Singular perturbations for port-Hamiltonian systems, normal hyperbolicity and non-hyperbolicity

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My MOR background

A control systems perspective:

- Long term: Theory for nonlinear balancing, realization theory, and nonlinear balanced truncation, around equilibrium points, around trajectories, related to stability and incremental stability, etc. Computationally not useful for very high order systems yet, need for numerical collaborators.
- Some balancing of linear systems with structure preservation.
- Recent: Reduction of linear networks, clustering based, structure preserving (first order and second order networks) and balancing, work on poster with Xiaodong Cheng.
- Recent: singularly perturbed systems, topic of today.

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Regular perturbation

Consider the algebraic problem

$$x^2 + \varepsilon x - 1 = 0, \qquad 0 < \varepsilon \ll 1.$$

It has solutions

$$x_{1,2} = \frac{-\varepsilon \pm \sqrt{4 + \varepsilon^2}}{2} = \pm 1 + O(\varepsilon)$$

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"The solutions of the limit equation are
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Singular perturbation

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It has solutions

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"The solutions of the limit equation are
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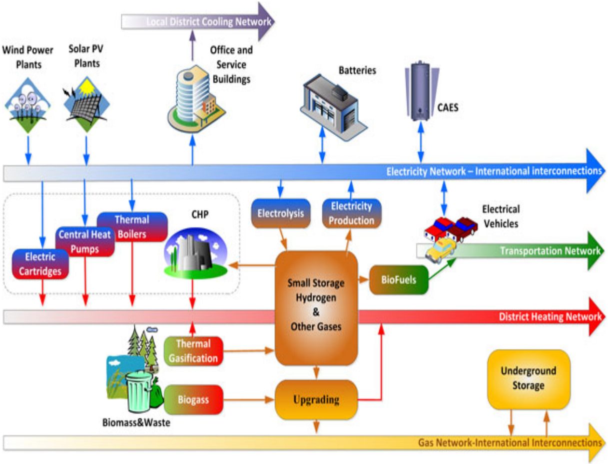
Motivation

Relevant considerations for differential equations with various time-scales. In many applications, e.g., energy grids:



Singularly perturbed ODEs (a.k.a. Slow-Fast Systems)

slow

$$\dot{x} = f(x, z, \varepsilon)$$
$$\varepsilon \dot{z} = g(x, z, \varepsilon)$$

$$\tau = \frac{t}{\varepsilon} \qquad 0 < \varepsilon \ll 1$$

$$x' = \varepsilon f(x, z, \varepsilon)$$
$$z' = g(x, z, \varepsilon)$$

fast

$\varepsilon \to 0$ $\qquad$ $\varepsilon \to 0$

DAE

$$\dot{x} = f(x, z, 0)$$
$$0 = g(x, z, 0)$$

$$x' = 0$$
$$z' = g(x, z, 0)$$

layer

$\mathcal{S}$

critical manifold

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Normal hyperbolicity

Definition (Critical manifold)

$$\mathcal{S} = \{(x,z) \in \mathcal{X} \times \mathcal{Z} \,|\, g(x,z,0) = 0\}$$

$\mathcal{S}$ is said to be *Normally Hyperbolic* if spec $\left\{ \frac{\partial g}{\partial z}(x,z,0) \right\}$ has nonzero real part.

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Recall the reduced systems

$$\dot{x} = f(x,z,0) \qquad\qquad x' = 0$$
$$0 = g(x,z,0) \qquad\qquad z' = g(x,z,0)$$

$\rightarrow$ The manifold $\mathcal{S}$ is the phase-space of the DAE and the set of equilibrium points of the layer equation.

$\mathcal{S}$ is NH if each point $(x,z) \in \mathcal{S}$ is a hyperbolic equilibrium point of the layer equation.

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If $\mathcal{S}$ is NH, then $\exists\, h_0(x)$ such that locally[1]

$$\mathcal{S} = \{(x,z) \in \mathcal{X} \times \mathcal{Z} \,|\, z = h_0(x)\}$$

[1] Implicit Function Theorem

Normal hyperbolicity

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$$\mathcal{S} = \{(x, z) \in \mathcal{X} \times \mathcal{Z} \,|\, z = h_0(x)\}$$

Then, the flow along $\mathcal{S}$ is given by the reduced *slow* system

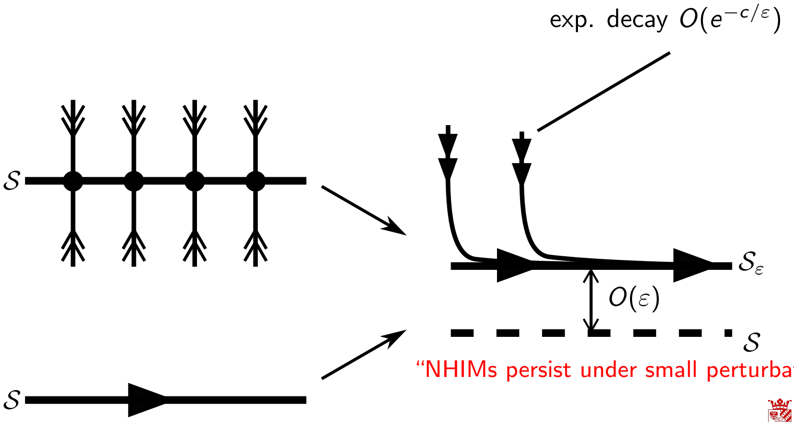
$$\dot{x} = f(x, h_0(x), 0)$$

[1] Implicit Function Theorem

Geometric Singular Perturbation Theory *N. Fenichel, 1979*

Let $\bar{\mathcal{S}}$ be NH and $\mathcal{S} \subseteq \bar{\mathcal{S}}$ be compact. Then, for $\varepsilon > 0$ sufficiently small

- $\exists$ an invariant manifold $\mathcal{S}_\varepsilon$ diffeomorphic to $\mathcal{S}$
- The flow along $\mathcal{S}_\varepsilon$ is ε-close to the flow along $\mathcal{S}$



"NHIMs persist under small perturbations"

Relies on Tikhonov's theorem (1935).

$$\dot{x} = f(x, z, \varepsilon, u) \qquad x' = \varepsilon f(x, z, \varepsilon, u)$$
$$\varepsilon \dot{z} = g(x, z, \varepsilon, u) \qquad z' = g(x, z, \varepsilon, u)$$

$$\dot{x} = f(x, z, 0, u) \qquad x' = 0$$
$$0 = g(x, z, 0, u) \qquad z' = g(x, z, 0, u)$$

Let $\mathcal{S}$ be NH and $u = u_s(x) + u_f(x, z), \quad u_f(x, z)|_{\mathcal{S}} = 0$

$$\dot{x} = f_r(x, z, u_s) \qquad z' = \bar{g}_x(z, u_f)$$

Stabilize reduced subsystems and combine for overall control.

Application to control - Composite control *Kokotović et al.*

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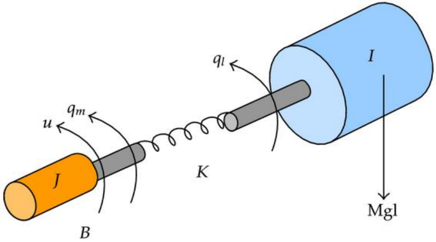
Stabilize reduced subsystems and combine for overall control.

However, so far model order reduction and composite control only hold around hyperbolic points. Furthermore, properties as passivity not always preserved.

Motivation

Flexible-joint robots are a standard example of two time scale mechanical systems



Goal: to follow a desired trajectory with only position measurements
Assumption: $|K|$ is large

Flexible-joint manipulator

Joint flexibility can be attributed to:

- Harmonic drives
- Transmission belts
- Long shafts
- Robotic hands
- Variable stiffness drives for safety/interaction purposes
- ⋮

Some preliminary remarks:

- Flexible-joint robots have been studied for many years
- Port-Hamiltonian systems + singular perturbations have a wide range of applicability

Port-Hamiltonian systems *Maschke, van der Schaft, 1992*

General description in x coordinates on some n dimensional manifold:

$$\begin{aligned}\dot{x} &= (J(x) - R(x))\frac{\partial H}{\partial x}(x) + g(x)u \\ y &= g^T(x)\frac{\partial H}{\partial x}(x)\end{aligned}$$

where
$J(x) = -J^T(x)$: interconnection structure (related to Dirac structures)
$R(x) = R^T(x) \geq 0)$: damping
$H(x) > 0$: is the Hamiltonian (total energy).

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Nice property: $\boxed{\dot{H} = -\frac{\partial^T H}{\partial x}(x)R(x)\frac{\partial H}{\partial x}(x) + y^T u \leq y^T u}$

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Passivity! Very useful for Passivity Based Control, control based on the port-Hamiltonian structure (e.g., energy shaping and damping injection).

Standard mechanical systems in the PH framework

Generalized coordinates q, generalized momenta p. Hamiltonian:

$$H(q,p) = \frac{1}{2}p^T M^{-1}(q)p + V(q)$$

$V(q) > 0$ potential energy, $M(q) = M^T(q) > 0$ mass inertia matrix.

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Model without damping:

$$\begin{aligned} \dot{q} &= \frac{\partial H}{\partial p}(q, p) \\ \dot{p} &= -\frac{\partial H}{\partial q}(q, p) + B(x)u \\ y &= B^T(x) \frac{\partial H}{\partial p}(q, p) \end{aligned}$$

Input is a generalized force, output is a generalized velocity, $u^T y$ is the supplied power.

$q_1 \in \mathbb{R}^n$ links' coordinate,

- Link's kinetic energy:

$$K_l(q_1, \dot{q}_1) = \frac{1}{2}\dot{q}_1^T M_l(q_1)\dot{q}_1$$

- Motor's kinetic energy:

$$K_m(\dot{q}_2) = \frac{1}{2}\dot{q}_2^T I \dot{q}_2$$

$q_2 \in \mathbb{R}^n$ motors' coordinate

- Potential energy due to gravity

$$P_g(q_1) = \sum_{i=1}^{n} (P_{g,l_i}(q_1) + P_{g,m_i}(q_1))$$

- Potential energy due to joint stiffness

$$P_s(q_1, q_2) = \frac{1}{2}(q_1 - q_2)^T K(q_1 - q_2),$$

where $K \in O(1/\varepsilon)$.

Total energy

$$H = \frac{1}{2}\dot{q}_1^T M_l(q_1)\dot{q}_1 + \frac{1}{2}\dot{q}_2^T I \dot{q}_2 + P_g(q_1) + \frac{1}{2\varepsilon}(q_1 - q_2)^T(q_1 - q_2)$$

Let

$$\varepsilon z = q_1 - q_2.$$

Then

$$\bar{H} = \boxed{\frac{1}{2}\dot{q}_1^T(M_l(q_1) + I)\dot{q}_1 + P_g(q_1)} + \varepsilon\left(-\dot{q}_1^T I \dot{z} + \frac{1}{2}\varepsilon \dot{z}^T I \dot{z} + \frac{1}{2}z^T z\right)$$

Rigid robot

Port-Hamiltonian model of a flexible-joint robot 3/3

Let $q = (q_1, z)$, $\bar{H}$ can be written as

$$\bar{H} = \frac{1}{2} p^T M_\varepsilon{}^{-1}(q) p + V_\varepsilon(q),$$

where

$$M_\varepsilon = \begin{bmatrix} M_l(q_1) + I & -\varepsilon I \\ -\varepsilon I & \varepsilon^2 I \end{bmatrix}, \qquad p = M_\varepsilon \dot{q}, \qquad V_\varepsilon(q) = P_g(q_1) + \frac{1}{2} \varepsilon z^T z$$

Port-Hamiltonian model of a flexible-joint robot 3/3

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Major obstruction for *good* model.

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What is good model? Consider

$$\begin{bmatrix} \dot{x}_1 \\ \varepsilon \dot{x}_2 \end{bmatrix} = J(x, \varepsilon) \frac{\partial H}{\partial x} + G(x, \varepsilon) u \qquad \begin{bmatrix} \dot{x}_1 \\ 0 \end{bmatrix} = \begin{bmatrix} \bar{J}_{11} & \bar{J}_{12} \\ \bar{J}_{21} & \bar{J}_{22} \end{bmatrix} \begin{bmatrix} \frac{\partial \bar{H}}{\partial x_1} \\ \frac{\partial \bar{H}}{\partial x_2} \end{bmatrix} + \begin{bmatrix} \bar{G}_1 \\ \bar{G}_2 \end{bmatrix} u$$

NH implies $x_2 = h_0(x_1, u)$. Then, reduced system is not necessarily in port-Hamiltonian format.

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Solution: use canonical change of coordinates[2][3] to obtain

$$\bar{H}_\varepsilon(\bar{q}, \bar{p}) = \frac{1}{2}\bar{p}^\top \bar{p} + \bar{V}_\varepsilon(\bar{q})$$

[3] *Fujimoto, K. and Sugie, T. (2001).*

[3] *Viola, G., Ortega, R., Banavar, R., Acosta, J.A., and Astolfi, A. (2007).*

Reduced models

Reduced slow (rigid):

$$\begin{bmatrix} \dot{\bar{q}}_1 \\ \dot{\bar{p}}_1 \end{bmatrix} = \begin{bmatrix} 0 & t_1^{-T} \\ -t_1^{-1} & j_1 \end{bmatrix} \begin{bmatrix} \frac{\partial H_0}{\partial \bar{q}_1} \\ \frac{\partial H_0}{\partial \bar{p}_1} \end{bmatrix} + \begin{bmatrix} 0_{n\times n} \\ g_1(\bar{q}_1, \bar{p}_1) \end{bmatrix} u_s$$

Reduced fast:

$$\begin{bmatrix} \bar{q}_2' \\ \bar{p}_2' \end{bmatrix} = \begin{bmatrix} 0 & t_4^{-T} \\ -t_4^{-1} & j_{32} \end{bmatrix} \begin{bmatrix} \frac{\partial \bar{H}}{\partial \bar{q}_2} \\ \frac{\partial \bar{H}}{\partial \bar{p}_2} \end{bmatrix} + \begin{bmatrix} 0_{n\times n} \\ g_2(\alpha, \bar{q}_2, \bar{p}_2) \end{bmatrix} u_f$$

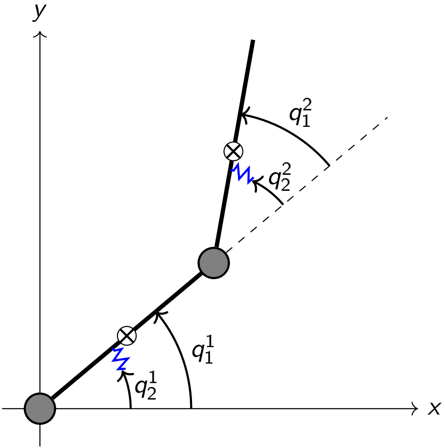
with

$$t_i = t_i(\bar{q}_k), \quad j_\bullet = j_\bullet(\bar{q}_k, \bar{p}_k), \quad k = 1, 2.$$

Both reduced systems are port-Hamiltonian

Simulation

Control of a 2DOF flexible joint robot with only position measurements



Goal: To make both links follow the desired trajectory

$$q_d = 0.1 + 0.05\sin(t)$$

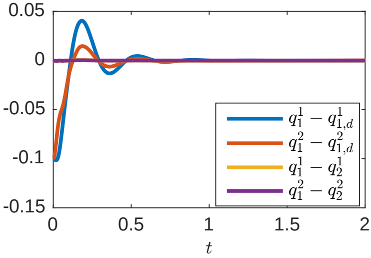
Composite control of the flexible model[4]

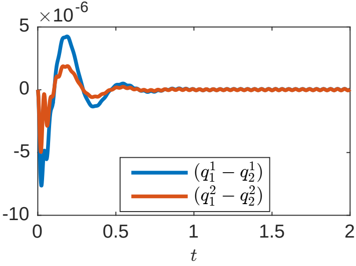
$$u = u_s + u_f,$$

where u_s is given by the existing control, and u_f stabilizes the fast subsystem with reference

$$z_d = \frac{1}{\varepsilon}(q_{1,d} - q_{2,d}) = (0,0).$$

$$u_f = -L_p z - L_c(z - z_c)$$
$$\dot{z}_c = L_d^{-1} L_c(z - z_c)$$





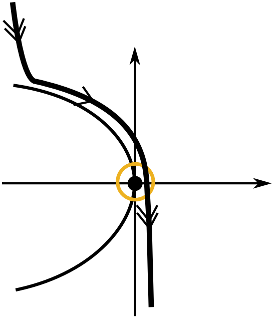
[4] *Jardón-Kojakhmetov, Munoz-Arias, Scherpen, 2016.*

Non-hyperbolic points

Examples

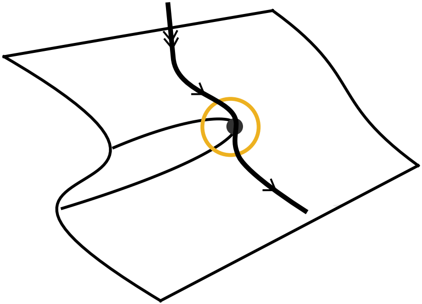
The fold

$$\dot{x} = 1$$
$$\varepsilon \dot{z} = -(z^2 + x)$$



The cusp

$$\dot{x}_1 = 1$$
$$\dot{x}_2 = 0$$
$$\varepsilon \dot{z} = -(z^3 + x_2 z + x_1)$$



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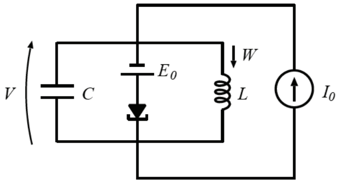
Why are non-hyperbolic points interesting?

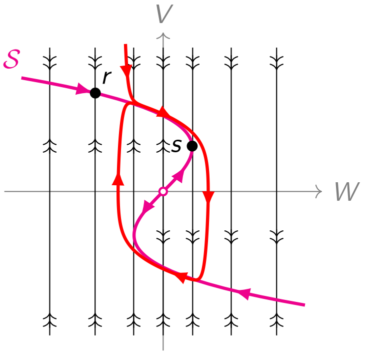
- They are responsible for relaxation oscillations
- They are responsible for hidden effects (canards)
- They model complicated phenomena (mixed-mode oscillations, canards explosion)
- They appear in many mathematical models of
  - Electric circuits (van der Pol oscillator)
  - Biology (cell division, heartbeat)
  - Chemistry (biochemical reactions)
  - Neuroscience (nerve impulse)
  - Classical mechanics

  ⋮

van der Pol oscillator



Source: `http://www.scholarpedia.org/article/Van_der_Pol_oscillator`



$r \equiv$ hyperbolic point $\rightsquigarrow$ "well understood"

$s \equiv$ non-hyperbolic point $\rightsquigarrow$ "?"

Goal: to stabilize a non-hyperbolic point

Geometric Desingularization

- Has its origins in algebraic geometry.

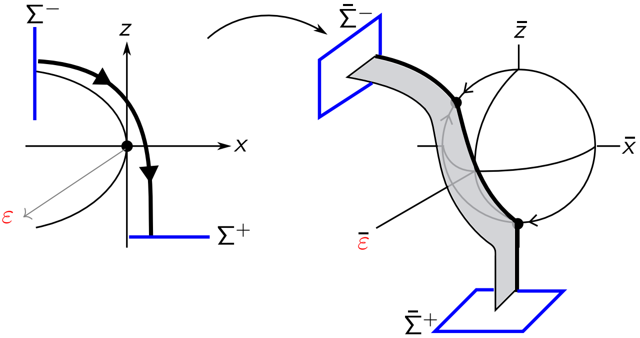


Figure: Schematic picture of a blow up of a fold point

- The blown up vector field is regular, hyperbolic
- The blown up vector field is equivalent to the original one

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Stabilization of a folded point *Jardon-Kojakhmetov, Scherpen, 2017*

$$
\begin{aligned}
x' &= \varepsilon(Ax + Bz + u) \\
z' &= -(z^2 + x) \\
\varepsilon' &= 0
\end{aligned}
\quad \longrightarrow \quad
\begin{aligned}
x &= r^2\bar{x} \\
z &= r\bar{z} \\
\varepsilon &= r^3
\end{aligned}
$$

Stabilization of a folded point *Jardon-Kojakhmetov, Scherpen, 2017*

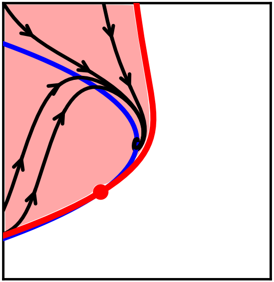
Design controller here!

$$\begin{aligned} x' &= \varepsilon(Ax + Bz + u) \\ z' &= -(z^2 + x) \\ \varepsilon' &= 0 \end{aligned} \quad \longrightarrow \quad \begin{aligned} x &= r^2\bar{x} \\ z &= r\bar{z} \\ \varepsilon &= r^3 \end{aligned} \quad \longrightarrow \quad \boxed{\begin{aligned} \bar{x}' &= Ar^2\bar{x} + Br\bar{z} + \bar{u} \\ \bar{z}' &= -(\bar{z}^2 + \bar{x}) \\ r' &= 0 \end{aligned}}$$

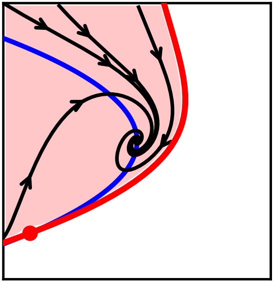
Stabilization of a folded point *Jardon-Kojakhmetov, Scherpen, 2017*

Design controller here!

$$\begin{aligned} x' &= \varepsilon(Ax + Bz + u) \\ z' &= -(z^2 + x) \\ \varepsilon' &= 0 \end{aligned} \quad \longrightarrow \quad \begin{aligned} x &= r^2 \bar{x} \\ z &= r\bar{z} \\ \varepsilon &= r^3 \end{aligned} \quad \longrightarrow \quad \boxed{\begin{aligned} \bar{x}' &= Ar^2\bar{x} + Br\bar{z} + \bar{u} \\ \bar{z}' &= -(\bar{z}^2 + \bar{x}) \\ r' &= 0 \end{aligned}}$$



$\longleftarrow$



closed-loop slow-fast system

$u = -Ax - Bz + \alpha\varepsilon^{-2/3}x + \beta\varepsilon^{-1/3}z$

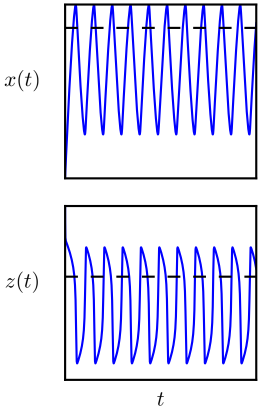
closed-loop blown up v.f.

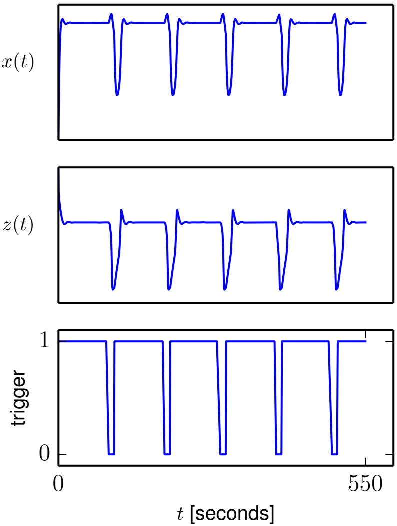
$\bar{u} = -Ar^2\bar{x} - Br\bar{z} + \alpha\bar{x} + \beta\bar{z}$

Application: Trigger control of the van der Pol oscillator

Jardón-Kojakhmetov, Scherpen 2016.

$$x' = \varepsilon(z+u), \qquad u = -z + O(\varepsilon^{-1/3})$$
$$z' = -z^3 + z - x$$





Adaptive stabilization of a non-hyperbolic point

Blow up + backstepping → injection of hyperbolicity, *Jardon-Kojakhmetov, del Puerto Flores, Scherpen, 2017*

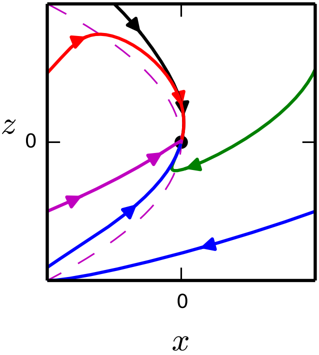
Consider the SFS

$$x' = \varepsilon(A_0 + Ax + Bz + u(x, z, \varepsilon))$$
$$z' = -(z^2 + x),$$

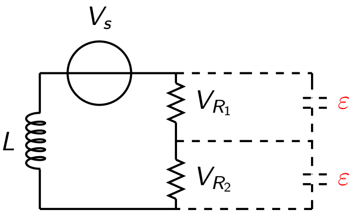
where A_0, A, B are *unknown*, together with the control

$$u = \frac{1}{\varepsilon}(-\hat{a}_0 + O(\varepsilon^{1/3}, z))$$
$$\hat{a}_0' = O(\varepsilon^{-2/3})$$

Then the origin is a locally a.s. equilibrium point.

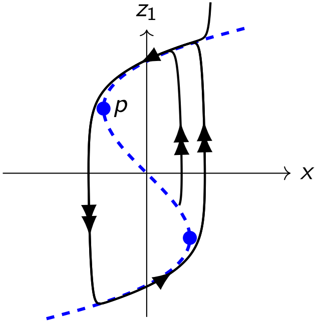


Adaptive control of an electrical circuit

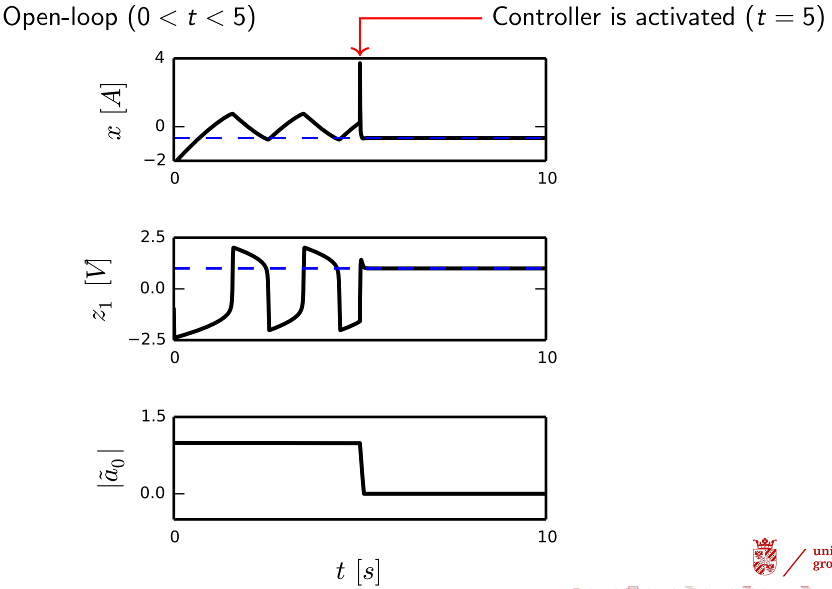


$$\dot{x} = -\alpha_1 z_1 - \alpha_2 z_2 + u$$
$$\varepsilon \dot{z}_1 = -f_1(z_1) + x$$
$$\varepsilon \dot{z}_2 = -f_2(z_2) + x,$$

R_1	–	non linear
R_2	–	linear
x	–	current through L
z_i	–	voltage at R_i



Adaptive control of an electrical circuit



Conclusions

- Starting from a slow fast PH system, we can rewrite it such that the slow and fast subsystems are both port-Hamiltonian.
- Model order reduction can be used to design a controller for a flexible-joint robot from a rigid one.
- We have presented a novel approach to stabilize non-hyperbolic points of slow-fast systems.
- The blow up technique allows us to desingularize a fold point and study the dynamics nearby.
- The "geometric desingularization" technique has been introduced into the control systems context.
- Geometric desingularization + well-known control strategies can be used to stabilize non-hyperbolic points of slow-fast systems.

Future research

For ODE systems

- Consideration of general "slow-fast PHSs".
- Influence of ε on the transient performance.
- Regularization of Differential Algebraic port-Hamiltonian systems.
- Path following and trajectory tracking along non-hyperbolic sets.
- More than 2 time scales.
- Etc.

For PDE systems (relevant for e.g. fast reaction- slow diffusion systems)

- Extension of Tikhonov's theorem.
- Normally hyperbolic and non-hyperbolic extensions?
- Well-posedness issues.
- ....
- Etc.

Groningen Autumn School on MOR (in COST action)

Main invited speakers

- Serkan Gugercin (Virginia Tech)
- Paolo Rapisarda (University of Southampton)

with in addition some local speakers.

Topics: Model reduction for design and optimization, data-based model reduction, and model reduction of networks.

30 October - 2 November 2017
University of Groningen, the Netherlands

http://www.math.rug.nl/gcsc/morschool.html

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