Developments in 6d SCFTs

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1. String theory constructions

- Discovery
- Constructions from string, M-, F- theories

2. Conventional QFT approaches

- Tensor branch & self-dual strings
- 5d Yang-Mills at weak/strong couplings
- 4d Yang-Mills & S-duality

3. 6d strings, S-duality & “non-Abelian” physics

- 2d QFT approaches
- Elliptic genera, modularity, etc.
- Relations to S-duality & non-Abelian degrees of freedom

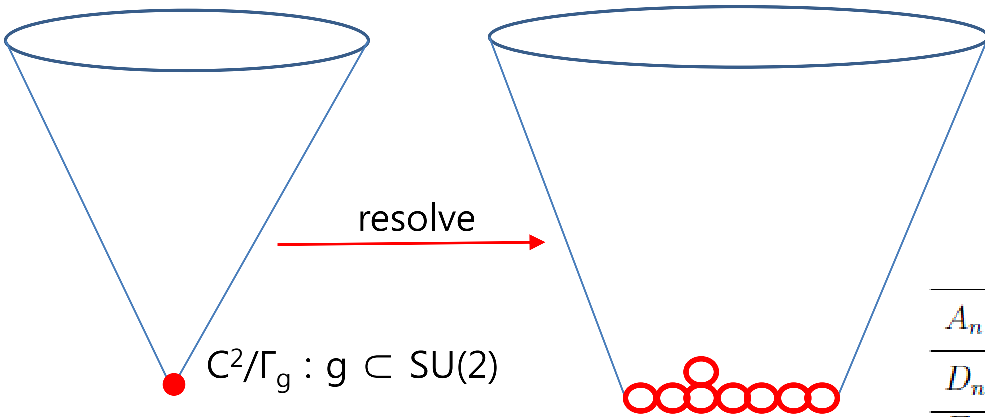
4. Summary and some future directions

I’ll focus on several key themes, related to one another:

self-dual strings, self-dual tensors, compactifications, S-duality, …

Discovery of 6d QFTs

- 2nd string revolution had a corollary: new interacting 6d QFTs "discovered"
- type IIB on $R^{5+1} \times C^2/\Gamma_g$ singularity, w/ g=ADE: [Witten] (1995)



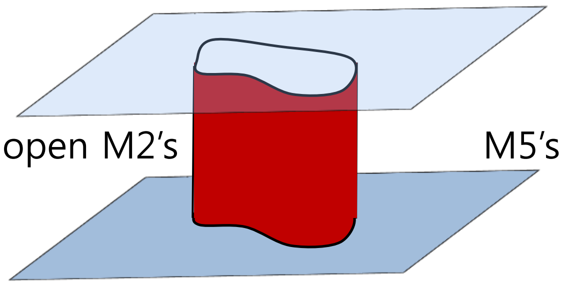
A_n	$(z_1, z_2) \to \left(e^{\frac{2\pi i}{n+1}} z_1, e^{-\frac{2\pi i}{n+1}} z_2\right)$
D_n	$(z_1, z_2) \to \left(e^{\frac{\pi i}{n-2}} z_1, e^{-\frac{\pi i}{n-2}} z_2\right)$, $(z_1, z_2) \to (z_2, -z_1)$
$E_6,\ E_7,\ E_8$	more involved...

- 6d system at singularity, decoupled from gravity at low E.
- D3's wrapped on 2-cycles ~ strings: tension ~ volume of 2-cycles (~ VEV of a 6d field)
- 6d QFT has strings "charged" with 2-form tensor fields.
- Like W-bosons charged with 1-form gauge fields. (mass ~ VEV)
- Singular limit: 6d maximal superconformal theory. $N = (2,0)$ SUSY.
- Should (abstractly) view this as a local QFT [Seiberg]
- Even without fully known quantum description (e.g. Lagrangian).

6d N=(2,0) theories

- Obeys an ADE classification

- A_{N-1} type may also be viewed as N M5s' worldvolume theory
- Strings given by open M2's. [Strominger] [Howe, Lambert, West]
- In a sense, QFT for these strings, not for particles…

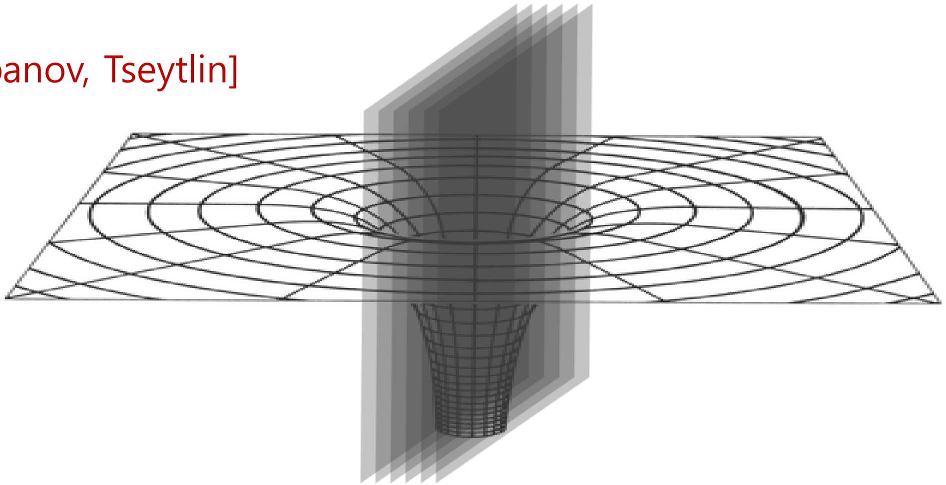


- Number of degrees of freedom scales like $\sim N^3$: much larger than Yang-Mills on D-branes

N≫1 M5's of M-theory make AdS_7:

black 5-branes at temperature T [Klebanov, Tseytlin]

$$\frac{S_{BH}}{(\text{volume})_5} \sim N^3 T^5$$



- D_N type may also be viewed as N M5s' probing $R^{5+1} \times R^5/Z_2$ orbifold of M-theory
- E_N type realized only as type IIB on $R^{5+1} \times C^2/\Gamma_{E_N}$

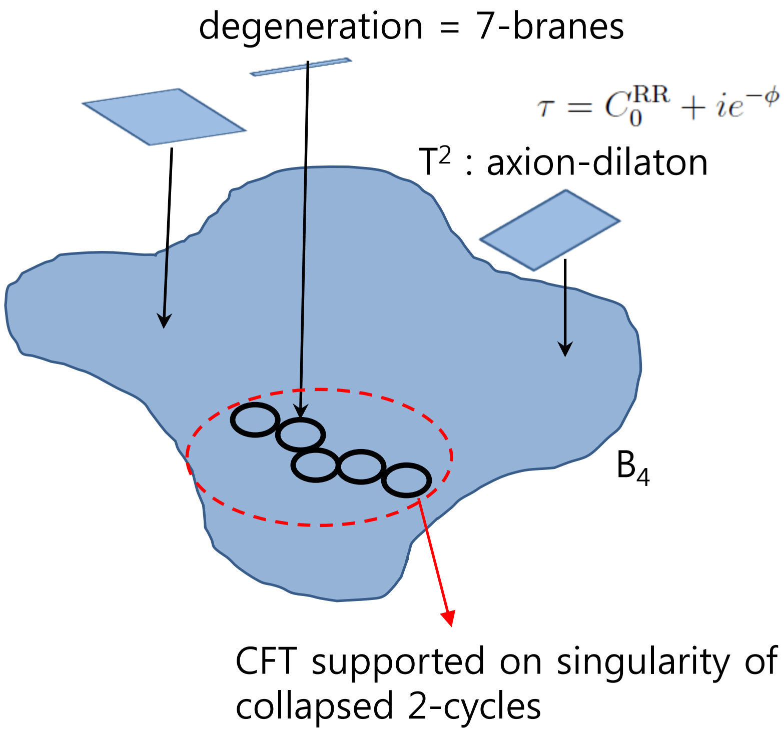
6d $N = (1,0)$ theories

- $N = (1,0)$ SCFTs: $R^{5+1} \times M_4$ w/ nontrivial axion-dilaton fields ~ "F-theory"
- F-theory on elliptically fibered CY_3
- Earlier examples [Morrison, Vafa] [Witten] 1996
- Recently, appeared a "classification"

[Morrison, Taylor] 2012, [Heckman, Morrison, Vafa] 2013

[Heckman, Morrison, Rudelius, Vafa] 2015

[Del Zotto, Heckman, Tomasiello, Vafa] 2014 , ………



- This is basically a classification of CY_3 with suitable properties to have 6d SCFTs.
- Like the (2,0) theories, D3-branes wrapping 2-cycles yield strings.

Aspects

- There is a universal "branch" in all known 6d SCFTs. "tensor branch"
- 6d self-dual tensor supermultiplet: comes with a real scalar in the multiplet

$$B_{\mu\nu} \text{ with } H = dB = \star dB \;, \quad \Psi^A \;, \quad \boxed{\Phi} \longrightarrow \text{VEV}$$

- Give VEVs to the real scalars: "tensor branch"
- Geometrically, in F-theory, VEV's are the volumes of resolved 2-cycles
- In M-theory w/ M5-branes, VEV's are the separations of branes.

- Effective action in the tensor branch is known, which only contains the "Abelian part" of the tensor multiplet.
- 6d supermultiplets in the tensor branch:
- Abelian tensor multiplets $B_{\mu\nu}$ with $H = dB = \star dB \;, \quad \Psi^A \;, \quad \Phi$
- non-Abelian vector multiplets: 6d vector + chiral fermions
- Hypermultiplets: Two (complex) scalars + anti-chiral fermions

- Vectors/hypers are optional, depending on models, but tensors are universal.

Some approaches from “conventional” QFTs

- Various conventional QFTs serve as either
- effective field theory (EFT) in certain limits,
- Exact description of the 6d QFT by taking limits (like “large N”… more later)
- Etc.

- We shall first discuss insights from these descriptions, to understand various “phenomenological” constraints on the true QFT to be found in the future.

- Key concepts to be explored today:
- 4d S-duality
- its 6d geometrization & its manifestations in various 6d, 5d, 4d descriptions
- N^3
- self-dual strings, and how they encode information on the other issues above

Tensor branch

- For simplicity, consider SCFTs with 1 dimensional tensor branch (i.e. 1 Abelian tensor multiplet in this branch)
- For example, A_1 type $\mathrm{N} = (2{,}0)$ has 1 tensor multiplet + 1 hypermultiplet, which makes $N = (2{,}0)$ tensor multiplet.

- $N = (1{,}0)$ theories having vector multiplets have subtler tensor branch action

$$S^{\text{bos}}_{\text{v+t}} = \int \left[\frac{1}{2}d\Phi \wedge \star d\Phi + \frac{1}{2}H \wedge \star H\right] + \sqrt{c}\int \left[-\Phi \text{tr}(F \wedge \star F) + B \wedge \text{tr}(F \wedge F)\right]$$

$$H \equiv dB + \sqrt{c}\ \text{tr}\left(AdA - \frac{2i}{3}A^3\right)$$

- “action” involving self-dual tensor: impose self-duality on e.o.m. later by hand.
- c is a model-dependent positive constant.
- Vector & tensor should couple according to the second term, to yield the classical gauge anomaly: 6d Green-Schwarz mechanism [Green, Schwarz, West] [Sagnotti]

Tensor branch & self-dual strings

- There are strings which are charged under $B_{\mu\nu}$ with equal electric/magnetic charge, thus called self-dual strings. [Strominger] [Howe, Lambert, West]

- $N = (2{,}0)$ theory, or more generally for tensors not coupled to vectors

$$dH_3 = d \star H_3 \sim \delta^{(4)}$$

BPS equation for the tensor multiplet fields $H = \mp \star_4 d\Phi$

- Generic $N = (1{,}0)$ theory: Yang-Mills instanton charge = string charge

$$d \star H \,(= dH) = \sqrt{c}\, \mathrm{tr}(F \wedge F)$$

BPS equations (valid only at the leading order in $1/\langle\Phi\rangle$ expansion)

$$F = \pm \star_4 F \qquad k = \frac{1}{8\pi^2} \int_{\mathbb{R}^4} \mathrm{tr}(F \wedge F) \text{ is quantized}$$

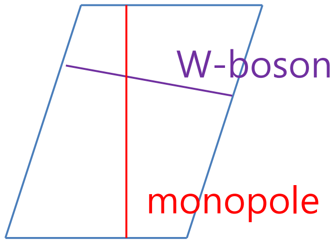
$$H = \mp \star_4 d\Phi$$

- These strings are analogous to the W-bosons of Yang-Mills theory: traces of non-Abelian d.o.f. in the broken phase. String tension $\propto \langle\Phi\rangle$.
- These strings contain information on "non-Abelian" natures of the 6d QFT. I'll later explain how to (partly) see them.

Self-dual strings & electromagnetic duality

- Consider the (2,0) theory, compactified on a torus.
- It is known that small torus reduction yields the 4d maximal SYM
- Since the 6d strings are self-dual, 4d electric/magnetic particles come from same self-dual string, wrapped on different sides of T^2.

$$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g_{YM}^2} \to -1/\tau$$



- Naturally explains S-duality, or more generally $SL(2,Z)$ duality [Montonen, Olive]
- This aspect is both a benefit and a challenge.

- Better geometric understanding of 4d S-duality (& its generalization to other 4d QFTs)
- Challenge for formulating 6d QFT, to realize S-duality manifestly upon compactification.

- Similar, and perhaps even more drastic, roles should by played by these strings for the $N = (1{,}0)$ theories compactified on torus. Also $N = (2{,}0), (1{,}0)$ theories on Riemann surfaces.
- I will mostly restrict the discussions to the rather better studied $(2{,}0)$.

5d Yang-Mills theory

- It is illustrative to make T^2 compactifiation in steps.
- First make S^1 compactification with radius R. Obtains maximal super-Yang-Mills

- 5d Yang-Mills at low energy: e.g. for A_{N-1} type, reducing N M5's yield N D4's

$$\text{Fields: } A_\mu,\ \lambda^i_\alpha\ (i=1,2,3,4 \text{ for } SO(5)_R),\ \phi^I\ \ (I=1,2,3,4,5)$$

$$S=\frac{1}{g^2_{YM}}\int d^5x\ \mathrm{tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{2}D_\mu\phi^I D^\mu\phi^I-\frac{i}{2}\bar{\lambda}_i\gamma^\mu D_\mu\lambda^i+\frac{1}{4}[\phi^I,\phi^J]^2-\frac{i}{2}\bar{\lambda}_i(\hat{\gamma}_I)^i{}_j[\phi^I,\lambda^j]\right]$$

- Parameters: $g^2_{\mathrm{YM}} \sim R$.
- Weakly-coupled when $E \ll 1/g^2_{YM} \sim 1/R$, i.e. when 6d KK modes are heavy.

- Naïve reduction of Lagrangian QFT at small circle yields strong coupling
- Morally, eletromagnetic duality is involved, from tensor to vector, to get 5d YM.

- This gauge theory description provides great intuitions on this system.

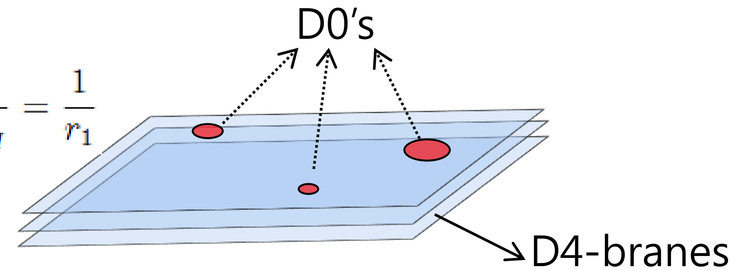
5d instanton solitons & KK modes

- D0-branes bound to D4's: "instanton" solitons see the KK tower.

$$F_{\mu\nu} = \pm \star_4 F_{\mu\nu} \quad \text{on} \quad R^4$$

$$k = \frac{1}{8\pi^2}\int \mathrm{tr}(F\wedge F) \in \mathbb{Z}$$

The radius of 6th circle: $\frac{4\pi^2}{g_{YM}^2} = \frac{1}{r_1}$



- It makes the studies of 5d (maximal) SYM very interesting. [Douglas] [Lambert, Papageorgakis, Schmidt-Sommerfeld] (2010)
- At least apparently, quantum calculus appears to see UV incompleteness, despite not always in the usual violent form of UV divergences.

- E.g. in SUSY observables, one should consider the moduli space dynamics of instantons, which has small instanton singularities, creating possible ambiguities of calculus.

small instanton singularity

single SU(N) instanton: $ds^2 = g_{MN}(X)dX^M dX^N = ds^2(\mathbb{R}^4) + d\lambda^2 + \lambda^2\left[ds^2(S^3/\mathbb{Z}_2) + ds^2(\mathcal{M}_{4N-8})\right]$

c.o.m. | instanton "size" | $SU(2)$ orientation | $\frac{SU(N)}{SU(2)\times U(N-2)}$

- Can be cured by simple UV completion, e.g. by uplifting to D0-D4 system. (But I didn't really use maximal SUSY, which might eliminate ambiguity within moduli space dynamics.)

Some instanton calculus

- The Witten indices of D0-branes (Nekrasov partition function)

- R-symmetry: $SO(5)$, or $SU(2)_R \times SU(2)_L$ in Coulomb branch (separate M5's along a line)

$$Z_k(\epsilon_{1,2}, a_i, m) = \mathrm{Tr}_k \left[(-1)^F e^{-\epsilon_1(J_1+J_R)-\epsilon_2(J_2+J_R)} e^{-a^i q_i} e^{-2mJ_L} \right]$$

- Can be computed using 5d instanton calculus, or D0-D4 quantum mechanics

$$Z_k = \sum_{\sum_i |Y_i|=k} \prod_{i=1}^{N} \prod_{s\in Y_i} \prod_{j=1}^{N} \frac{2\sinh\frac{E_{ij}(s)+m-\epsilon_+}{2} \cdot 2\sinh\frac{E_{ij}-m-\epsilon_+}{2}}{2\sinh\frac{E_{ij}(s)}{2} \cdot 2\sinh\frac{E_{ij}(s)-2\epsilon_+}{2}}$$

$$E_{ij}(s) = a_i - a_j - \epsilon_1 h_i(s) + \epsilon_2(v_j(s)+1)$$

- Its grand partition function, $Z(q = e^{2\pi i\tau}) = \sum_{k=0}^{\infty} Z_k q^k$, is a partition function on $R^4 \times T^2$

- For instance, single M5-brane theory, shows the expected 6d spectrum

$$Z(q, \epsilon_{1,2}, m) = PE\left[Z_1(\epsilon_{1,2}, m)\frac{q}{1-q} \right] \equiv \exp\left[\sum_{n=1}^{\infty} \frac{1}{n} Z_1(n\epsilon_{1,2}, nm)\frac{q^n}{1-q^n} \right] \qquad Z_1(\epsilon_{1,2}, m) = \frac{\sinh\frac{m+\epsilon_-}{2}\sinh\frac{m-\epsilon_-}{2}}{\sinh\frac{\epsilon_1}{2}\sinh\frac{\epsilon_2}{2}}$$

- For multiple M5-branes, one needs more technical controls. (later)

- d.o.f. carrying momenta, bound to 6d strings: $N^2 \to N^3$ enhancement as $q \to 1^-$?

4d Yang-Mills theory

- Further reduction on the second circle, to 4d.
- We have important implications from 6d self-dual QFT

- Electric/magnetic particles/fields should have same origin.
- Montonen-Olive electromagnetic duality: [Montonen, Olive] [Osborne]

- For special 4d gauge theories, such as $N = 4$, there is a chance of realizing S-duality because both W-boson & monopoles are in the same type of supermultiplet.

- Actual studies of quantum spectrum of dyons in Coulomb branch justified S-duality. [Sen], …
- Many other observables studied in the unbroken phase (such as curved space partition function [Vafa, Witten], …) justifies S-duality more generally.

S-duality from gauge theory

- However, in the current formulation in terms of gauge theory, electromagnetic duality is not manifest at all.

- E.g., to show that the dyon spectra respect $SL(2,Z)$ in SU(2) Yang-Mills, one has to show that the spectrum is the same for all (p,q) magnetic/electric charges, for coprime p,q.
- This is because these (p,q) are related to the W-boson at $(0,1)$ charge.
- A. Sen & others had to study the moduli space of magnetic monopoles, making a hard study of their bound state wave functions. On the other hand, elementary W-boson's spectrum is so easy to get.

- The results support $SL(2,Z)$, but it is "emergent" in the gauge theory.
- If one can make a covariant formulation of the 6d theory, one can naturally imagine that this duality should be more manifest.

6d self-dual strings

- So far, I talked about:

1) universality of tensor branch & self-dual strings,
2) Spectrum calculus using 5d approach (whose structures, e.g. N^3, are left unaddressed yet)
3) 4d S-duality

- Now, I'll explain that 2) and 3) can be better addressed by studying 1).

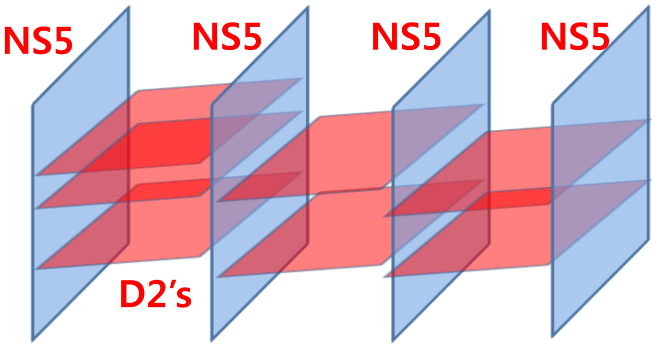
- Tensionless string at conformal point? Worldsheet descriptions of them? (difficult)
- I shall explain aspects of tensionful strings in the tensor branch. Even here, many aspects of these strings are different from fundamental strings.

- The approach I'll explain uses 2d QFTs living on these strings, which are expected to flow to 2d SCFTs on these strings.

- Somewhat like studying massive 4d dyons using quiver quantum mechanics. [Denef], …
- Depending on how one embeds the 2d CFTs in UV, various calculations become easier/difficult. So it involves a bit of "art" of constructing good UV descriptions.

Approaches from 2d QFT & RG flow

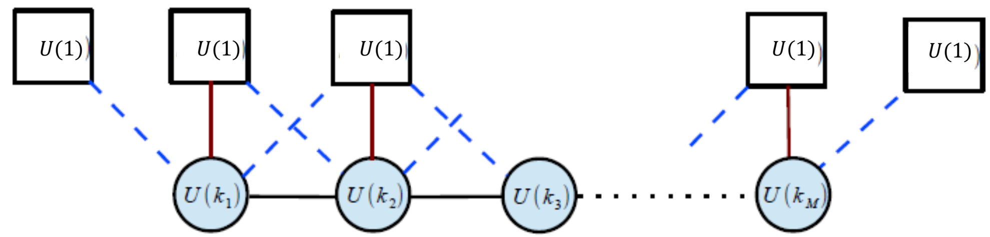
- The case of $N=(2,0)$ theory: uplift to IIA. M2-M5 → D2-NS5



yields 2d N=(4,4) quiver

tricky UV theory use: only sees $SO(3)_R \subset SO(4)_R$
(similar to 3d $N=8$ SYM vs. M2: $SO(7) \subset SO(8)$)

	0	1	2	3	4	5	6	7	8	9
NS5	•	•	•	•	•	•	–	–	–	–
D6	•	•	•	•	•	•	•	–	–	–
D2	•	•	–	–	–	–	•	–	–	–

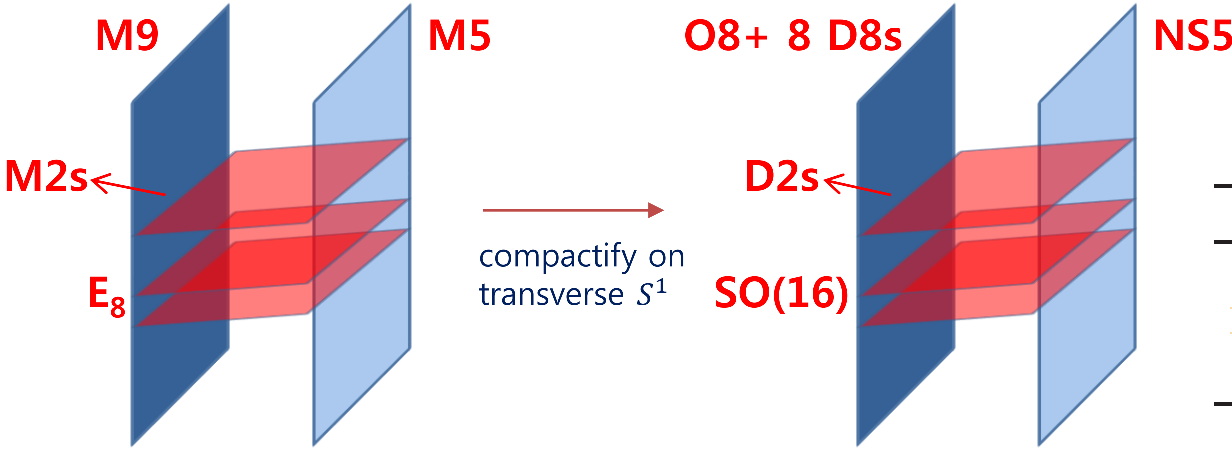


Sees $SU(2) \times U(1) \subset SO(4)_R$: similar to "mirror dual" of 3d $N=8$ SYM $U(1)^2 \times SU(2)^2 \subset SO(8)$ [Haghighat,Iqbal,Kozcaz,Lockhart,Vafa]

- $k_1, \cdots, k_{N-1}$ are string numbers in various $U(1)^{N-1}$ tensors.
- String theory asserts that these QFTs flow to those on self-dual string worldsheet in IR.

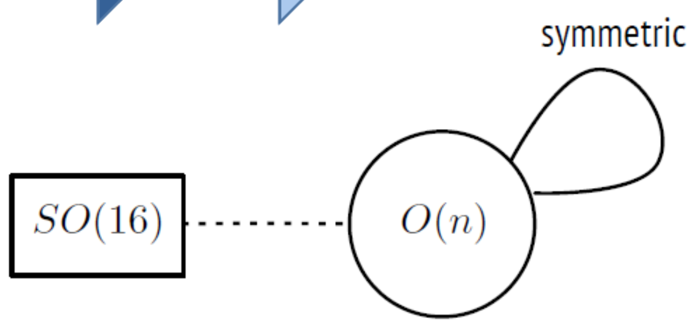
Strings of $N = (1,0)$

- Similar 2d constructions can be made for the strings of various $N = (1,0)$ theory.
- E.g. the "E-string theory" (w/ rank 1 tensor branch)



	0	1	2	3	4	5	6	7	8	9
NS5	•	•		•	•	•	•			
D8-O8	•	•		•	•	•	•	•	•	•
D2	•	•	•							

- The 2d quiver with (0,4) SUSY



- Many other (1,0) strings are explored from different approaches, related to the dynamics of instanton string worldsheet dynamics. [Haghighat, Klemm, Lockhart, Vafa] [J.Kim,SK,K.Lee,Park,Vafa] [Gadde,H,JK,SK,L,V] [Del Zotto, Lockhart] [Del Zotto, Gu, Huang, Kashani-Poor, Klemm, Lockhart] [H.-C.Kim, SK, Park] [] ……

Elliptic genera

- Some invariant observables under RG flow: Can be computed from UV.
- Elliptic genus: T^2 partition function w/ SUSY

$$Z_k(\tau, \epsilon_{1,2}, m_a) = \mathrm{Tr}\left[(-1)^F e^{2\pi i\tau H_+} e^{2\pi i\bar{\tau} H_-} e^{2\pi i\epsilon_1(J_1+J_R)} e^{2\pi i\epsilon_2(J_2+J_R)} \cdot \prod_{a\in\text{flavor}} e^{2\pi i m_a F_a}\right]$$

$$H_\pm \equiv \frac{H\pm P}{2} \qquad H_- \sim \{Q, \overline{Q}\}$$

- The elliptic genus at given string winding numbers, $n_1, \cdots, n_{N-1}$ on spatial S^1

$$Z_{(n_i)} = \sum_{Y_1,\cdots,Y_{N-1};|Y_i|=n_i} \prod_{i=1}^{N} \prod_{s\in Y_i} \frac{\theta_1(\tau|\frac{E_{i,i+1}(s)-m+\epsilon_-}{2\pi i})\theta_1(\tau|\frac{E_{i,i-1}(s)+m+\epsilon_-}{2\pi i})}{\theta_1(\tau|\frac{E_{i,i}(s)+\epsilon_1}{2\pi i})\theta_1(\tau|\frac{E_{i,i}(s)-\epsilon_2}{2\pi i})}$$

$$E_{i,j}(s=(a,b)) = (Y_{i,a}-b)\epsilon_1 - (Y^T_{j,b}-a)\epsilon_2$$

- Coefficient of the grand partition function, which sums over the string numbers

$$Z(v_i, \tau, \epsilon_{1,2}, m) = e^{-\varepsilon_0} Z^N_{U(1)} \sum_{n_1,\cdots,n_N}^{\infty} e^{-n_i(v_i-v_{i+1})} Z_{(n_i)}(\tau, m, \epsilon_{1,2})$$

- This is the same partition function as those computed from 5d approach

Aspects of elliptic genus

- These elliptic genera behave very differently from the partition functions of fundamental strings, which obey the Hecke transformation formula

- Single fundamental string's partition function $Z_1(\tau, z)$.
- Multiple strings' partition function $Z_n(\tau, z)$ is given by the Hecke transformation

$$Z(w,\tau,z) \equiv \sum_{n=0}^{\infty} w^n Z_n(\tau,z) = \exp\left[\sum_{n=1}^{\infty} \frac{1}{n} w^n \sum_{ad=n;a,d\in\mathbb{Z}} \sum_{b (\mathrm{mod}\ d)} Z_1\left(\frac{a\tau+b}{d}, az\right)\right]$$

- A consequence of multi-particle statistics & twisted sectors.
- Self-dual strings' elliptic genera don't obey this formula. More nontrivial bound states.

- S-modular property: modular anomaly factor $\propto n^2$ is incompatible w/ Hecke formula

$$Z_{(n_i)}\left(-\frac{1}{\tau}, \frac{m}{\tau}, \frac{\epsilon_{1,2}}{\tau}\right) = \exp\left[\frac{1}{4\pi i\tau}\left(\epsilon_1\epsilon_2 \sum_{i,j=1}^{N-1} \Omega^{ij} n_i n_j + 2(m^2 - \epsilon_+^2) \sum_{i=1}^{N-1} n_i\right)\right] Z_{(n_i)}(\tau, m, \epsilon_{1,2})$$

$$\Omega^{ii} = 2\ , \quad \Omega^{i,i+1} = \Omega^{i,i-1} = -1$$

- The last modular property is connected to both 4d S-duality & 6d N^3

Modular property & S-duality

- The grand partition function: Nekrasov's paritition function of QFT on $R^4 \times T^2$

$$Z(v_i,\tau,\epsilon_{1,2},m) = e^{-\varepsilon_0} Z_{U(1)}^N \sum_{n_1,\cdots,n_N}^{\infty} e^{-n_i(v_i-v_{i+1})} Z_{(n_i)}(\tau,m,\epsilon_{1,2})$$

- Reduces to 4d or 5d Coulomb branch partition function of mass-deformed maximal super-Yang-Mills theory by taking suitable limits
- In the limit $\epsilon_{1,2} \to 0$, yields Seiberg-Witten prepotential $-\log Z \to f(v,m,\tau)/\epsilon_1\epsilon_2$
- f is "quantum prepotential" related to full prepotential by $F = \pi i \tau v^2 + f$.
- S-modular property of $Z_{(n_i)}$ determines S-duality of (mass-deformed) N=4 SYM.

- In 4d limit, expects Legendre transformation of E/M prepotentials under S-duality.
- $F(-1/\tau, a_D) = F(\tau,a) - a\, a_D = F(\tau,a) - a\, dF/da$
- In full 6d, from physics reasoning we don't expect such a simple S-duality.
- $\tau \to i\infty$: small spatial S^1 of T^2. Or low T. Expect $\log Z \sim N^2$ from 5d Yang-Mills
- $\tau \to i0^+$: large spatial S^1. Or high T. Expect $\log Z \sim N^3$ from 6d (2,0) physics
- Therefore, we expect certain anomaly of S-duality of full 6d system on T^2.

Details

- τ dependence via quasi-modular forms: $\theta_1(\tau|z) = 2\pi i z\ \eta(\tau)^3 \exp\left[\sum_{k=1}^{\infty} \frac{B_{2k}}{(2k)(2k)!} E_{2k}(\tau)(2\pi i z)^{2k}\right]$

- 3 generators: $E_2(-1/\tau) = \tau^2 \left(E_2 + \frac{6}{\pi i \tau}\right)$, $E_4(-1/\tau) = \tau^4 E_4(\tau)$, $E_6(-1/\tau) = \tau^6 E_6(\tau)$

  → causes modular anomaly of $Z_{(n_i)}(\tau, m, \epsilon_{1,2})$

- modular anomaly equation:

$$\frac{\partial}{\partial E_2} Z_{(n_i)}(\tau, m, \epsilon_{1,2} : E_2) = \frac{1}{24}\left[\epsilon_1\epsilon_2 \Omega^{ij} n_i n_j - 2\Omega^{ij}(m^2 - \epsilon_+^2)\rho_i n_j\right] Z_{(n_i)}$$

$$\hat{Z}(\tau, v, m, \epsilon_{1,2}) = \sum_{n_1,\cdots,n_r=0}^{\infty} e^{-\sum_{i=1}^{r} n_i \alpha_i(v)} Z_{(n_i)}$$

$$\frac{\partial \hat{Z}}{\partial E_2} = \frac{1}{24}\left[\epsilon_1\epsilon_2\Omega^{ij}\partial_i\partial_j + 2(m^2 - \epsilon_+^2)\Omega^{ij}\rho_i\partial_j\right]\hat{Z} \equiv \frac{1}{24}\left[\epsilon_1\epsilon_2\Omega^{ij}\partial_i\partial_j + 2\Omega^{ij} I_i(m, \epsilon_+)\partial_j\right]\hat{Z}$$

- some manipulations:

$$Z_{S-dual} \equiv \hat{Z}\ \exp\left[\# \frac{\Omega^{ij} I_i v_j}{\epsilon_1\epsilon_2} + \# \frac{\Omega^{ij} I_i I_j}{\epsilon_1\epsilon_2} E_2(\tau)\right]$$

($\hat{Z} \to Z/Z_{U(1)}^N$, $\# \frac{\Omega^{ij} I_i v_j}{\epsilon_1\epsilon_2} \to e^{-\varepsilon_0}$)

$$\frac{\partial Z_{\text{S-dual}}}{\partial E_2} = \frac{\epsilon_1\epsilon_2}{24}\Omega^{ij}\partial_i\partial_j Z_{\text{S-dual}}$$: heat equation

- dividing modular anomaly: "standard" one + anomaly of "standard" one

$\Omega^{ij} I_i I_j \sim \rho^2 \sim c_2|G|$
$= N^3 - N$

Modular property & N^3

- S-duality of Z_{S-dual} : ($\delta = \frac{6}{\pi i \tau}$) $Z_{\text{S-dual}}\left(-\frac{1}{\tau}, v, \frac{m}{\tau}, \frac{\epsilon_{1,2}}{\tau}; E_2(-\frac{1}{\tau})\right) = Z_{\text{S-dual}}(\tau, v, m, \epsilon_{1,2}, E_2(\tau)+\delta)$

$$Z_{\text{S-dual}}(\tau, v, m, \epsilon_{1,2}; E_2(\tau)+\delta) = \int_{-\infty}^{\infty} \prod_{i=1}^{N} dv_i' \; K(v, v') Z_{\text{S-dual}}(\tau, v', m, \epsilon_{1,2}; E_2(\tau))$$

$$K(v, v') = \left(\frac{i\tau}{\epsilon_1\epsilon_2}\right)^{\frac{N}{2}} \exp\left[-\frac{\pi i \tau}{\epsilon_1\epsilon_2}(v-v')^2\right]$$

- Small $\epsilon_{1,2}$: RHS computed by saddle point approx. $Z \sim \exp\left[-\frac{f(\tau, v, m)}{\epsilon_1\epsilon_2}\right]$, $Z_{\text{S-dual}} \sim \exp\left[-\frac{f_{\text{S-dual}}(\tau, v, m)}{\epsilon_1\epsilon_2}\right]$

$$\tau^2 F_{\text{S-dual}}\left(\tau_D = -\frac{1}{\tau}, v_D = v + \frac{1}{2\pi i \tau}\frac{\partial f}{\partial v}, \frac{m}{\tau}\right) = F_{\text{S-dual}}(\tau, v, m) - v\frac{\partial F_{\text{S-dual}}}{\partial v}(\tau, v, m)$$

4d limit (small T^2)

$$\sim F_{\text{S-dual}}\left(-\frac{1}{\tau}, v_D \equiv \tau v + \frac{1}{2\pi i}\frac{\partial f}{\partial v}, m\right)$$

"Standard" S-duality in 4d Seiberg-Witten:
"magnetic dual prepotential ~ S-dual prepotential"

- "S-duality anomaly": extra anomaly. E.g. for ADE (2,0),

$$f(\tau, v, m) = f_{\text{S-dual}}(\tau, v, m) + r f_{U(1)}(\tau, m) + \frac{c_2|G|}{288} m^4 E_2(\tau)$$

$$f_{U(1)} = m^2\left(\frac{1}{2}\log m - \frac{3}{4} + \frac{\pi i}{2} + \log\phi(\tau)\right) + \sum_{n=1}^{\infty}\frac{m^{2n+2}B_{2n}}{2n\cdot(2n+2)!}E_{2n}(\tau)$$

- note: The last S-duality anomaly vanishes in the 4d limit (after restoring S^1 radius)

Asymptotic free energy

- One can use S-dual low T ($\tau \to i\infty$) setting to study the decompactifying limit ($\tau \to i0^+$): contribution from anomalous part + 5d perturbative part

- Results: $$-\log Z \sim \frac{f(\tau \to 0, v, m)}{\epsilon_1 \epsilon_2} \sim \frac{i}{\epsilon_1 \epsilon_2 \tau}\left[\frac{N^3 m^4}{48\pi} - \frac{\pi N m^2}{12} \mp i \frac{N^2 m^3}{12}\right] \quad \text{for } 0 < \pm \mathrm{Im}(m) < \frac{2\pi}{N}$$

from anomalous parts

from "low T" dual perturbative part

- At $m = 0$, SUSY enhances, #(boson) – #(fermion) = 0. Obstruct full cancelation by $m \neq 0$
- Mechanism, w/ light D0's…? Interpretation?

- In a sense, one has observed "deconfined" degrees of freedom coming from light instantons/D0-branes, in an asymptotic limit
- These structures can be extended to study black holes in AdS_7, and more generally in AdS_5, AdS_3 (work in progress)

Challenges

- Attempts towards microscopic formulations (many of them to be discussed in this conference)

- Fully satisfactory descriptions should explain the following challenging problems.

- 4d S-duality
- Building in N^3 in a consistent manner
- $N^2 \rightarrow N^3$ interpolation of d.o.f. by changing S^1 radius

- I also tried to explain that all these aspects are connected to the physics of 6d self-dual strings, especially their anomaly structures. Some connections are quantitatively established recently.

- Hopefully, these should put strong constraints on the higher structures relevant for 6d SCFTs, thus providing useful guidance.

Challenges

- 4d S-duality:

- Highly nontrivial in 4d Yang-Mills, but geometrical in 6d. What makes 4d S-duality possible should be built-in in the 6d QFT, if one seeks for a manifestly Lorentz-covariant description.
- There may be other ways to formulate these QFTs, without manifest covariance. [Zwanziger]
- Perhaps equivalently, w/ auxiliary fields. [Pasti, Sorokin, Tonin] [Lambert, Papageorgakis]

- 6d self-duality:

- Lagrangian is always unfriendly to self-dual p-form potentials in $d = 2p + 2$. One way of dealing with it is to use suitable auxiliary fields, e.g. the PST formalism.
- Another way was recently found in the context of type IIB SUGRA [Sen] (2015)

$$S = S_1' + S_2$$

$$S_1' = \frac{1}{2}\int dP^{(4)} \wedge *dP^{(4)} - \int dP^{(4)} \wedge Q^{(5)} - \int B^{(2)} \wedge F^{(3)} \wedge Q^{(5)} + \frac{1}{2}\int *\left(B^{(2)} \wedge F^{(3)}\right) \wedge \left(B^{(2)} \wedge F^{(3)}\right) .$$

which replaces the "conventional action" in which self-duality is imposed later by hand

$$S_1 \equiv -\frac{1}{2}\int \widehat{F}^{(5)} \wedge *\widehat{F}^{(5)} + \int F^{(5)} \wedge B^{(2)} \wedge F^{(3)}$$

- Sort of "guaranteed" to work at quantum level, being inspired by string field theory
- Once we find the right equation of motion, a covariant action may be available.