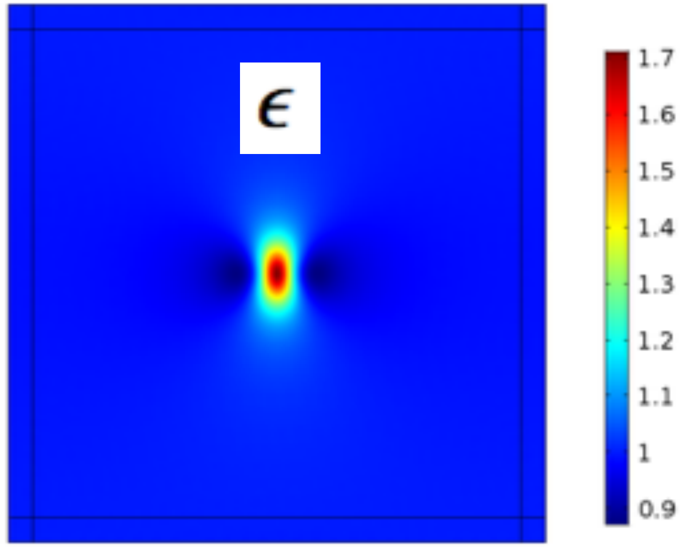
$$A = 1 - f \exp(-r^2/R^2),$$

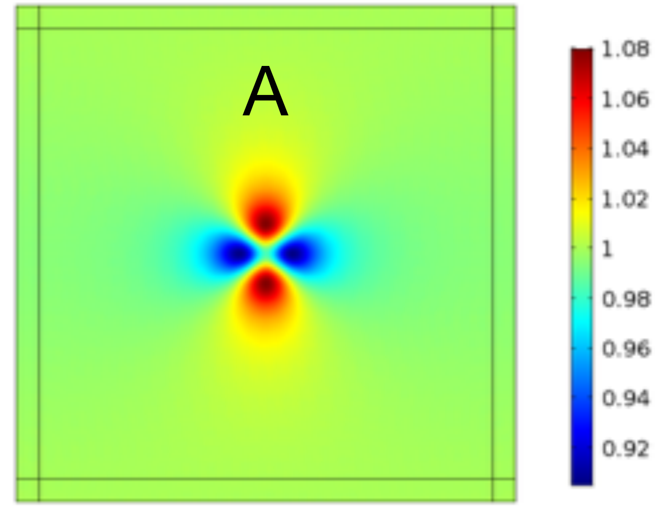
TE->TM?

TM: amp= A, instead define

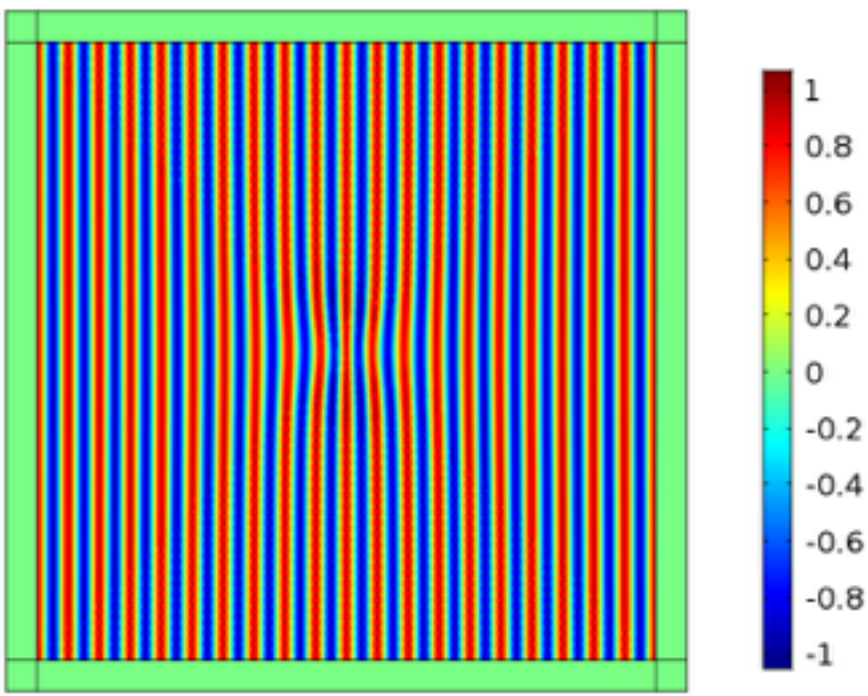
$$a = 1 - f \exp(-r^2/R^2)$$

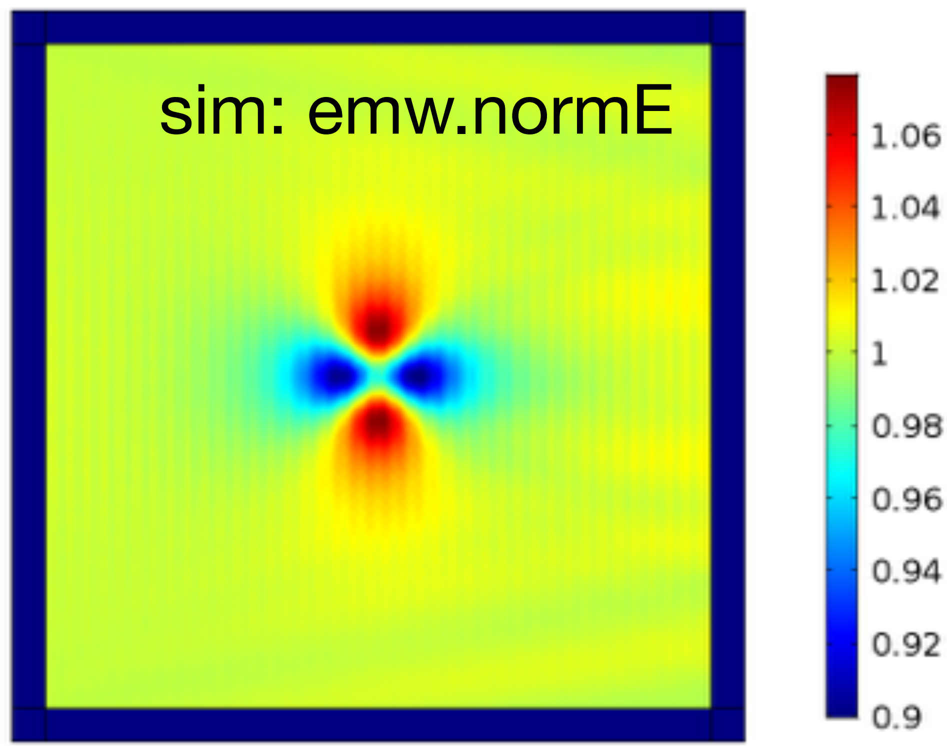
$$a^2 = A^2/\varepsilon$$





f cannot be large! An issue.





5. Conclusion and outlook

- a class of invisible material for planar wave or point source (2D) — amplitude controlling;
- besides, phase modulating.
- isotropic, inhomogeneous, TE and TM, 3D…

Thank you. Q & A

Yangjié

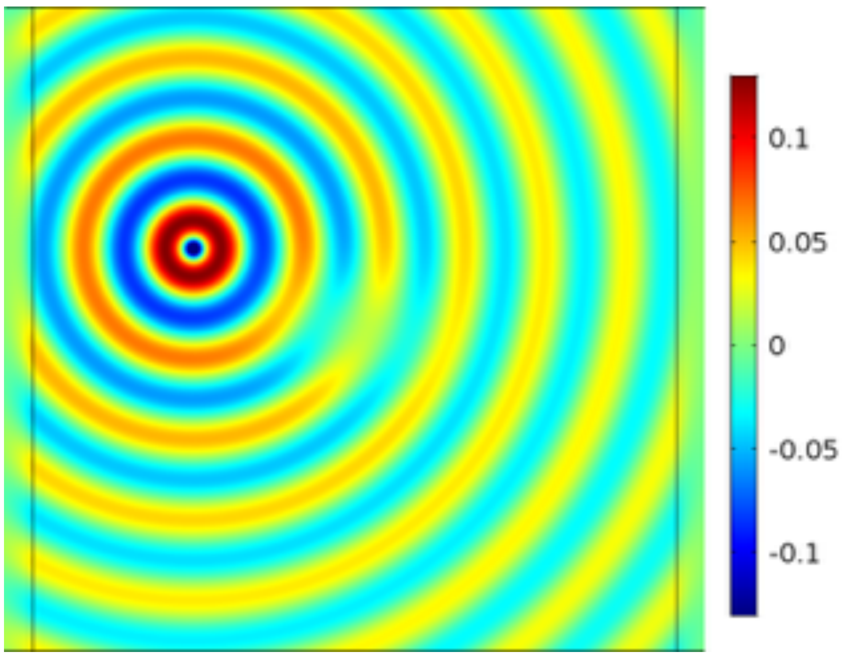
ANTENNAS & ELECTROMAGNETICS RESEARCH GROUP

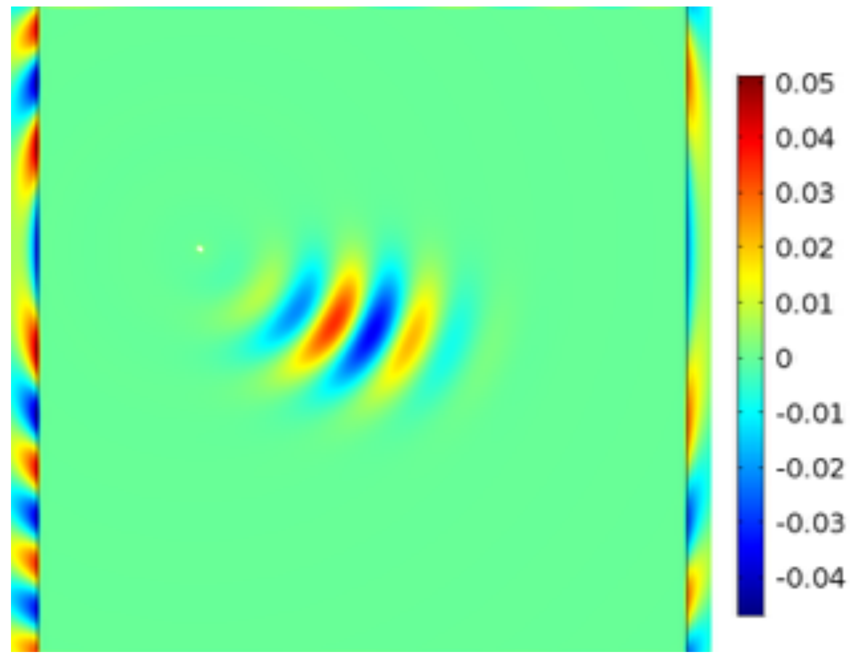
—*Workshop: THz Sources , 31 May, Queen Mary.*
—*London Plasmonics Forum, 9 Jun., King's College of London.*
—talk, Metamaterials 2016 Greece, Sept 2016;
—submitted, OSA Frontiers in Optics: 17 Oct 2016 - 21 Oct 2016.

M.C. Escher, Circle Limit III (1959), Hyperbolic plane, from http://www.maths.dur.ac.uk/Ug/projects/highlights/CM3/Hayter_Hyperbolic_report.pdf

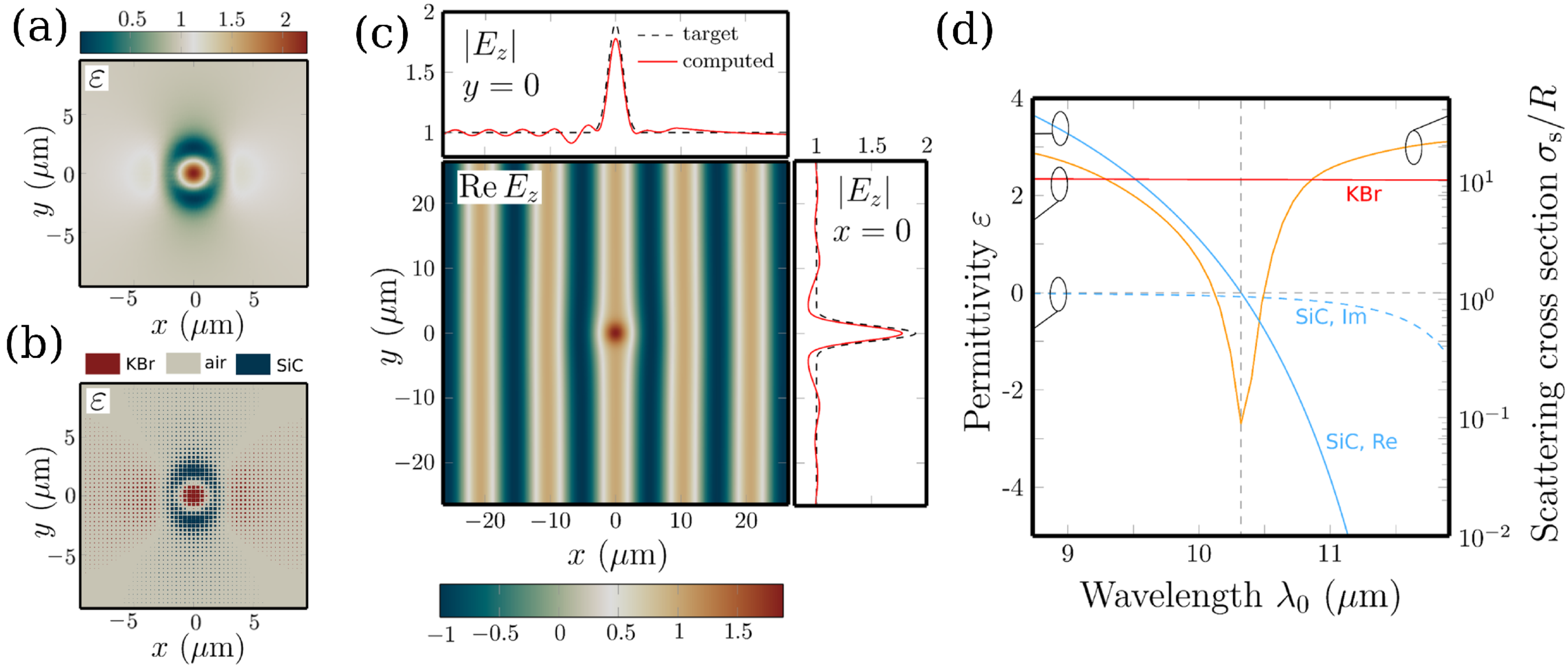
Appendix 2.2 cylindrical wave keeper:

— break the rotational symmetry





Appendix 2.3 metamaterial structure (credit: Ben Vial)



—B. Vial, Y. Liu, etc, paper1 submitting

Appendix 3. phase shaper: cylindrical into planar—smooth profile

$$\begin{cases} \nabla \cdot \left(A^2 \nabla \phi\right) = 0, \\ \epsilon = \frac{1}{k_0^2}\left(|\nabla \phi|^2 - \frac{\nabla^2 A}{A}\right). \end{cases} \Rightarrow \nabla A^2 \approx -\frac{\nabla^2 \phi}{|\nabla \phi|^2} \nabla \phi. \Rightarrow A = \frac{1}{S}$$

$$S_{\mathrm{i}} = \arctan[Y_0(k_0\rho), -J_0(k_0\rho)],$$

$$\phi(x,y) = \left(S_{\mathrm{i}}(x,y)[1 - sx(x)] + sx(x)\rho[(x+b)\cos\theta + y\sin\theta]\right), sx(x) := \frac{1 + \tanh(\beta x)}{2}, \qquad (29)$$

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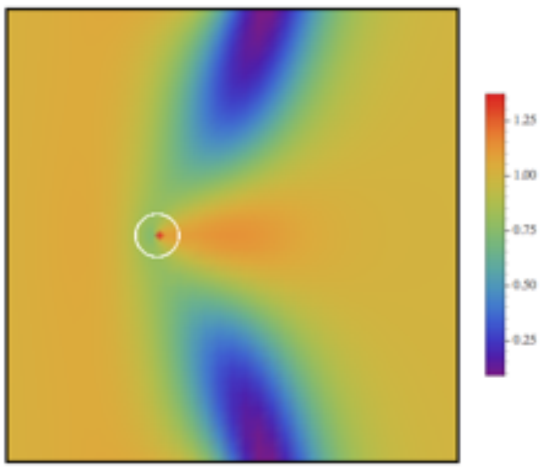
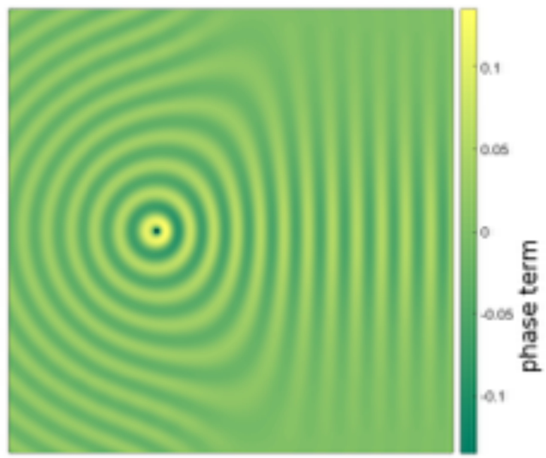


$F = Ae^{i\phi}$



a point source