Weyl Anomaly and Induced String Current in Boundary CFT in 6d

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Outline

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M5-branes

- String and M-theory is a source of new structures and new ideas.
- The studies of multiple M2-branes has inspired the construction of a new class of gauge theory based on Lie 3-algebras. However, multiple M5-brane system remains mysterious. (see Lambert, Kim's talk)
  - Gauge symmetry for multiple M5-branes ?
  - Quantized geometry for M5-brane in a large constant 3-form C-field?
  - Interacting self-dual dynamics on M5-branes worldvolume?
  - Entropy counting N^3 for N number of M5-branes?
- In this talk, I want to talk about the exact understanding of certain new physical phenomena associated with boundary.
  We will apply it to boundary M5-branes system and learn something about it.

Boundary systems

- The existence of boundary often give rise to interesting and powerful mathematical and physical relations that allows not just deeper understanding of the system but also revelation of novel relations. e.g. for mathematical boundary operator: cocycles and homotopies, descent relations, anomaly, Schwinger term, Wess-Zumino Lagrangian.
- For physical boundaries, e.g. Casimir effects, edge modes in topological insulator, Hall effects.

Most of the time we study boundary phenomena with perturbation method. It would be nice to have handle on the nontrivial boundary phenomena beyond perturbation theory. In this talk, I will talk about

1. Some new exact and universal results of BCFT from field theory
2. A nonperturbative approach via AdS/BCFT holography

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2.1.1. Casmir effect in BCFT

- In general, for a d-dimensional BQFT, vev of renormalized stress tensor has the asymptotic behaviour near boundary:

$$\langle T_{ij} \rangle = x^{-d} T^{(d)}_{ij} ... + x^{-1} T^{(1)}_{ij} + \cdots , \quad x \sim 0,$$

  x is proper distance from boundary.

- For a d-dimensional BCFT, conformal symmetry improves it to:

$$T^{(d)}_{ij} = 0, \qquad T^{(d-1)}_{ij} = 2\alpha_1 \bar{k}_{ij},$$

$$T^{(d-2)}_{ij} = \frac{-4\alpha_1}{d-1} n_{(i} h^l_{j)} \nabla_l k - \frac{4\alpha_1}{d-2} n_{(i} h^l_{j)} n^p R_{lp} + \frac{2\alpha_1}{d-2} (n_i n_j - \frac{h_{ij}}{d-1}) \mathrm{Tr} \bar{k}^2 + t_{ij},$$

$$t_{ij} := \left[\beta_1 C_{ikjl} n^k n^l + \beta_2 \mathcal{R}_{ij} + \beta_3 k k_{ij} + \beta_4 k^l_i k_{lj} \right],$$

  where n_i, h_{ij} and $\bar{k}_{ij}$ are respectively the normal vector, induced metric and the traceless part of extrinsic curvature of the boundary P.

The Casimir coefficients (α, β_i) fixes the shape dependence of the leading and subleading Casimir effects of BCFT.

- Known for free BCFT:

Table: Casimir coefficients for 4d free BCFT

	α_1	β_1	β_2	β_3	β_4
Scalar, Dirichlet B.C	$-\frac{1}{480\pi^2}$	0	0	$-\frac{19}{10080\pi^2}$	$-\frac{1}{420\pi^2}$
Scalar, Robin B.C	$-\frac{1}{480\pi^2}$	0	0	$-\frac{1}{288\pi^2}$	0
Maxwell field	$-\frac{1}{40\pi^2}$	(0)	0	$-\frac{43}{840\pi^2}$	$\frac{1}{70\pi^2}$

- Unknown in general!

2.1.2. Casimir effects from Weyl Anomaly

Boundary Weyl anomaly

- In general, quantum mechanics breaks conformal symmetry and give rises to the Weyl anomaly

$$\mathcal{A} := \partial_\sigma W[e^{2\sigma} g_{ij}]|_{\sigma=0} = \int_M \langle T_i^i \rangle,$$

  where

$$\left\langle T_i^i \right\rangle = \left\langle T_i^i \right\rangle_M + \delta(x_\perp) \langle T_a^a \rangle_P .$$

  $\left\langle T_i^i \right\rangle_M$ = bulk Weyl anomaly, $\langle T_a^a \rangle_P$ = boundary Weyl anomaly. (Dowker, Schofield 90; Moss, Poletti 94, also D'Eath, Esposito 95; Herzog, Huang, Jensen 15; Solodukhin 15; Fursaev 15 etc.)

- Weyl anomaly are classified in terms of curvature invariants.
  3d BCFT: $\left\langle T_i^i \right\rangle = \delta(x)[b_1 \mathcal{R} + b_2 \mathrm{Tr}\bar{k}^2]$
  4d BCFT:
  $\left\langle T_i^i \right\rangle = \frac{c}{8}\mathrm{Tr}C^2 - \frac{a}{16\pi^2} E_4 + \delta(x)[\frac{a}{16\pi^2} E_4^{\mathrm{bdy}} + b_3 \mathrm{Tr}\bar{k}^3 + b_4 C^{ac}{}_{bc}\bar{k}^b{}_a]$
  Bulk central charges c do not depend on BC. Boundary central charges b_i depend on BC in general.

Claim: the energy momentum tensor $T_{\mu\nu}$ and current J_μ has universal behaviour near the boundary. (Chu, Miao 17, 18)

- Consider a BCFT with a well defined effective action. It is easy to show that the Weyl anomaly $\mathcal{A}$ can be obtained as the logarithmic UV divergent term of the effective action,

$$I = \cdots + \mathcal{A} \log(\frac{1}{\epsilon}) + I_{\rm finite}, \quad \epsilon = \text{UV cutoff}.$$

- Using this one can establish the "integrability condition":

$$\boxed{(\delta \mathcal{A})_{\partial M} = \left(\frac{1}{2} \int_{x \geq \epsilon} \sqrt{g}\, T^{ij} \delta g_{ij} \right)_{\log(1/\epsilon)}}$$

  where $(\delta \mathcal{A})_{\partial M}$ is the boundary terms in the variations of Weyl anomaly and T^{ij} is the renormalized bulk stress tensor.

- Note that the right hand side must give an exact variation, this imposes strong constraints on the possible form of the stress tensor near the boundary.

- E.g. For 3d BCFT, Weyl anomaly has only boundary contributions

$$\mathcal{A} = \int_{\partial M} \sqrt{h}(b_1 \mathcal{R} + b_2 \text{Tr}\bar{k}^2),$$

$$LHS = (\delta\mathcal{A})_{\partial M} = b_2 \int_{\partial M} \sqrt{h}\Big[(\frac{\text{Tr}\bar{k}^2}{2} h^{ab} - 2\bar{k}^a_c k^{cb})\delta h_{ab} + 2\bar{k}^{ab}\delta k_{ab}\Big].$$

On the other hand, we can use the near-boundary expression of $T_{\mu\nu}$, integrate over x and pick up the log divergent term

$$RHS = -\alpha_1 \int_P \sqrt{h}[(\frac{\text{Tr}\bar{k}^2}{2} h^{ab} - 2\bar{k}^a_c k^{cb})\delta h_{ab} + 2\bar{k}^{ab}\delta k_{ab}] + \int_P \sqrt{h}[(\frac{\beta_3}{2} - \alpha_1) k\bar{k}^{ab}\delta h_{ab} + \frac{\beta_4}{2} \lceil k^a_c k^{cb} \rceil \delta h_{ab}].$$

The integrability condition then give

$$\boxed{\alpha_1 = -b_2, \quad \beta_3 = -2b_2, \quad \beta_4 = 0.}$$

- E.g. For 4d BCFT, similarly we obtain

$$\boxed{\alpha_1 = \frac{b_4}{2}, \quad \beta_1 = \frac{c}{2\pi^2} + b_4, \quad \beta_2 = 0, \quad \beta_3 = 2b_3 + \frac{13}{6} b_4, \quad \beta_4 = -3b_3 - 2b_4.}$$

- One can check that these relations are indeed satisfied by free BCFT. But never noticed before.
- It is remarkable that the Casimir coefficients are completely determined by the boundary central charges.
- The relations between them are universal and independent of BC and theory.

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2.2.1. Chiral Anomaly and Transport

- The quantum transportation of charges induced by anomaly is an interesting phenomena.
- The famous ones are:

  - The chiral magnetic effect (CME) which refers to the generation of currents parallel to an external magnetic field **B**.

  (Vilenkin 80; Giovannini Shaposhnikov 98; Froehlich etal 98)

$$\mathbf{J}_V = \sigma_{(\mathcal{B})V}\mathbf{B} \quad \mathbf{J}_A = \sigma_{(\mathcal{B})A}\mathbf{B},$$

  where the chiral magnetic conductivities are

$$\sigma_{(\mathcal{B})V} = \frac{e\mu_A}{2\pi^2}, \quad \sigma_{(\mathcal{B})A} = \frac{e\mu_V}{2\pi^2}$$

  - The chiral vortical effect (CVE) refers to the generation of a current due to rotational motion in the charged fluid. (Kharzeev, Zhitnitsky 07; Erfmemger etal 09; Son etal 09; Landsteiner etal 11)

$$\mathbf{J}_V = \sigma_{(\mathcal{V})V}\omega, \quad \mathbf{J}_A = \sigma_{(\mathcal{V})A}\omega,$$

  where the chiral vortical conductivities are

$$\sigma_{(\mathcal{V})V} = \frac{\mu_V\mu_A}{\pi^2}, \quad \sigma_{(\mathcal{V})A} = \frac{\mu_V^2+\mu_A^2}{2\pi^2} + \frac{T^2}{6}$$

- Note that these anomalous transport occurs only in a material system where the chemical potentials are non-vanishing.

Q. In the presence of boundary, can the phenomena of anomalous transport occur in vacuum like Casimir effect? if so, how is it possible?

2.2.2. Weyl Anomaly and induced current

- Just as energy momentum tensor has an universal expansion near boundary, the renormalized current is also divergent near the boundary:

$$\langle J_i \rangle = x^{-3} J_i^{(3)} + x^{-2} J_i^{(2)} + x^{-1} J_i^{(1)}, \quad x \sim 0,$$

where x is the proper distance from the boundary and $J_i^{(n)}$ depend only on the background geometry and the background vector field strength.

- Imposing current conservation

$$\nabla_i \langle J^i \rangle = 0,$$

we obtain the gauge invariant solutions

$$J_\mu^{(3)} = 0, \quad J_\mu^{(2)} = 0,$$
$$J_\mu^{(1)} = \alpha_1 F_{\mu\nu} n^\nu + \alpha_2 \mathcal{D}_\mu k + \alpha_3 \mathcal{D}_\nu k_\mu^\nu + \alpha_4 \star F_{\mu\nu}\, n^\nu$$

- Like the case of $T_{\mu\nu}$, these current coefficients can also be determined in terms of central charge

- One has similarly the integrability condition

$$\boxed{(\delta\mathcal{A})_{\partial M} = \left(\int_M \sqrt{g} J^\mu \delta A_\mu\right)_{\log\frac{1}{\epsilon}}}$$

- Consider background $U(1)$ gauge field, e.g. QED

$$\mathcal{A} = \int_M \sqrt{g}[b_1 F_{\mu\nu}F^{\mu\nu} + \text{metric part}], \quad b_1 = -\beta(e)/(2e^3), \text{ centeral charge.}$$

- This implies that

$$(\delta\mathcal{A})_{\partial M} = -4b_1 \int_{\partial M} \sqrt{h} F^b{}_n \, \delta a_b.$$

Matching it with the RHS of the integrability condition, we obtain

$$\alpha_1 = 4b_1, \quad \alpha_2 = \alpha_3 = \alpha_4 = 0$$

and for the expectation value of the current

$$\boxed{J_b = \frac{4b_1 F_{bn}}{x}, \quad x \sim 0}$$

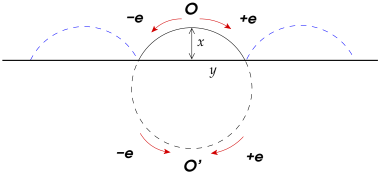
- Spelling it out, we obtain the induced current near the boundary of a BQFT:

$$\boxed{\mathbf{J} = \frac{e^2 c}{\hbar} \frac{4 b_1 \mathbf{n} \times \mathbf{B}}{x}, \quad x \sim 0}$$

- remark: the current is independent of the choices of BC and hence is more universal than the renormalized stress tensor near the boundary.

Q. What is the physics of this current?

An intuitive picture: The current is a result of charge separation due to vacuum fluctuation in the presence of external B-field.



Outline

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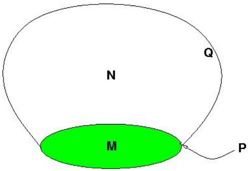
3.1.1. Takayanagi's Proposal of AdS/BCFT

- Consider d dimensional CFT defined on $R^{1,d-1}$. There is a $SO(2,d)$ conformal symmetry. This is realized in holography as isometries of the bulk of AdS_{d+1} space (Takayanagi 11)

$$ds^2 = \frac{dz^2 + dx_i^2}{z^2}, \quad z \geq 0.$$

- When a boundary is introduced, the full conformal symmetry is reduced at the boundary. Takayanagi proposed to extend the d dimensional manifold M to a $d+1$ dimensional manifold N so that $\partial N = M \cup Q$, where Q is a d dimensional manifold which satisfies $\partial Q = \partial M = P$.



- The bulk grav action is given by

$$I = \int_N \sqrt{G}(R - 2\Lambda) + 2\int_M \sqrt{g}K + 2\int_Q \sqrt{h}(K - T) + 2\int_P \sqrt{\sigma}\theta,$$

  T measures the boundary degrees of freedom (g-function).

- The central issue is the determination of the location of Q in the bulk. Takayanagi proposed to impose Neumann boundary condition on Q to fix its position:

$$\boxed{\text{EOM of } Q: \quad K_{\alpha\beta} - (K - T)h_{\alpha\beta} = 0} \qquad (*1)$$

  This gives a second order differential equation for Q.

- In standard AdS/CFT, the QFT is dual to a asymptotic AdS theory whose metric admits a Fefferman-Graham (FG) expansion,

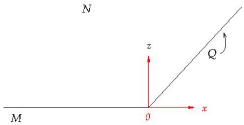
$$ds^2 = \frac{dz^2 + g_{ij} dx^i dx^j}{z^2},$$

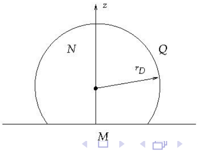
where $g_{ij} = g^{(0)}_{ij} + z^2 g^{(1)}_{ij} + \cdots$. $g^{(0)}_{ij}$ is the metric of BCFT on M. $g^{(1)}_{ij}$ is fixed by the PBH (Penrose-Brown-Henneuax) transformation:

$$g^{(1)}_{ij} = -\frac{1}{d-2}\left(R^{(0)}_{ij} - \frac{R^{(0)}}{2(d-1)} g^{(0)}_{ij}\right).$$

- For M with high symmetry such as the case of half-plane, $(*1)$ fixes the location of Q and produces correct results for BCFT.
- eg. For flat 2d BCFT on the half plane $x < 0$, N is given by the following portion of AdS_3:

$$x \le z \sinh(\rho/R), \quad \text{where} \quad T = \frac{d-1}{R} \tanh\frac{\rho}{R}.$$





3.1.2. Comments

- Note that Q is of co-dimension 1 and the location of Q is determined by a single embedding function:

  $$z = z(x^i), \quad \text{here } x^i = \text{coordinates of } M$$

  The embedding equation

  $$K_{\alpha\beta} - (K - T)h_{\alpha\beta} = 0, \qquad (*1)$$

  generally imposes too many constraints and (*1) does not has solution for general shape P of BCFT.

This has been a puzzle and it has been unclear what to do with BCFT with general shape of boundary. In the following, I propose a consistent formulation of holographic BCFT and discuss some of its applications. (Chu, Guo, Miao 17; Chu, Miao 17)

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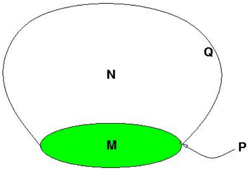
3.2.1. Non FG expansion

- In the standard AdS/CFT, FG expansion of the bulk metric is assumed. However in the case of AdS/BCFT, the dual manifold N has discontinuity at the corner P where Q and M meet.



- To allow for this, we have to give up the assumption that the dual space has a metric that can be FG expanded.
- The problem is to solve Einstein equation for the dual metric, but without the assumption that the metric can be expanded in small z near M. Hard!

- Convenient to use the Gauss normal coordinates. The metric $g_{ij}^{(0)}$ of the BCFT takes the form

$$ds_0^2 = dx^2 + (\sigma_{ab} + 2xk_{ab} + x^2 q_{ab} + \cdots)dy^a dy^b,$$

where P is located at $x = 0$.

- We found a new systematic construction of non-FG expanded metric by employing k_{ab}, q_{ab} etc as expansion parameter, but keeping both the z and x dependence as exact. In this way, we are able to construct a perturbative solution to the bulk Einstein equation: (Chu, Miao 2017)

$$ds^2 = \frac{dz^2 + dx^2 + \left(\delta_{ab} - 2x\bar{k}_{ab} f\left(\frac{z}{x}\right)\right) dy^a dy^b}{z^2} + \cdots .$$

- At the order $O(k)$, we obtain a single equation

$$s\left(s^2+1\right)f''(s)-(d-1)f'(s)=0$$

with $s := z/x$.

- It has the solution

$$f(s)=1-\lambda_1\frac{s^d\,{}_2F_1\left(\frac{d-1}{2},\frac{d}{2};\frac{d+2}{2};-s^2\right)}{d},$$

where λ_1 is free parameter.

- $\lambda_1=0$ gives FG. More free constants appear in higher order terms of the solution.

3.2.2. A more general proposal for AdS/BCFT

- Let Q be described by the embedding $x = X(z, y)$. Expanding it in z,
$$x = a_1 z + a_2 z^2 + a_3 z^3 \cdots ,$$
The embedding equation is now solvable, with
$$a_1 = \sinh\rho, \quad a_2 = -\frac{\cosh^2\rho \,\mathrm{Tr}k}{2(d-1)}, \qquad T = (d-1)\tanh\rho$$
These are independent of λ_1. But not so for higher coefficients.
- e.g. for $d = 3$, wrtting $k_{ab} = \mathrm{diag}(k_1, k_2)$ etc, we obtain
$$\begin{aligned} a_3 = \frac{\sinh\rho}{24}\Big[&7k_1^2 + 4k_2k_1 + 7k_2^2 - 4(q_1+q_2) \\ &+ \left(5k_1^2 + 2k_2k_1 + 5k_2^2 - 2(q_1+q_2)\right)\cosh(2\rho) \\ &+\lambda_1^2(k_1-k_2)^2\left((2+\cosh(2\rho))\log(\coth^2\rho) - 1\right)\Big]. \end{aligned}$$
- With this, the dual gravity description
$$I = \int_N \sqrt{G}(R - 2\Lambda) + 2\int_M \sqrt{g}K + 2\int_Q \sqrt{h}(K - T) + 2\int_P \sqrt{\sigma}\theta,$$
is now well defined.

- Note that instead of imposing the Neumann BC $(*1)$, one may also impose on Q a mixed BC so that there is only ONE constraint for Q:

$$(K^{\alpha\beta} - (K - T)h^{\alpha\beta})h_{\alpha\beta} = 0.$$

i.e.

$$\boxed{(1-d)K + dT = 0} \qquad (*2)$$

This is just the trace part of Takayanagi's condition

$$(K^{\alpha\beta} - (K - T)h^{\alpha\beta}) = 0. \qquad (*1)$$

- (*2) is natural as there is only one embedding function for Q and we expect one condition for it.
- With this, the coefficients λ_n's in the non-FG metric are not fixed. This is a consistent proposal and describes the duals for a wide class of BCFTs. The original proposal of Takayanagi is a special case.

Outline

3.3 Application: Induced current near boundary

- To investigate the renormalized current in holographic models of BCFT, we consider the following gauge invariant action for holographic BCFT

$$I = \int_N \sqrt{G}[R - 2\Lambda - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}] + 2\int_Q \sqrt{\gamma}[K - T]$$

- Consider 4d and planar boundary, one can solve for the bulk Maxwell equation and BC, and obtain

$$\mathcal{A}_a = F_{xa}\sqrt{x^2 + z^2}, \tag{2}$$

where F_{xa} is the field strength at the boundary.

- The holographic current is

$$\langle J^a \rangle = \lim_{z\to 0} \frac{\delta I}{\delta A_a} = \lim_{z\to 0} \sqrt{G}\mathcal{F}^{za} = -\frac{F_{ax}}{x} + \cdots \tag{3}$$

- For higher dimensions, we obtain

$$\mathcal{A}_a = xf(\frac{z}{x})F_{xa},$$

with

$$f(s) = 1 + \beta_d \frac{s^{d-2}\, {}_2F_1\left(\frac{d-3}{2}, \frac{d-2}{2}; \frac{d}{2}; -s^2\right)}{d-2},$$

and β_d some constant depending on ρ and d.
For example, we have for $d = 4, 5$,

$$\beta_4 = 1, \qquad \beta_5 = \frac{2}{\pi + 4\tan^{-1}\left(\tanh\left(\frac{\rho}{2}\right)\right)}.$$

- We obtan the hologrphic current

$$\langle J_a \rangle = -\beta_d \frac{F_{ax}}{x^{d-3}} + O(\frac{1}{x^{d-4}}).$$

Agrees with field theory!

Outline

4.1 Boundary string current from holography

- Consider a charged particle moving on the worldline C: $x^\mu = x^\mu(\tau)$. The motion gives a current

$$J^\mu(x) = \delta^{(d-1)}(x - x(\tau))\frac{dx^\mu(\tau)}{d\tau}.$$

It couples to the gauge field as

$$\int_M J_\mu A^\mu = \int_C A_\mu dx^\mu$$

- Similarly, movement of strings gives the higher 2-form current

$$J_{\mu\nu} = \delta^{(d-2)}(x - x(\sigma,\tau))\epsilon^{\alpha\beta}\frac{\partial x^\mu}{\partial \sigma^\alpha}\frac{\partial x^\nu}{\partial \sigma^\beta}.$$

It couples to the 2-form potential $B_{\mu\nu}$ as

$$\int_M J_{\mu\nu}B^{\mu\nu} = \int_\Sigma B_{\mu\nu}dx^\mu dx^n$$

over the worldsheet.

Q. Any implication of knowing the existence of such a coupling?

- Consider a BCFT in 6d and denote the Weyl anomaly as $\mathcal{A}$. The gravational part is well understood.
  Q. What about the contribution from background gauge field?
- One can similarly establish the relation

$$\boxed{(\delta\mathcal{A})_{\partial M} = \Big(\int_{M_\epsilon} J_{\mu\nu}\delta B^{\mu\nu}\Big)_{\log 1/\epsilon}.}$$

Thus, knowing the current, or vice versa, would allow us to learn something about the anomaly structure of the 6d CFT.

- Consider a 6d BCFT dual to the the bulk action with an H-field

$$I = \int d^7x\sqrt{G}(R - 2\Lambda - \frac{1}{12}H^2_{\mu\nu\lambda})$$

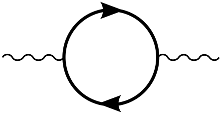
Using our BCFT holography, one finds a string current parallel to the boundary when a H-field strength is turned on

$$J_{ab} = b_1 \frac{H_{abx}}{x}.$$

- The relation $(\delta\mathcal{A})_{\partial M} = \left(\int_{M_\epsilon} J_{\mu\nu}\delta B^{\mu\nu}\right)_{\log 1/\epsilon}$ predicts the Weyl anomaly in the 6d CFT:

$$\boxed{\mathcal{A} = \int_M \frac{b_1}{12}H^2}$$

- It is interesting to understand how matter fields would couple to the $B_{\mu\nu}$ field (covariant derivatives?) and give rises to the Weyl anomaly.



Prediction for the Weyl anomaly on M5-branes theory

- For a system of N M5-branes, the field strength is self dual and the Weyl anomaly cannot be given by H^2.
  Perry-Schwarz/Henneaux-Teitelboim (also Pasti-Sorokin-Tonin) has shown that a action formulation for a single M5-brane can be written down if one give up manifest Lorentz invariance.
- In the Perry-Schwarz formulation, x_5 is singled out and the self-dual tensor gauge field is represented by a 5×5 antisymmetric tensor field $B_{\mu\nu}$. i.e. $B_{\mu 5}$ never appear.
- Denote the 5d and 6d coordinates by x^μ and $x^\mu = (x^a, x^5)$. The Perry-Schwarz action is

$$S_{PS}(B) = \frac{1}{2}\int d^6x \left(-\tilde{H}^{ab}\tilde{H}_{ab} + \tilde{H}^{ab}\partial_5 B_{ab}\right)$$

where

$$\tilde{H}^{ab} := \frac{1}{6}\epsilon^{abcde}H_{cde}, \qquad H^{abc} = -\frac{1}{2}\epsilon^{abcde}\tilde{H}_{de}.$$

- The equation of motion

$$\epsilon^{abcde}\partial_c(\tilde{H}_{ab} - \partial_5 B_{ab}) = 0$$

is second order and has the general solution

$$\tilde{H}_{ab} - \partial_5 B_{ab} = \partial_a \alpha_b - \partial_b \alpha_a, \quad \text{for arbitrary } \alpha_a.$$

- The action is invariant under the gauge symmetry

$$\delta B_{ab} = \partial_a \varphi_b - \partial_b \varphi_a, \quad \text{for arbitrary } \varphi_a.$$

This allows one to reduce the general solution to the EOM to the first order form

$$\tilde{H}_{ab} = \partial_5 B_{ab}.$$

i.e. the self-duality equation.

- Using the PS variables, one can show that the expression

$$\boxed{\mathcal{A} = b_1 \int_M \tilde{H}_{ab} \partial_x B^{ab},} \quad x^\mu = (x^a, x).$$

satisfies the relation $(\delta\mathcal{A})_{\partial M} = \left(\int_{M_\epsilon} J_{\mu\nu} \delta B^{\mu\nu} \right)_{\log 1/\epsilon}$ for the current predicts by holography. We conjecture that this is the contribution of the self-dual field strength to the Weyl anomaly of M5-branes.

- Restoring the units, b_1 is given by

$$b_1 = \frac{R^5}{16\pi G_7} = \frac{N^3}{3\pi^3},$$

where $G^{(7)} = G^{(11)}/R_S^4$, $R_S = l_P(\pi N)^{1/3}$ is the 4-sphere radius and $R = 2R_S$ is the AdS_7 radius.

- Therefore for a system of N M5-branes with boundary, one finds for the singlet current

$$J_{ab} = \frac{N^3}{3\pi^3} \frac{H_{abx}}{x}.$$

This is another way to see that there is N^3 degrees of freedom in the theory.

Outline

Conclusions and Discussions

1. We have obtained universal relations for near boundary behaviour of stress tensor and electric current.
2. Based on the original work of Takayanagai, we have provided complete proposal of holographic BCFT.
3. Applying the BCFT holography to 6d, we find an induced string current. This predicts a Weyl anomaly for the M5-branes system.

Open questions:

- What is the origin of the Weyl anomaly?
- It is known that a constant magnetic field is related to a noncommutative gauge theory with space-space noncommutativity

$$[x^i, x^j] = i\theta^{ij}.$$

  What is the relation between NCG and our induced electric current?
- Similarly, can our result of the induced string current help us to understand what kind of higher NCG on the M5-brane worldvolume

$$[x^i, x^j, x^k] = i\theta^{ijk}$$

  when a constant 3-form C-potential is turned on?

Thank you!