A stochastic McKean–Vlasov equation arising in finance

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The financial motivation

- Credit risk is the risk of default on a payment by an obligor.
- Portfolio credit derivatives, such as CDOs, were constructed to repackage default risk of many obligors for sale to those with different risk appetites.
- A portfolio consists of $N \geq 1$ defaultable assets with random default times $\{\tau^i\}_{1 \leq i \leq N}$.
- Options on the proportional loss process are

$$L_t^N = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}_{\tau^i \leq t}, \qquad \text{payoff} = \Psi\Big((L^N)_{t \in [0,T]} \Big)$$

  For CDO tranches the payoff Ψ is piecewise linear.
- Correlations matter: defaults tend to cluster.

Model framework

- Want a model for generating τ^i
- Structural model: assign *distance-to-default* process, X^i
- When X^i hits zero, default is triggered:

$$\tau^i := \inf\{t > 0 : X_t^i \leq 0\}.$$

A Simple model

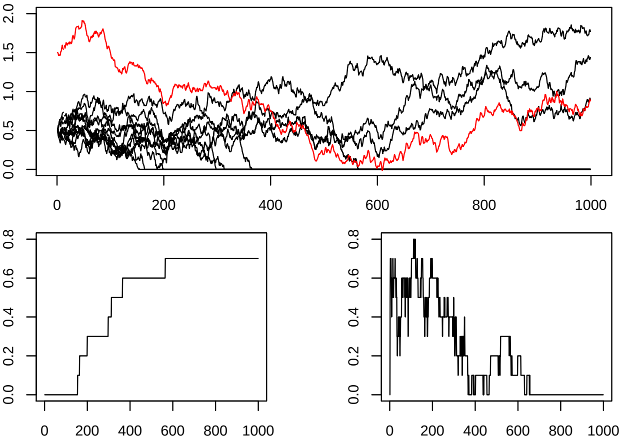
$$dX_t^i = \mu dt + \rho dW_t + \sqrt{1-\rho^2} dW_t^i$$
$$X_0^i \sim \nu_0$$

- Take a limit as $N \to \infty$,
- Study the empirical processes

$$\nu_t^N = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}_{t<\tau^i} \delta_{X_t^i} \in \mathcal{M}$$

- $L_t^N = 1 - \nu_t^N((0, \infty))$.

Basic model


0.0
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0.0
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1000

Limit $N \to \infty$

- $\nu_t^N(S) = \frac{1}{N}\sum_{i=1}^N \mathbf{1}_{X_t^i \in S; t<\tau^i} \to \mathbb{P}(X_t^1 \in S; t < \tau^1 | W)$,
- If we write $\nu_t(\phi) = \int \phi d\nu_t$

The SPDE in weak form

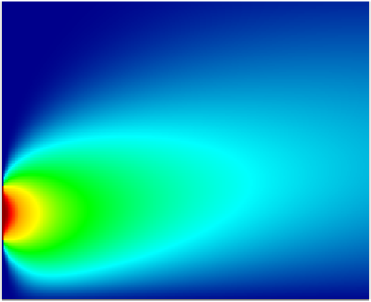
$$\begin{aligned} d\nu_t(\phi) &= \mu\nu_t(\partial_x\phi)dt + \frac{1}{2}\nu_t(\partial_{xx}\phi)dt + \rho\nu_t(\partial_x\phi)dW_t \\ \phi(0) &= 0. \end{aligned}$$

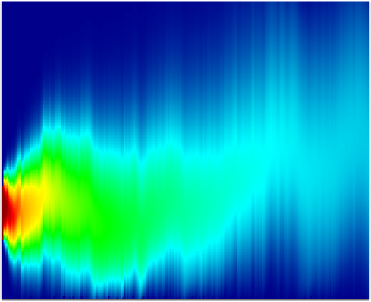
- If ν has a density V it will satisfy

The SPDE

$$\begin{aligned} dV_t &= -\mu\partial_x V_t dt + \frac{1}{2}\partial_{xx} V_t dt - \rho\partial_x V_t dW_t \\ V_t(0) &= 0. \end{aligned}$$

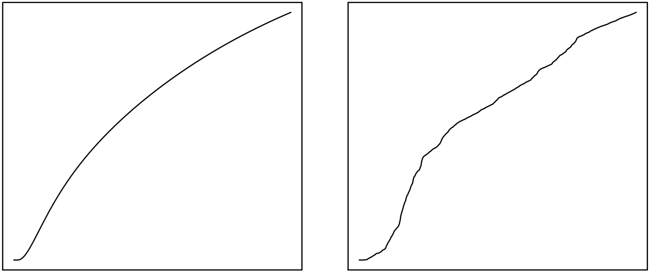
The heat map for the evolution started from a dirac mass when $\rho = 0$ and $\rho > 0$.





Limit $N \to \infty$

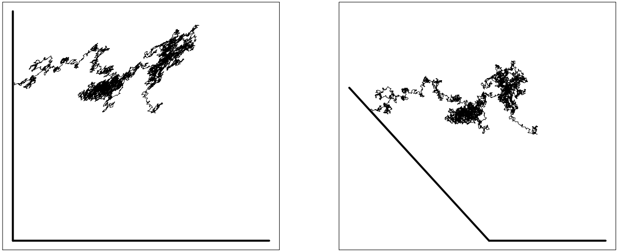
The loss function in the two cases



If ν_0 has an L^2 density, then ν_t has an H^1 density V but $xV_{xx} \in L^2$ (first observed by Krylov).

Analysis

- Convergence of the particle system to the L^2 density using kernel smoothing methods needs estimates on $\mathbb{E}[\nu_t(0,\varepsilon)^2]$.
- Need asymptotics for 2d Brownian motion near the apex of a cone.



- Ledger (2014) - the regularity of the SPDE at 0 is a function of ρ.

Regularity

Let $w_c(x) = x^c \exp(-x)$ for $x > 0$ be a weight function.
Let $\alpha = \pi/2 + \arcsin \rho$.

Theorem (Ledger)

If V_0 is bounded there exists a unique solution to the SPDE in the class of finite measure valued processes. For almost all $(\omega, t) \in \Omega \times [0, T]$, ν_t has a density V_t on $(0, \infty)$. Furthermore, suppose V_0 is n times weakly differentiable in $(0, \infty)$ and that for $k = 0, 1, \dots, n$ we have

$$\|w_{k-\beta/2} V_0\|_2 < \infty, \quad \forall \beta \in (-\infty, \pi/\alpha - 1).$$

Then, for almost all $(\omega, t) \in \Omega \times [0, T]$ we have V_t is $n+1$ times weakly differentiable and for $k = 0, 1, \dots, n+1$

$$\mathbf{E} \int_0^T \|w_{k-\beta/2} V_t\|_2^2 dt < \infty, \quad \forall \beta \in (-\infty, \pi/\alpha - 1).$$

Extensions

- Model very simple, lots of extensions:

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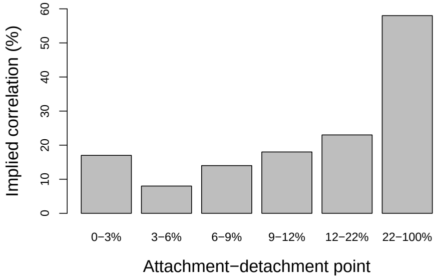
- More general coefficients,
- *Jump processes*, Bujok and Reisinger (2012)
- *Stochastic volatility*, H. and Kolliopoulos (2017)
- *Numerical problems* (MLMC): Giles and Reisinger (2012); Reisinger and Wang (2016)
- *Mortgage-backed securities model*, Ahmad, H and Ledger (2016)
- *CLT/Fluctuations*, Giesecke, Spiliopoulos, Sirignano (2014)
- Our interest will be incorporating loss-dependent correlation and contagion effects in such structural models.

Skew

- The model is too simple as we cannot choose one ρ to match all traded tranche spreads, there is *correlation skew* or *smile*,

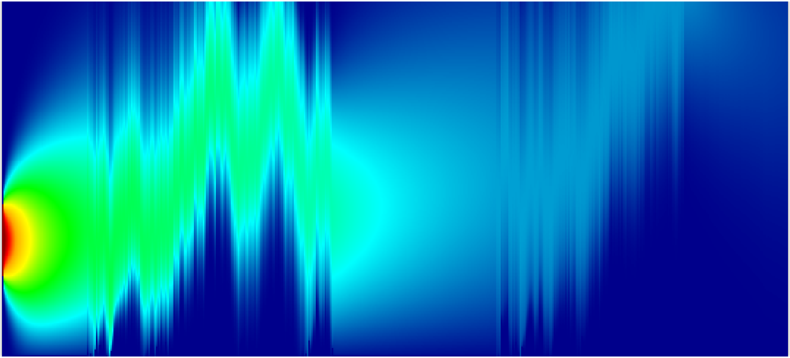


- A practioner approach is to make ρ a function of the loss in the system.

Loss-dependent model

$$dX_t^{i,N} = \rho(L_t^N)dW_t + \sqrt{1-\rho^2(L_t^N)}dW_t^i$$

Loss dependent example



Here we have an exaggerated loss dependent correlation

$$\rho(\ell) = \begin{cases} 0 & \text{if } \ell \in \left[0, \frac{1}{5}\right) \cup \left[\frac{2}{5}, \frac{3}{5}\right) \cup \left[\frac{4}{5}, 1\right] \\ \frac{9}{10} & \text{if } \ell \in \left[\frac{1}{5}, \frac{2}{5}\right) \cup \left[\frac{3}{5}, \frac{4}{5}\right). \end{cases}$$

Conditions

- We can consider general case

$$X_t^{i,N} = X_0^i + \int_0^t \mu(s, X_s^{i,N}, L_s^N)ds + \int_0^t \sigma(s, X_s^{i,N})\rho(s, L_s^N)dW_s + \int_0^t \sigma(s, X_s^{i,N})(1 - \rho(s, L_s^N)^2)^{\frac{1}{2}} dW_s^i. \quad (1)$$

.

- Piecewise constant ρ across tranches desirable.
- Allow finitely many discontinuities: piecewise Lipschitz ρ
- Need $0 \leq \rho(\ell) \leq \rho_{\max} < 1$, to prevent degeneracy
- Challenges: need to deal with boundary effects but correlation too complicated to do explicit calculations.
- For convergence, discontinuous ρ is bad. The key is to show limit points must have strictly increasing loss process.

Coefficient assumptions

Let $\mu : [0,T] \times \mathbb{R} \times [0,1] \to \mathbb{R}$, $\sigma : [0,T] \times \mathbb{R} \to [0,\infty)$ and $\rho : [0,T] \times [0,1] \to [0,1)$ be the coefficients in (1) and ν_0 be the common law of the initial values of the distance-to-default processes. We assume that we have a sufficiently large constant, $C \in (1,\infty)$, such that all the following hold:

(i) (Initial condition) The probability measure ν_0 is supported on $(0,\infty)$, has a density $V_0 \in L^2(0,\infty)$ and satisfies for every $\alpha > 0$,

$$\nu_0(\lambda,\infty) = o(\exp\{-\alpha\lambda\}), \qquad \text{as } \lambda \to +\infty.$$

(ii) (Spatial regularity) For all fixed $t \in [0,T]$ and $\ell \in [0,1]$, $\mu(t,\cdot,\ell), \sigma(t,\cdot) \in C^2(\mathbb{R})$ with

$$|\partial_x^n \mu(t,x,\ell)|, |\partial_x^n \sigma(t,x)| \leq C$$

for all $t \in [0,T]$, $x \in \mathbb{R}$, $\ell \in [0,1]$ and $n = 0,1,2$,

(iii) (Non-degenerate) For all $t \in [0, T]$, $x \in \mathbb{R}$, $\ell \in [0, 1]$

$$\sigma(t, x) \geq C^{-1} > 0, \qquad 0 \leq \rho(t, \ell) \leq 1 - C^{-1} < 1,$$

(iv) (Piecewise Lipschitz in loss) There exists $0 = \theta_0 < \theta_1 < \cdots < \theta_k = 1$ such that

$$|\mu(t, x, \ell) - \mu(t, x, \bar{\ell})|, |\rho(t, \ell) - \rho(t, \bar{\ell})| \leq C|\ell - \bar{\ell}|,$$

whenever $t \in [0, T]$, $x \in \mathbb{R}$ and both $\ell, \bar{\ell} \in [\theta_{i-1}, \theta_i)$ for some $i \in \{1, 2, \ldots, k\}$,

(v) (Integral constraint) $\sup_{s \in [0,T]} \int_0^\infty |\partial_t \sigma(s, y)| dy < \infty$.

Regularity conditions

Let ν be a càdlàg process taking values in the space of sub-probability measures on $\mathbb{R}$. The regularity condition on ν are

(i) (Loss function) The loss $L_t := 1 - \nu_t(0, \infty)$ is non-decreasing at all times and is strictly increasing when $L_t < 1$,

(ii) (Support) For every $t \in [0, T]$, ν_t is supported on $[0, \infty)$,

(iii) (Exponential tails) For every $\alpha > 0$

$$\mathbf{E} \int_0^T \nu_t(\lambda, +\infty) dt = o(e^{-\alpha\lambda}), \qquad \text{as } \lambda \to \infty,$$

(iv) (Boundary decay) There exists $\beta > 0$ such that

$$\mathbf{E} \int_0^T \nu_t(0, \varepsilon) dt = O(\varepsilon^{1+\beta}), \qquad \text{as } \varepsilon \to 0,$$

(v) (Spatial concentration) There exists $C > 0$ and $\delta > 0$ such that

$$\mathbf{E} \int_0^T |\nu_t(a, b)|^2 dt \leq C|b - a|^\delta, \qquad \text{for all } a < b.$$

Results

We state these for the simple case of $\mu = 0, \sigma = 1$.

Theorem (Tightness/Weak existence)

The sequence of triples $(\nu^N, L^N, W)_{N\geq 1}$ *are tight (with suitable topology). If* (ν^*, L^*, W) *realises a limiting law, then*

$$d\nu_t^*(\phi) = \frac{1}{2}\nu_t^*(\partial_{xx}\phi)dt + \rho(L_t)\nu_t^*(\partial_x\phi)dW_t$$

$$L_t^* = 1 - \nu_t^*(0, \infty),$$

[+ regularity conditions.] where $\phi \in C^{test} = \{f \in C^2 : f(0) = 0\}$.

Results

Theorem (Pathwise uniqueness/LLN)

Under the assumptions on regularity, for a given W, the SPDE has at most one solution ν in $(D_{\mathcal{S}'}, M1)$. The limit for the associated loss process L is unique in $(D_{\mathbb{R}}, M1)$. Hence there is a unique law of a solution (ν, L, W) and we have the sequence (ν^N, L^N, W) converges to (ν, L, W) as $N \to \infty$.

Corollary

With probability 1, for every $t \in [0, T]$, there exists $V_t \in L^2([0, \infty))$ such that

$$\nu_t(\phi) = \int_0^\infty \phi(x) V_t(x) dx, \qquad \phi \in L^2(0, \infty).$$

Results

The result can be expressed as a stochastic M–V problem.

M–V problem

For any independent B.M. $W^\perp$ there exists a process X satisfying

$$dX_t = \rho(L_t)dW_t + \sqrt{1-\rho(L_t)^2}\, dW_t^\perp$$
$$\tau := \inf\{t > 0 : X_t \leq 0\}$$
$$\nu_t(\phi) = \mathbb{E}[\phi(X_t)\mathbf{1}_{t<\tau}|W], \qquad L_t = \mathbb{P}(\tau \leq t|W).$$

The law of (X, W) is unique.

Methods

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- Need weaker methods
- *Existence*: Skorokhod M1 topology gives tightness because loss function is monotone
- Adapt the topology to space of distributions (Ledger 2016)
- Gives limit points solving the limiting equations

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- Need weaker methods
- *Existence*: Skorokhod M1 topology gives tightness because loss function is monotone
- Adapt the topology to space of distributions (Ledger 2016)
- Gives limit points solving the limiting equations
- *Uniqueness*: Work in a weaker Sobolev space, H^{-1}, so that only first moment estimates are needed. There is no need for correlations.
- Additional stopping and regularity arguments are needed for discontinuous coefficients.

Contagion model

- If a default occurs, each particle receives a kick of $\frac{\alpha}{N}$ towards the boundary, $\alpha > 0$ interesting case — positive feedback
- We drop the common noise term for simplicity

Discrete model

$$X_t^i = X_0^i + B_t^i - \alpha L_t^N, \qquad L_t^N = \frac{1}{N} \sum_{i=1}^N \mathbf{1}_{\tau^i \le t}$$

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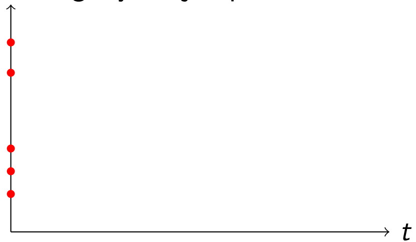

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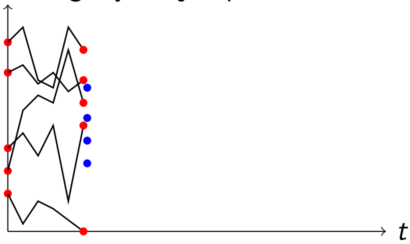

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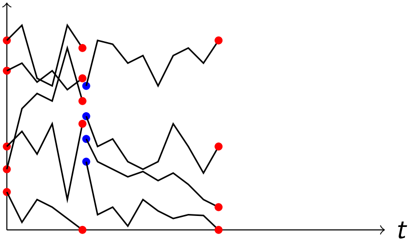

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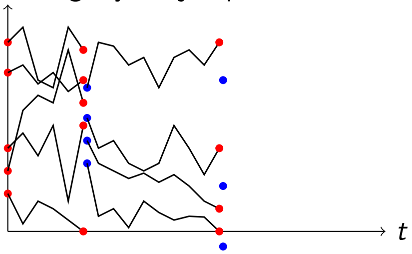

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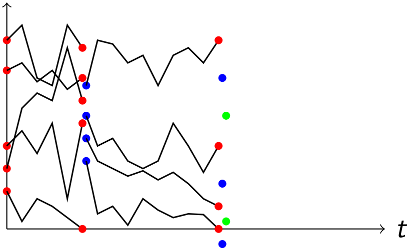

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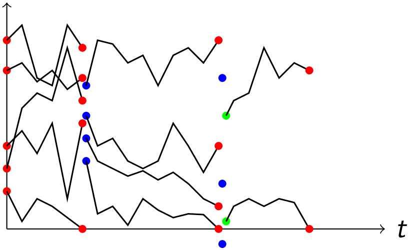

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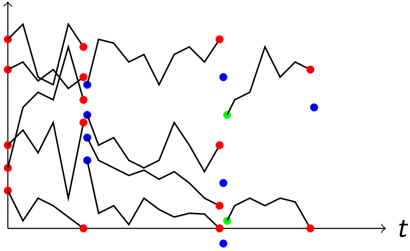

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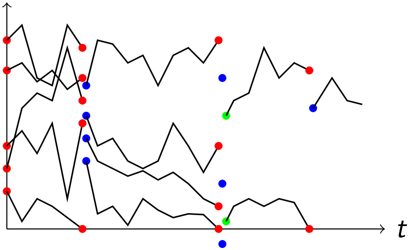

Limiting equations

McKean–Vlasov problem (MV)

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$$\tau = \inf\{t > 0 : X_t \leq 0\}$$
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PDE problem

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Fixed-point problem

Let $\Gamma : L. \to P(\tau^L < .)$ be the map taking the input loss function to its output. The fixed point satisfies

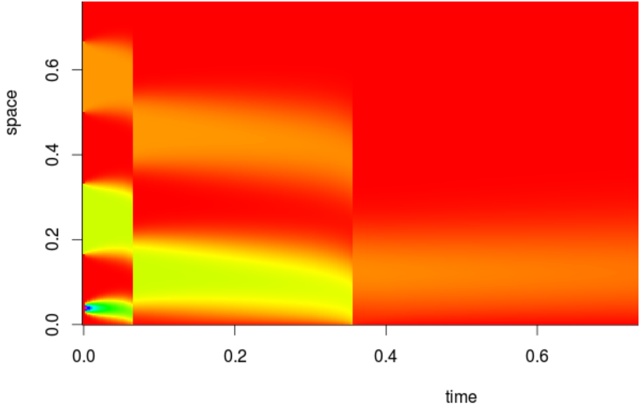
$$\int_0^\infty \Phi\Big(-\frac{x - \alpha\ell_t}{t^{1/2}}\Big)\nu_0(dx) = \int_0^t \Phi\Big(\alpha\frac{\ell_t - \ell_s}{(t-s)^{1/2}}\Big)d\Gamma[\ell]_t.$$

Positive results

- Model in neuroscience: Delarue, Inglis, Rubenthaler, Tanré, 2015
- Essential difficulties are the same
- Show unique C^1 solution for small enough α, $\nu_0 = \delta_x$ $x > 0$
- In another paper, Delarue, Inglis, Rubenthaler, Tanré, 2015, also give existence for all α, as limit points of particle system, with *physical jump condition*
- Initial ν_0 zero near zero
- Related financial model: Nadtochiy, Shkolnikov, 2017. Uniqueness up to a blow-up where L^2 norm of derivative blows-up, ν_0 has H^1 density V_0 with $V_0(0) = 0$, so $V_0(x) = O(x^{1/2})$

Blow-ups

- If α is large enough, no solution can be continuous for all times, Cáceres, Carrillo, Perthame (2011)
- Jump in loss must occur



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- *Claim*: If $\alpha > 2m_0$ where $\nu_0 = \delta_{m_0}$, then L cannot be continuous for all time.
- *Proof*:

$$0 \leq X_{t\wedge\tau} = X_0 + B_{t\wedge\tau} - \alpha L_{t\wedge\tau}$$

- Take expectation

$$m_0 \geq \alpha\mathbb{E}[L_{t\wedge\tau}].$$

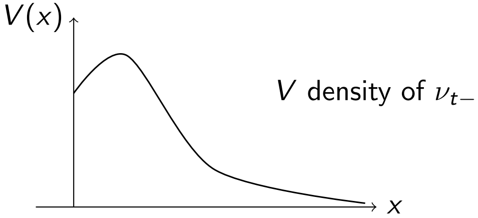
- By comparison with B.M. $\tau < \infty$ a.s. $L_\infty = 1$

$$m_0 \geq \alpha\mathbb{E}[L_\tau] = \alpha\int_0^\infty L_s dL_s = \frac{\alpha}{2}(L_\infty^2 - L_0^2) = \frac{\alpha}{2}.$$

□

What is happening at jumps?

- Before jump ν_{t-}
- If jump in loss is ΔL_t, then push-down by $-\alpha \Delta L_t$



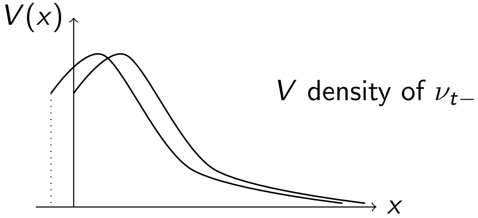
- Mass lost must equal ΔL_t, so

$$\nu_{t-}(0, \alpha \Delta L_t) = \Delta L_t.$$

- Choose smallest jump allowing càdlàg solution
- $\Delta L_t = \inf\{x > 0 : \nu_{t-}(0, \alpha x) < x\}$

What is happening at jumps?

- Before jump ν_{t-}
- If jump in loss is ΔL_t, then push-down by $-\alpha \Delta L_t$



- Mass lost must equal ΔL_t, so

$$\nu_{t-}(0, \alpha \Delta L_t) = \Delta L_t.$$

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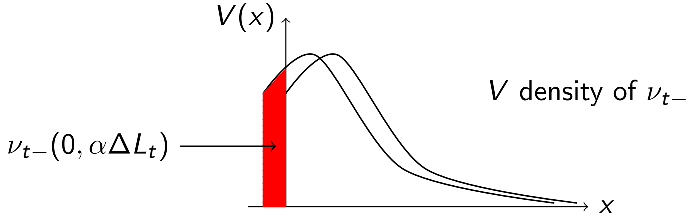

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Global uniqueness?

Conjecture

There exists a unique solution to (MV) satisfying the natural-jump/minimal-jump condition. Jumps according to rule, between jumps C^1 with $\sqrt{t}$ singularities.

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- Obstruction: after a jump the solution is $\asymp 1$ near the boundary
- $\Rightarrow L_t$ grows at least as fast as $\sqrt{t}$ near $t = 0$
- $L'_t \asymp t^{-1/2}$, Girsanov tricks just fail in this case

Main problem

Show small time uniqueness for (MV) started from initial law ν_0 satisfying only $\inf\{\nu_0(0, \alpha x) < x\} = 0$.

- Cannot yet attack problem started from density V_0 with $V_0(x) \geq \delta > 0$ near zero, for δ as small as you like.

Currrent work

- With S. Ledger, A. Søjmark, we can start with density $O(x^\beta)$, for $\beta > 0$, and we can add in the coefficients
- $O(x^\beta)$ implies $L'_t = O(t^{-\frac{1-\beta}{2}})$
- Uniqueness in small time for $\beta > 0$, uniqueness in small α for $\beta = 0$, but don't know solution lives there
- Would like to add a common noise term

$$X_t = X_0 + B_t + \beta(t) - \alpha L_t$$

  with β a Brownian sample path, for example.
- For any fixed α, β can be bad enough to cause a blow-up.
- Methods relying on differentiability of the loss function are really broken!
- H, Søjmark: In the case where we mollify L, we can add loss-dependent coefficients and common noise

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