Propagation enhancement by a line of fast diffusion: a property of the level sets

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Joint work with L. Caffarelli.

The hero

Probably introduced $\sim$ 2004, in Chinese bonsai pots.

The outcome
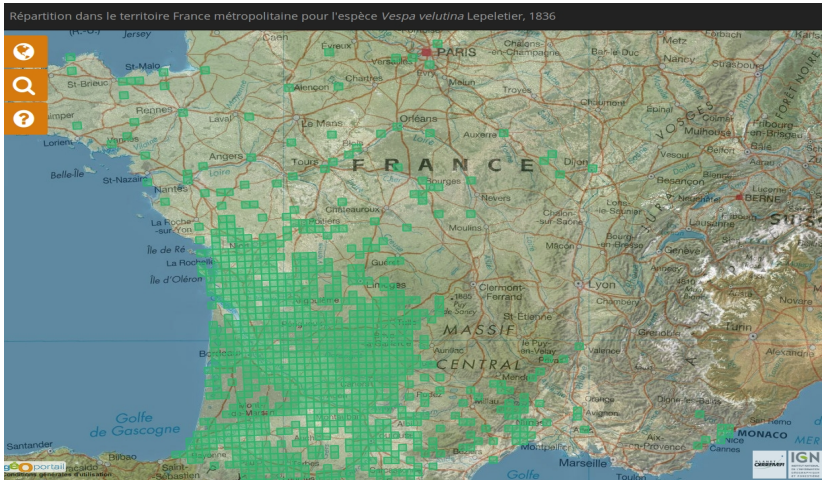
Source: Museum d'Histoire Naturelle.
Green squares indicate first date of observation (avaliable online)
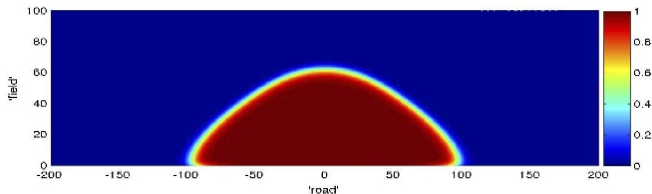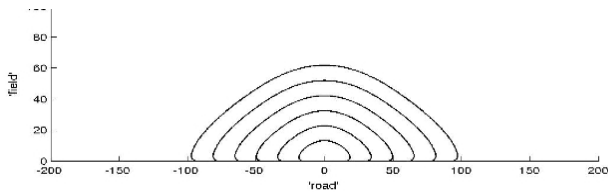
The system under study

$$
\begin{aligned}
v_t - \Delta v &= f(v) \quad (t > 0, \ (x, y) \in \mathbb{R}^2_+) \\
u_t - D\Delta u + \mu u - \nu v(t, x, 0) &= 0 \quad (t > 0, \ x \in \mathbb{R})
\end{aligned}
$$

$$
-v_y = \mu u - \nu v \quad (t > 0, \ x \in \mathbb{R})
$$

The parameters

- $f(v) = v - v^2$
- Initial datum: $u(0, x) = \mathbf{1}_{[-1,1]}(x)$, $v(0, x, y) \equiv 0$.
- Various sizes for D: $D = 10, 50, 100, 500, 1000$.

The general picture

Top: level set 0.5 of v at times 10, 20, 30, 40. **Bottom:** $v(40, x, y)$.

A closer look

- 0.5 level set of v at $t = 10, 20, 30, 40$.
- $D = 10, 50, 100, 500, 1000$.