University of Durham

Department of Mathematical Sciences

Mathematics Workbook

Preparing to come to Durham

Instructions

Read the preamble below and then attempt the questions in this workbook. Please complete the questionnaire at the back. If you are doing Mathematics only, please hand in the questionnaire at the inaugaural (Core A) lecture or to any of your Maths lecturers. If you are taking the Foundations of Physics module (e.g. if you are doing Natural Sciences, Physics or Joint Maths/Physics) please hand the questionnaire to your tutor at the earliest opportunity. The questions are graded as follows;

- ♡ Questions from areas that should be familiar from the core A-level. Good knowledge of these topics will be assumed.
- ♣ More difficult questions from these areas.
- ◊ Questions from areas outside the core A-level that will have been seen by many. These topics will be covered in the lectures.
- ♠ More difficult questions from these areas.

Why a workbook?

It is important to realise before starting your degree that Universities are not centres of teaching; they are centres of learning. In this respect they differ from Schools, where your learning has probably been directed fairly closely by teaching staff. At Durham University we will expose you to lecture courses that provide a framework *on which to base your own study.* You will be expected to take control of your own degree, direct your own learning and above all be aware of your own strengths and weaknesses. The students who flourish are those who can independently identify and correct gaps in their knowledge; this requires a degree of introspection and reflection that may be new to you.

You will have to adapt to this new environment during the first term of study. How well you do so will to a great extent determine the outcome of your degree. (Those who believe that they can coast during the first terms and catch up later invariably do badly.) That process begins with this workbook, where you can identify and deal with any areas of weakness in order to lay down firm foundations for a successful period of study at Durham. Mathematics is above all other disciplines one in which advanced knowledge builds upon more basic knowledge. It is essential therefore that before coming to Durham you make sure that you are familiar with the basic skills. The workbook presents a set of material which it would be useful to know before starting your degree to help you do that. It is not *necessary* to know *all* this material, but it would make your first term more comfortable if you did know it. Rather than presenting a list of topics, the workbook contains questions on each topic that should be straightforward; by tackling these questions, you will see how much knowledge is expected.

In order to make it less daunting the questions are graded as above. We estimate that completing all the straightforward core questions (those marked with a ♡) will probably take you 6 hours or more. These are areas with which you should be very familiar; the better you know them the more comfortable your adjustment to the degree program will be. If you find that some of the core material is unfamiliar, you should take the time now to look it up. You may wish to look up some of the non-core material too. Although this will be covered in lectures it will give you a head start. All the material here is covered in the standard A-level texts. If a particular core area is really unfamiliar to you, it would be worth doing more exercises from a text book to supplement those given here. You may also like to ask your current director of studies for help while they are still available to you. If you have difficulties with some questions, don't worry; you will have oppportunities to cover the material when you get to Durham.

Most of you will be very familiar with some of the material covered here, especially if you have done Further Maths. However it is still worth sketching out a solution to these questions even if you do not go through them in detail: it will be good revision and anyway you never really know that you have understood a problem until you do it. The answers are given at the end of the workbook. Note that none of the questions requires the use of a calculator.

There is a secondary more specific reason for providing a workbook. It is important for us that we keep a track of what our incoming students have done at A-level. This has become increasingly difficult in recent years as the A-level syllabus has broadened significantly. Half of any A-level syllabus are the 'common core' modules (e.g. P1 P2 P3 in the Edexcel board), but the material chosen for the other half varies between examining boards and according to the choice of modules. The information we gather is passed on to our first year lecturers so that they can tailor their courses accordingly in order to provide as smooth a transition as possible from secondary education. Thankyou for helping us do this.

Please e-mail comments or corrections to S.A.Abel@durham.ac.uk

24th June 2004

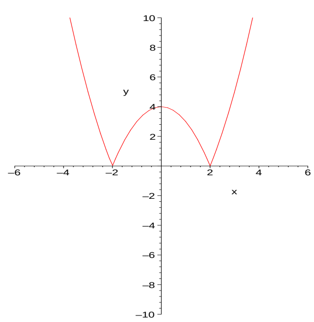
Functions of a real variable

You should be familiar with basic functions of a real variable. You should also be familar with the definitions of domain and range and the notation (that for a function f of a real variable x) that $f : \mathbb{R} \to \mathbb{R} : x \to f(x)$.

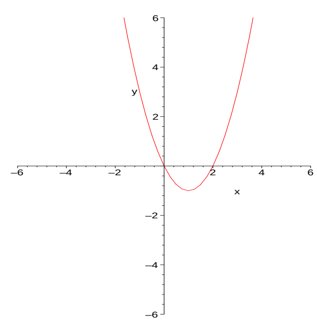
1. Match the following functions to the graphs below ♡

(a) $f_1(x) = \ln x$	(g) $f_7(x) = \cos 2x + 1$
(b) $f_2(x) = \arcsin x$	(h) $f_8(x) = x^2 - 2x$
(c) $f_3(x) = 1/x$	(i) $f_9(x) = x^3$
(d) $f_4(x) = \|3 - x\|$	(j) $f_{10}(x) = e^{-x^2}$
(e) $f_5(x) = x - \|x\|$	(k) $f_{11}(x) = \|4 - x^2\|$
(f) $f_6(x) = \sin(x + \pi)$	(l) $f_{12}(x) = \tan x$

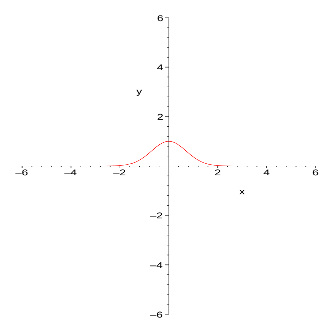
Note You will be expected to be able to draw the graphs of elementary functions, and to perform basic manipulations of graphs (horizontal and vertical translation, symmetry w.r.t. the y-axis, the x-axis and the origin, dilatations and contractions).



graph 1



graph 2



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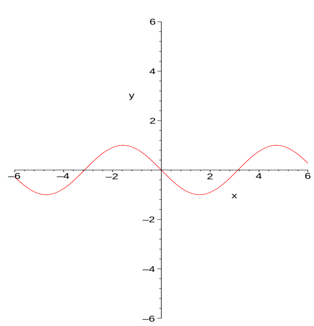

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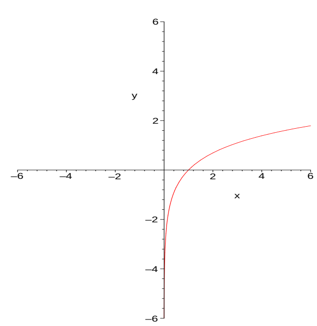
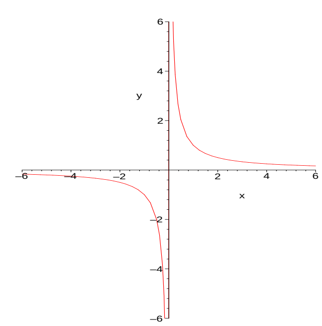


graph 5



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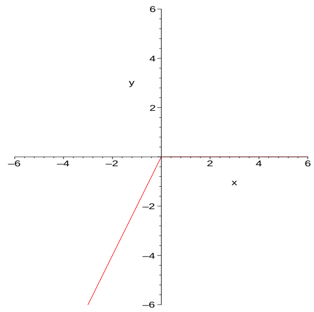
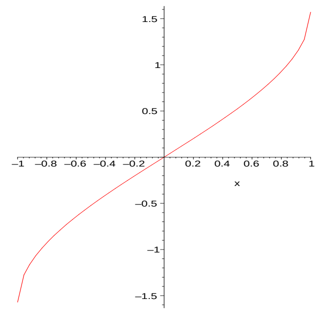
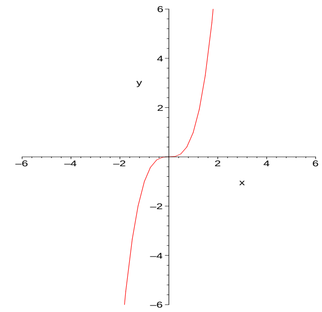


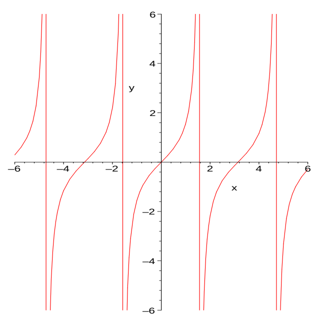
graph 7



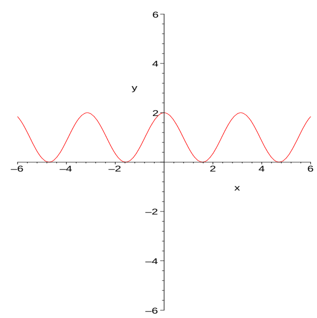
graph 8



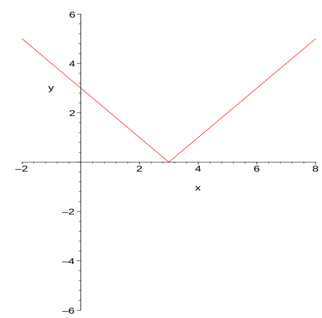
graph 9



graph 10



graph 11



graph 12

Algebra

Although computers and even calculators are very good at algebra, all mathematicians agree that it is important to be able to do routine algebra quickly and accurately. You should be able to state elementary series expansions including binomial, sine and cosine, and ln series.

2 Factorization ♡

Factorize the following polynomials:

(i) $x^2 - 3x + 2$;

(ii) $3x^3 - 3x^2 - 6x$;

(iii) $x^2 - x - 1$;

(iv) $x^3 - 1$;

(v) $x^4 - 3x^3 - 3x^2 + 11x - 6$.

Notes In part (iii) you will need the quadratic formula to find the factors; part (iv) has one linear and one quadratic factor; for part (v) you can use the factor theorem.

3 More factorization ♡

Find the values of x for which $x^3 < 2x^2 + 3x$.

4 Partial fractions ♡

Express the following in partial fractions:

(i) $\dfrac{2}{(x+1)(x-1)}$;

(ii) $\dfrac{1}{x^3+1}$;

(iii) $\dfrac{4x+1}{(x+1)^2(x-2)}$;

(iv) $\dfrac{x^2-7}{(x-2)(x+1)}$.

Note It is best for these purposes not to use the 'cover-up rule'; there are at least two other ways which involve elementary mathematics, whereas the cover-up rule works for more sophisticated reasons and to most users is simply a recipe (which does not always work).

5 Completing the square ♡

Find the smallest value (for real x and y) of:

(i) $x^2 - 2x + 6$;

(ii) $x^4 + 2x^2 + y^4 - 2y^2 + 3$;

(iii) $\sin^2 x + 4\sin x$.

Note Of course, you could find the smallest value by calculus, but expressing the function as a perfect square plus a remainder term is a surprisingly useful technique – for example, when integrating a function with a quadratic denominator.

6 Exponentials and logs ♢

(i) The exponential function e^x can be defined by the series expansion

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots.$$

Use this definition to show that $\dfrac{\mathrm{d}e^x}{\mathrm{d}x} = e^x$.

(ii) The natural log function $\ln t$ can be defined (for $t > 0$) as the inverse of the exponential function, so that $\ln e^x = x$. Set $t = e^x$ and use the relationship $\dfrac{\mathrm{d}x}{\mathrm{d}t} = 1\Big/\dfrac{\mathrm{d}t}{\mathrm{d}x}$ to show that

$$\frac{\mathrm{d}\ln t}{\mathrm{d}t} = \frac{1}{t}.$$

(iii) Assuming that the exponential function has the property $e^s e^t = e^{s+t}$, prove that $\ln(xy) = \ln x + \ln y$.

(iv) The definition of a^x for any a is $e^{x \ln a}$. Prove that $a^x a^y = a^{x+y}$ and $a^x b^x = (ab)^x$.

Note If you have not thought of defining a^x in this way, it is worth considering how else you could give it a meaning when x is not an integer.

7 Binomial expansions ♡

(i) Find the coefficient of x^k (for $0 \leqslant k \leqslant 10$) in the binomial expansion of $(2 + 3x)^{10}$.

(ii) Use the binomial theorem to find the expansion in powers of x up to x^4 of $(1 + x + x^2)^6$, by writing it in the form $\big(1 + (x + x^2)\big)^6$.

(iii) Use binomial expansions to find the expansion in powers of x up to x^4 of $(1 - x^3)^6(1 - x)^{-6}$.

(iv) Find the first four terms in the binomial expansion of $(2 + x)^{\frac{1}{2}}$.

8 Series expansions of elementary functions ♠

Using only the series expansions $\sin x = x - x^3/3! + x^5/5! + \cdots$, $\cos x = 1 - x^2/2! + x^4/4! + \cdots$, $\mathrm{e}^x = 1 + x + x^2/2! + x^3/3! + \cdots$ and $\ln(1 + x) = x - x^2/2 + x^3/3 + \cdots$, find the series expansions of the following functions:

(i) $\tan x$ (up to the x^5 term);

(ii) $\sin x \cos x$ (up to the x^5 term);

(iii) $\dfrac{e^x + e^{-x}}{2}$ (up to the x^5 term);

(iv) $\ln(e^x)$ (up to the x^3 term);

(v) $\dfrac{1 - \cos^2 x}{x^2}$ (up to the x^2 term).

Notes Do part (ii) without using a trig. formula, and compare your answer with the expansion for $\sin(2x)$. The function in part (iii) is $\cosh x$, of which more later. The interesting thing about the function in part (v) is that the series shows it is ‘well-behaved’ at $x = 0$, despite appearances.

9 Proof by induction ♡

Prove by induction that the following results are valid.

(i) $$a + ar + ar^2 + \cdots + ar^{n-1} = a\left(\frac{1-r^n}{1-r}\right).$$

(ii) $$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2.$$

Note Most students should have met mathematical induction ; if you haven't, you will probably want to try out this straightforward but important method of proof.

10 Arithmetic and geometric progressions ♢

(i) Find the sum of all the odd integers from 11 to 99.

(ii) Evaluate $6 + 3 + \frac{3}{2} + \frac{3}{4} + \cdots$.

(iii) Find $\sin\theta + 2\sin^3\theta + 4\sin^5\theta + \cdots$. (What ranges of values of θ are allowed?).

(iv) Estimate roughly the approximate number of times a piece of paper has to be torn in half, placing the results of each tearing in a stack and then doing the next tearing, for the stack of paper to reach the moon.

Note The dots in (ii) and (iv) indicate that the series has an infinite number of terms. For part (iii), recall that the expansion $a(1 + r + r^2 + \cdots)$ only converges if $-1 < r < 1$. You may find the approximation $10^3 = 2^{10}$ useful for part (iv). The distance from the Earth to the Moon is about 4×10^5 km.

Trigonometry

It is not necessary to learn all the various trigonometrical formulae; but you should certainly know what is available. The double-angle formulae (such as $\tan 2x = 2\tan x/(1-\tan^2 x)$) are worth learning, as are the basic formulae for $\sin(A\pm B)$, $\cos(A\pm B)$ and $\tan(A\pm B)$, the Pythagoras-type identities (such as $\sec^2 x = 1+\tan^2 x$) and a few special values (such as $\sin\pi/4 = 1/\sqrt{2}$) that can be deduced from right-angled triangles with sides $(1,1,\sqrt{2})$ or $(1,\sqrt{3},2)$.

11 Basic identities ♡

Starting from the identity $\sin(A+B) = \sin A\cos B + \cos A\sin B$, use the basic properties of the trig. functions (such as $\sin(-A) = -\sin A$) to prove the following:

(i) $\sin(A-B) = \sin A\cos B - \cos A\sin B$;

(ii) $\cos(A+B) = \cos A\cos B - \sin A\sin B$;

(iii) $\tan(A+B) = \dfrac{\tan A + \tan B}{1-\tan A\tan B}$;

(iv) $\sin C + \sin D = 2\sin\frac{1}{2}(C+D)\cos\frac{1}{2}(C-D)$;

(v) $\tan^{-1} a + \tan^{-1} b = \tan^{-1}\left(\dfrac{a+b}{1-ab}\right)$.

Notes For part (ii), recall that $\cos A = \sin(\pi/2 - A)$. You can use part (iii) to help with part (v).

12 Trig. equations ♡

(i) Solve the following equations:

(a) $\sin(x+\frac{\pi}{6}) + \sin(x-\frac{\pi}{6}) = \dfrac{\sqrt{3}}{2}$; (b) $\cos x + \cos 2x + \cos 3x = 0$.

(ii) Write down the value of $\cot(\pi/6)$ and use a double angle formula to show that $\cot(\pi/12)$ satisfies the equation $c^2 - 2\sqrt{3}c - 1 = 0$. Deduce that $\cot(\pi/12) = 2+\sqrt{3}$.

13 Trig. identities using Pythagoras ♣

Prove the following identities:

(i) $\tan\theta + \cot\theta = \sec\theta\,\text{cosec}\,\theta$; (ii) $(\cot\theta + \text{cosec}\,\theta)^2 = \dfrac{1+\cos\theta}{1-\cos\theta}$;

(iii) $\cos\theta = \dfrac{1-t^2}{1+t^2}$; $\sin\theta = \dfrac{2t}{1+t^2}$; $\tan\theta = \dfrac{2t}{1-t^2}$, where $t = \tan\frac{1}{2}\theta$.

Complex numbers

No prior knowledge of complex numbers is assumed in our first year course because some A-level syllabuses do not cover complex numbers. Nevertheless, the material is lectured quite fast, so if you have not met complex numbers before, working through the following examples with the help of a textbook would make the first few weeks of the course more comfortable. A complex number z can be written as $x + iy$, where x is the real part, y is the imaginary part and $i^2 = -1$. The modulus of z (written $|z|$ or r) is $\sqrt{(x^2 + y^2)}$ and the argument (written $\arg z$ or θ) is defined by $x = r\cos\theta$, $y = r\sin\theta$ and $-\pi < \theta \leqslant \pi$. The complex conjugate of z (written z^*) is $x - iy$. The inverse, z^{-1}, of z is the complex number that satisfies $z^{-1}z = 1$ (for $z \neq 0$).

14 Algebra of complex numbers ♢

Use the definitions above with $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ to show:

(i) $z_1 z_2 = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + y_1 x_2)$ (ii) $|z|^2 = zz^*$;

(iii) $z^{-1} = \dfrac{z^*}{|z|^2}$; (iv) $|z_1 z_2| = |z_1||z_2|$;

(v) $\arg(z_1 z_2) = \arg z_1 + \arg z_2$ (assume that $0 < \arg z_1 \leqslant \pi/4$ and $0 < \arg z_2 \leqslant \pi/4$).

Give a sketch of the x–y plane (called also the complex plane or the Argand diagram) showing the points representing the complex numbers $z_1 = 1 - i$, $z_2 = -\sqrt{3} + i$. Verify results (iii), (iv) and (v) for these numbers.

15 De Moivre's theorem ♠

(i) Show by means of series expansions that

$$\cos\theta + i\sin\theta = e^{i\theta}$$

and deduce that $\cos\theta - i\sin\theta = e^{-i\theta}$ and that $z = re^{i\theta}$.
Deduce also that $\cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$ and $\sin\theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$.

(ii) Use the above result to show that $re^{i\theta} = 1$ (for real θ and $r > 0$) if and only if $r = 1$ and $\theta = 2n\pi$ for some integer n.

(iii) Use part (ii) to find the three distinct roots of the equation $z^3 = 1$. Draw them on the complex plane and convert them from modulus-argument form to real–imaginary form.

Hyperbolic functions

Prior knowledge of hyperbolic functions is not assumed for our mathematics course, but it is worth getting to know the definitions and basic properties, which are given below. Hyperbolic functions are very similar to trigonometric functions, and many of their properties are direct analogues of the properties of trigonometric functions. The definitions are

$$\sinh x = \frac{e^x - e^{-x}}{2}, \qquad \cosh x = \frac{e^x + e^{-x}}{2}, \qquad \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

and $\operatorname{cosech} x = (\sinh x)^{-1}$, $\operatorname{sech} x = (\cosh x)^{-1}$, $\coth x = (\tanh x)^{-1}$.

16 Basic properties ♢

Give a rough sketch of the graphs of the six hyperbolic functions. Show from the above definitions that

(i) $\cosh^2 x - \sinh^2 x = 1\,;$

(ii) $\dfrac{\mathrm{d}(\sinh x)}{\mathrm{d}x} = \cosh x\,; \qquad \dfrac{\mathrm{d}(\cosh x)}{\mathrm{d}x} = \sinh x\,;$

(iii) $\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y\,;$

(iv) $\cosh(ix) = \cos x, \qquad \sinh(ix) = i \sin x, \qquad \cos(ix) = \cosh(x), \qquad \sin(ix) = i \sinh x.$

Notes You can use (iv) to prove (i) and (iii) using the corresponding trig. identities. In fact, (iv) is behind the rule which says that any formula involving trig. functions becomes the corresponding formula involving hyperbolic functions if you change the sign of every product of two odd functions (such as $\sin x$ or $\tan x$).

17 Further properties ♠

Use the definitions, and the results of the previous question, to show that

(i) $\operatorname{sech}^2 x = 1 - \tanh^2 x$

(ii) $\dfrac{\mathrm{d}(\tanh x)}{\mathrm{d}x} = \operatorname{sech}^2 x$

(iii) $\dfrac{\mathrm{d}^2(\sinh x)}{\mathrm{d}x^2} = \sinh x$

(iv) $\sinh^{-1} x = \ln\big(x + \sqrt{(x^2+1)}\big).$

Note For part (iv), note that $\sinh^{-1} x$ is the inverse function, not the reciprocal.

Differentiation

Differentiation of standard functions, products, quotients, implicit function expressions, and functions of a function (using the chain rule) should be routine.

18 Direct differentiation ♡

Differentiate $y(x)$ with respect to x in the following cases:

(i)	$y = (5x+2)^4$;	(ii)	$y = \sin(3x+2)$;
(iii)	$y = e^{(4x-1)}$;	(iv)	$y = 5\cos(2x+3)$;
(v)	$y = \cos^3 x$;	(vi)	$y = \log_e(4x-5)$.

19 More direct differentiation ♣

Differentiate $y(x)$ with respect to x in the following cases:

(i)	$y = \ln\left(x + \sqrt{(1+x^2)}\right)$;	(ii)	$y = a^x$;
(iii)	$y = x^x$;	(iv)	$y = \sin^{-1} \dfrac{x}{\sqrt{1+x^2}}$.

Notes Simplify your answer to part (i). For (ii), see the definition in question 6(iv). One way to do (iv) is to take the sine on both sides and use the chain rule. Can you see why the answer to (iv) is surprisingly simple?

20 Parametric differentiation ♠

Show that if $x = a\cos\theta$, $y = b\sin\theta$ then $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} < 0$ for $y > 0$.

21 More Chain and Product rule ♣

Using the chain and product rules etc, find the derivatives of the following (where a is a positive constant)

(i) $y = \sin(x^2)$ (ii) $y = \log_e(x^a + x^{-a})$

22 Stationary points ♡

Find the stationary points of the function

$$\mathrm{f}(x) = \frac{x}{x^2 + a^2},$$

where $a > 0$, classifying them as either maximum or minimum. Sketch the curve (without using a calculator).

Integration

You need to be able to recognise standard integrals (without having to leaf through a formula book) and evaluate them (referring to your formula book, if necessary). You need to be familiar with the techniques of integration by parts and by substitution.

23 Indefinite integrals ♡

Calculate the following integrals:

(i) $\displaystyle\int x^4 \, dx$; (ii) $\displaystyle\int 3\sin x \, dx$;

(iii) $\displaystyle\int 4e^x \, dx$; (iv) $\displaystyle\int 6 \, dx$;

(v) $\displaystyle\int 3x^{\frac{1}{2}} \, dx$; (vi) $\displaystyle\int 5^x \, dx$;

(vii) $\displaystyle\int x^{-4} \, dx$; (viii) $\displaystyle\int \frac{4}{x} \, dx$;

(ix) $\displaystyle\int 3\cos x \, dx$; (x) $\displaystyle\int 2\sec^2 x \, dx$.

24 More indefinite integrals ♣

Calculate the following integrals:

(i) $\displaystyle\int a^x \, dx$; (ii) $\displaystyle\int \frac{1}{x^2 - 2x + 6} \, dx$;

(iii) $\displaystyle\int e^{ax}\cos(bx) \, dx$; (iv) $\displaystyle\int e^{ax}e^{ibx} \, dx$;

(v) $\displaystyle\int \frac{1}{1 - x^3} \, dx$; (vi) $\displaystyle\int \operatorname{cosec} x \, dx$;

(vii) $\displaystyle\int \sec x \, dx$; (viii) $\displaystyle\int \frac{1}{\sqrt{(c^2 + m^2y^2)}} \, dy$;

(ix) $\displaystyle\int \tan^{-1} x \, dx$; (x) $\displaystyle\int x^3 e^{x^2} \, dx$.

Notes The difficulty of these varies. For part (i), see question 6(iv). For part (ii), see question 5(i). Note that you can obtain (iii) from (iv) by taking the real part. Use partial fractions for part (v). For (vi) and (vii), use the substitution $t = \tan(x/2)$ rather than the trick of multiplying top and bottom by e.g. $\sec x + \tan x$. Try also deriving (vii) from (vi) by means of the substitution $y = \pi/2 - x$. For (viii), you can substitute $my = c\sinh\theta$ if you are familar with hyperbolic functions. Use integration by parts for (ix) and (x).

25 Definite integrals ♣

Calculate the following integrals:

(i) $\displaystyle\int_0^\infty xe^{-x} \, dx$; (ii) $\displaystyle\int_0^{\pi/2} \sin 3\theta \cos\theta \, dx$;

(iii) $\displaystyle\int_0^1 \frac{x^2 + 1}{x^3 + 3x + 2} \, d\theta$; (iv) $\displaystyle\int_0^{\pi/2} \frac{1}{3 + 5\cos\theta} \, d\theta$;

Notes The difficulty of these varies. For part (iv) use the substitution $t = \tan(\theta/2)$.

Answers

1. $f_1(x)$: graph 5; $f_2(x)$: graph 8; $f_3(x)$: graph 6; $f_4(x)$: graph 12;
$f_5(x)$: graph 7; $f_6(x)$: graph 4; $f_7(x)$: graph 11; $f_8(x)$: graph 2;
$f_9(x)$: graph 9; $f_10(x)$: graph 3; $f_11(x)$: graph 1; $f_12(x)$: graph 10

2. (i) $(x-2)(x-1)$ (ii) $3x(x-2)(x+1)$
(iii) $(x-\frac{1}{2}(1-\sqrt{5}))(x-\frac{1}{2}(1+\sqrt{5}))$ (iv) $(x-1)(x^2+x+1)$
(v) $(x-3)(x-1)^2(x+2)$.

3. $x<-1$ or $0<x<3$.

4. (i) $\dfrac{1}{x-1}-\dfrac{1}{x+1}$ (ii) $\dfrac{1}{3}\left(\dfrac{1}{x+1}-\dfrac{x-2}{x^2-x+1}\right)$
(iii) $\dfrac{1}{(x+1)^2}-\dfrac{1}{x+1}+\dfrac{1}{x-2}$ (iv) $1-\dfrac{1}{x-2}+\dfrac{2}{x+1}$.

5. (i) 5; (ii) 2; (iii) -3 (smallest when $\sin x=-1$).

7. (i) $\dfrac{3^k 2^{10-k} 10!}{k!(10-k)!}$ (ii) $1+6x+21x^2+50x^3+90x^4$
(iii) Same as (ii) (iv) $\sqrt{2}\left(1+\frac{1}{4}x-\frac{1}{32}x^2+\frac{1}{128}x^3\right)$.

8. (i) $x+\frac{1}{3}x^3+\frac{2}{15}x^5$ (ii) $x-\frac{2}{3}x^3+\frac{2}{15}x^5$
(iii) $1+\frac{1}{2}x^2+\frac{1}{24}x^4$ (iv) x
(v) $1-\frac{1}{3}x^2$.

10. (i) 2475 (ii) 12
(iii) $\dfrac{\sin\theta}{\cos 2\theta}$ $(-\pi/4+n\pi<\theta<\pi/4+n\pi)$ (iv) about 42 times.

12. (ia) $n\pi+(-1)^n\pi/6$ (ib) $n\pi/2+\pi/4,\ 2n\pi\pm 2\pi/3$.

14. $|z_1|=\sqrt{2}$, $\arg z_1=-\pi/4$, $|z_2|=2$, $\arg z_2=5\pi/6$.
15. (iii) 1, $e^{2i\pi/3}$, $e^{-2i\pi/3}$ or 1, $(-1\pm i\sqrt{3})/2$.

18. (i) $20(5x+2)^3$ (ii) $3\cos(3x+2)$
(iii) $4e^{(4x-1)}$ (iv) $-10\sin(2x+3)$.
(v) $-3\sin x\,\cos^2 x$ (vi) $\frac{4}{4x-5}y$.

19. (i) $\dfrac{1}{\sqrt{x^2+1}}$ (ii) $a^x\log_e a$
(iii) $x^x(1+\ln x)$ (iv) $\dfrac{1}{1+x^2}$ $\left(\sin^{-1}\dfrac{x}{\sqrt{x^2+1}}=\tan^{-1}x\right)$.

21. (i) $2x\cos(x^2)$, (ii) $\frac{a(x^{a-1}-x^{-a-1})}{x^a+x^{-a}}$

22. Maximum at $\left(a, \dfrac{1}{2a}\right)$, minimum at $\left(-a, -\dfrac{1}{2a}\right)$.

23. (i) $\frac{x^5}{5} + C$ (ii) $-3\cos x + C$

(iii) $4e^x + C$ (iv) $6x + C$

(v) $2x^{\frac{3}{2}} + C$ (vi) $\frac{5^x}{\log_e x} + C$

(vii) $-\frac{x^3}{3} + C$ (viii) $4\log_e + C$

(ix) $3\sin x + C$ (x) $2\tan x + C$

24. (i) $\dfrac{1}{\ln a} a^x + C$ (ii) $\dfrac{1}{\sqrt{5}} \tan^{-1} \dfrac{x-1}{\sqrt{5}} + C$

(iii) $\dfrac{1}{a^2+b^2}(a\cos bx + b\sin bx)e^{ax} + C$ (iv) $\dfrac{1}{a+ib} e^{(a+ib)x} + C$

(v) $\dfrac{1}{6} \ln \dfrac{x^2+x+1}{(x-1)^2} + \dfrac{1}{\sqrt{3}} \tan^{-1}\left(\dfrac{2x+1}{\sqrt{3}}\right) + C$

(vi) $\ln\tan(x/2) + C$ (vii) $\ln\left(\dfrac{1+\tan(x/2)}{1-\tan(x/2)}\right) + C$

(viii) $m^{-1}\left(\ln(my) + \sqrt{m^2y^2 + c^2}\right) + C$ or $m^{-1}\sinh^{-1}(my/c) + C$

(ix) $x\tan^{-1} x - \frac{1}{2}\ln(1+x^2) + C$ (x) $\frac{1}{2}(x^2-1)e^{x^2} + C$

25. (i) 1 (ii) $\frac{1}{2}$ (iii) $\frac{1}{3}\log_e 3$ (iv) $\frac{1}{4}\log_e 3$

QUESTIONNAIRE

Please return this questionnaire at the induction lecture or to any of your lecturers.

Name ....

Examination board and modules taken ....

What is your Degree Course? ....

Has this workbook been useful? Please comment ....

Tick one box in each table for each question

Table A		
Material covered at school		
	Yes	No
Qn. 1		
Qn. 2		
Qn. 3		
Qn. 4		
Qn. 5		
Qn. 6		
Qn. 7		
Qn. 8		
Qn. 9		
Qn. 10		
Qn. 11		
Qn. 12		
Qn. 13		
Qn. 14		
Qn. 15		
Qn. 16		
Qn. 17		
Qn. 18		
Qn. 19		
Qn. 20		
Qn. 21		
Qn. 22		
Qn. 23		
Qn. 24		
Qn. 25		

Table B					
Difficulty of questions					
	Easy		$\longrightarrow$		Difficult
Qn. 1					
Qn. 2					
Qn. 3					
Qn. 4					
Qn. 5					
Qn. 6					
Qn. 7					
Qn. 8					
Qn. 9					
Qn. 10					
Qn. 11					
Qn. 12					
Qn. 13					
Qn. 14					
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Qn. 19					
Qn. 20					
Qn. 21					
Qn. 22					
Qn. 23					
Qn. 24					
Qn. 25					